Statistics 100B

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32/100

Name:

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**Problem 1 (25 points)**

Answer the following questions:

a. Let $X \sim \chi^2_p$. Let $g(x)$ be a function of $X$. Show that $E(g(x)) = pE\left(\frac{g(x)}{x}\right)$.

$\hookrightarrow \underbrace{\Gamma(\frac{p}{2}, 2)}_{\text{chi-squared}}$

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$$E(g(x)) = \int_{-\infty}^{\infty} \underbrace{g(x)f(x)\,dx}_{g(x)\cdot\frac{p}{x}}$$

$$pE\left(\frac{g(x)}{x}\right)$$

b. Show that if $Q \sim beta(\frac{1}{2}\alpha, \frac{1}{2}\beta)$ then $\frac{\beta Q}{\alpha(1-Q)} \sim F_{\alpha,\beta}$.

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$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$