

Statistics 100a Midterm

Rick Paik Schoenberg, 2/5/15, Young cs24, 11am-12:15pm.

PRINT YOUR NAME:

Victor Lai

SIGN YOUR NAME:

Victor Lai

Do not turn the page and start the exam until you are told to do so.

There are 15 multiple choice questions worth 6.7 points each.

Final answers are rounded to 3 significant digits.

No partial credit is given for multiple choice questions. Circle one answer only. If your answer is none of the above, then indicate the correct answer.

For problems 1-6, assume that you are guaranteed to be all-in next hand, no matter what. Also, assume that all of these questions relate specifically to the *next hand only*. In other words, when I ask what the probability is that you will be dealt a certain type of hand, I don't mean *eventually*; I mean on the very next hand.

7s means the seven of spades. Qh means the queen of hearts, etc.

$e \approx 2.718$.

1. What is the probability that you will be dealt a red king and a black queen?

- a) 0.208%
- b) 0.274%
- c) 0.302%
- d) 0.401%
- e) 0.533%
- f) None of the above.

$$\frac{\binom{2}{1} \cdot \binom{2}{1}}{\binom{52}{2}}$$

2. What is the probability that both of your hole cards will be face cards (jacks, queens or kings)?

- a) 4.98%
- b) 5.12%
- c) 5.49%
- d) 6.21%
- e) 7.18%
- f) None of the above.

$$\frac{\binom{12}{2}}{\binom{52}{2}}$$

3. What is the probability that you will flop exactly one pair? That is, what is the probability of you flopping 1 pair, and not 2 pairs or 3 of a kind or 4 of a kind?

- a) 8.12%
- b) 9.24%
- c) 9.53%
- d) 11.2%
- e) 13.0%
- f) None of the above.

$$13 \cdot \binom{4}{2} \cdot \binom{48}{3} / \binom{52}{5}$$

AA J K Q

4. Suppose you know that your opponent would go all in before the flop with 100% probability if she had AK or AQ, and she would go all in before the flop with 60% probability if she had a pocket pair. With other hands, there is 0% probability that she would go all in before the flop. Given that she has gone all in before the flop, and given no other information about her cards or her opponents' cards, what is the probability that she has a pocket pair?

- a) 56.2%
- b) 57.0%
- c) 58.1%
- d) 59.4%
- e) 60.2%
- f) 63.0%
- g) 65.9%
- h) 67.2%
- i) None of the above.

A = goes all in
 B = has AK or AQ
 C = has pocket pair
 D = has other hand

$$P(A|B) = 1$$

$$P(A|C) = 0.6$$

$$P(A|D) = 0$$

$$P(C) = \frac{P(A|C)}{P(A|C) + P(A|D)} = \frac{0.6}{0.6}$$

$$\binom{13}{2} \cdot \binom{4}{1} \cdot \binom{4}{1}$$

$$P(A|B) = \frac{\binom{13}{2} \cdot \binom{4}{1} \cdot \binom{4}{1}}{\binom{52}{2}}$$

$$P(C|A) = \frac{P(A|C) \cdot P(C)}{P(A|C) \cdot P(C) + P(A|B) \cdot P(B) + P(A|D) \cdot P(D)}$$

$$\binom{13}{2} \cdot \binom{4}{1} \cdot \binom{4}{1} \cdot \binom{4}{1} \cdot \binom{10}{1} \cdot \binom{4}{2} \quad \text{AK or AQ}$$

$$= \frac{0.6 \cdot (13 \cdot \binom{4}{2}) / \binom{52}{2}}{0.6 \cdot (13 \cdot \binom{4}{2}) / \binom{52}{2} + \frac{32}{27}}$$

$$\binom{13}{1} \cdot \binom{4}{2} \cdot \binom{10}{2}$$

$$\binom{4}{1} \cdot \binom{4}{1}$$

1 2 3

3 3 J Q K

5. What is the probability that you are dealt at least one ace?

- a) 14.9% b) 15.4% c) 16.2%
 d) 18.1% e) 19.9% f) 21.3%
 g) 22.0% h) 23.2% i) None of the above.

$$1 - \frac{48}{52} \cdot \frac{47}{51}$$

6. Given that you are dealt the J and Q of hearts, what is the probability that you will eventually make a straight flush or royal flush in hearts, when all 5 board cards are revealed?

- a) 0.122% b) 0.149% c) 0.172%
 d) 0.197% e) 1.12% f) None of the above.

$A = \text{dealt } JH \text{ } QH$

$B = \text{make straight flush of hearts}$

$$P(B|A) = \frac{11}{47 \cdot 46}$$

7. Suppose X is a Poisson random variable with mean $\lambda = 2.0$. What is $P(X > 1.5)$??

- a) 31.7% b) 42.5% c) 59.4%
 d) 65.0% e) 68.1% f) None of the above.

