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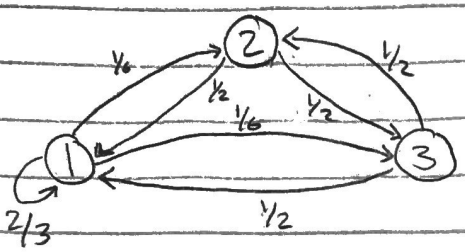
Midterm Stats 100A (page 1)

webpages 1, 2, 3



markov chain matrix

start \ end	1	2	3
1	2/3	1/6	1/6
2	1/2	0	1/2
3	1/2	1/2	0

X_t ~ webpage person is browsing at time t

assume $X_0 = 1$

1) $K =$

	1	2	3
1	2/3	1/6	1/6
2	1/2	0	1/2
3	1/2	1/2	0

2) $p_i^{(t)} = P(X_t = i)$. Let $p^{(t)} = (p_i^{(t)}, i = 1,2,3)$

$\star\ p^{(t)} = p^{(0)} K^t$, $p^{(0)} = \langle 1, 0, 0 \rangle$

$t=1$: $p^{(1)} = p^{(0)} K^1 = \langle 1,0,0 \rangle \begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \langle 2/3, 1/6, 1/6 \rangle$

$p^{(1)} = \langle 2/3, 1/6, 1/6 \rangle = \langle .\bar{6}, .1\bar{6}, .1\bar{6} \rangle$

$t=2$: $p^{(2)} = p^{(0)} K^2 = \langle 1,0,0 \rangle \begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}^2 = \langle 11/18, 7/36, 7/36 \rangle$

$p^{(2)} = \langle \frac{11}{18}, \frac{7}{36}, \frac{7}{36} \rangle = \langle .611, .194, .194 \rangle$

$t=3$: $p^{(3)} = p^{(0)} K^3 = \langle 1,0,0 \rangle \begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}^3 = \langle \frac{65}{108}, \frac{43}{216}, \frac{43}{216} \rangle$

$p^{(3)} = \langle \frac{65}{108}, \frac{43}{216}, \frac{43}{216} \rangle = \langle .602, .199, .199 \rangle$

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3) π_i ~ stationary distribution at webpage i

$\pi_j = \sum_{i=1}^{3} \pi_i k_{ij}$, $\pi = \pi K$

essentially the matrix multiplication will no longer affect anything once $p_i^{(t)} \to \pi_i$ that is why $\pi = \pi K$

$$\pi = \langle .6, .2, .2 \rangle = \langle \tfrac{3}{5}, \tfrac{1}{5}, \tfrac{1}{5} \rangle$$

for instance

$$\pi K = \langle \tfrac{3}{5}, \tfrac{1}{5}, \tfrac{1}{5} \rangle \begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \langle \tfrac{3}{5}, \tfrac{1}{5}, \tfrac{1}{5} \rangle$$

$p^{(3)}$ is very close to π

4) distribution with 1 million people at $t = 1, 2, 3$ assuming they all start at page 1

★ note: $1{,}000{,}000 = 10^6 = 1e6$

$t = 1$: $p^{(1)} = \langle 2/3, 1/6, 1/6 \rangle$

$(1e6) \langle 2/3, 1/6, 1/6 \rangle$

★★★ slight rounding error, for all times $t = 1, 2, 3$

webpage 1: 666,666

webpage 2: 166,667

webpage 3: 166,667

$t = 2$: $p^{(2)} = \langle \tfrac{11}{18}, \tfrac{7}{36}, \tfrac{7}{36} \rangle$

$(1e6) \langle \tfrac{11}{18}, \tfrac{7}{36}, \tfrac{7}{36} \rangle$

webpage 1: 611,112

webpage 2: 194,444

webpage 3: 194,444

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cont. 4) $t=3$: $p^{(3)} = \left\langle \frac{65}{108}, \frac{43}{216}, \frac{43}{216} \right\rangle$

$(1e6) \left\langle \frac{65}{108}, \frac{43}{216}, \frac{43}{216} \right\rangle$

webpage 1 : 601,852

webpage 2 : 199,074

webpage 3 : 199,074

☆ note : cannot have a percentage (decimal) of a person ∴ some rounding was taken into consideration to have proper sum of 1,000,000 people

webpage 1 is the most popular