For the next two questions, suppose you will play Texas Holdem until you are dealt AK, and let $X =$ the number of hands you play. (So, for instance, if on the first hand you are dealt K8, and on the second hand you are dealt AK, then $X = 2$.)

$$\frac{4 \cdot 4}{\binom{52}{2}}$$

8. What is the expected value of X ?

- a) 17.0 b) 49.1 c) 82.9
 d) 102.3 e) 990 f) None of the above.

9. What is the standard deviation of X ?

- a) 16.5 b) 43.5 c) 82.4
 d) 101 e) 901 f) None of the above.

$A = \{K, Q, 10, 9, 8\}$

K, Q, J

Q

J

10

9

8

$7, 6, 5, 4, 3$

$6, 5, 4, 3, 2$

$5, 4, 3, 2, A$

For the next three problems, suppose the following:

X is Uniform(0,1).

Y is exponential with $\lambda = 2$, so $E(Y) = 1/2$.

X and Y are independent.

Z = X+Y.

$$E(Z) = E(X+Y) = E(X) + E(Y) = \frac{1}{2} + \frac{1}{2}$$

W = min{X,Y}.

W.P.C. $P(X > c) = P(Y > c)$
 $0.25 = (1 - (1 - e^{-2c}))$

10. What is $E(Z)$?

- a) 0.25.
- b) 0.5.
- c) 0.75.
- d) 1.0.
- e) 2.0.
- f) None of the above.

11. What is $P(W \leq 0.25)$?

- a) 12.1%.
- b) 27.1%.
- c) 29.7%.
- d) 34.2%.
- e) 54.5%.
- f) None of the above.

$P(\min\{X,Y\} \leq 0.25)$

$1 - P(X > 0.25) P(Y > 0.25)$
 $0.25 (1 - e^{-0.5})$

$P(Z \geq 0.25) = 1 - P(Z \leq 0.25)$
 $1 - P(X > 0.25) P(Y > 0.25)$
 $0.25 (1 - e^{-0.5})$

12. What is the moment generating function of Z?

- a) $\frac{\exp(t)}{t^2-1}$.
- b) $\frac{t-1}{\exp(t)-t^2}$.
- c) $\frac{2\exp(t)-2}{2t-t^2}$.
- d) $\frac{\exp(t)}{t-t^2}$.
- e) $\frac{\exp(t)-1}{2-t^2}$.
- f) None of the above.

$e^t = 1 + 2t + \dots$
 $(1-2t)e^t = 1 + 2t - 2t^2 - 2t^3 - \dots$
 $2(1-2t) = 2 - 4t$
 $\frac{(1-2t)e^t - 2(1-2t)}{2(1-2t)}$
 $\frac{e^t - 1}{2 - 4t}$
 $\frac{e^t - 1}{2(1-2t)}$

$E(e^{tZ}) = \frac{e^t - 1}{t}$

$E(e^{2tZ}) = \frac{0.5}{0.5 - t}$

$\frac{1}{2(\frac{1}{2} - t)}$

$\frac{e^t - 1}{t} = \frac{1}{1-2t}$

$\frac{e^t - 1}{t} = \frac{0.5}{0.5 - t}$

$(0.5 + 2t^2 + 4t + 2t^3 + \dots) = 0.5 + 2t^2 + \dots$
 $0.5 + 2t^2 = 0.5 + 2t^2$
 $\frac{0.5 + 2t^2 + 4t + 2t^3 + \dots}{0.5 - t}$

For the next two questions, suppose you play 50 hands of holdem per day. Let Y denote the number of times per day you get dealt 2 cards of the same suit.

$$\binom{4}{1} \binom{48}{2}$$

1 day 50 hands

$$\binom{4}{2}$$

13. You'd expect Y typically to be about

- a) 7.09. b) 8.11. c) 9.2.
 d) 11.8. e) 13.5. f) None of the above.

14. How much would Y typically be off from your answer to the question above? D

- a) 1.21. b) 1.51. c) 1.83.
 d) 2.51. e) 3.00. f) None of the above.

15. Suppose you are in a winner take all tournament where you have 7s 7d and your opponent has Ks Kc. You are heads up, and the pot size at this point is \$300. The flop comes Qh 3h 2c. Your opponent now goes all in, betting an additional \$30. You have more than \$30 left, so it is \$30 more for you to call. Assume you know your cards, your opponents cards, and the flop. Let p be your probability of winning the hand if you call. In order to maximize your expected number of chips, should you call?

- a) No, because $p = 8.38\%$ which is $< 30/330$.
 b) Yes, because $p = 8.38\%$ which is $> 30/360$.
 c) No, because $p = 8.52\%$ which is $< 30/330$.
 d) Yes, because $p = 8.52\%$ which is $> 30/360$.
 e) No, because $p = 8.93\%$ which is $> 30/330$.
 f) Yes, because $p = 8.93\%$ which is $> 30/360$.
 g) None of the above.

$$\frac{30}{330}$$

7s 7d Ks Kc

Qh 3h 2c

7 (over 2)

$$\binom{4}{2} \binom{45-1}{44-2}$$