4.3/7

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EXAM

UCLA ID: : 9 A.M. _ _ _ _ _

LAST NAM -- **(Put ID on your desk)**

FIRST NA

ENGLISH - --

(a) **WRITE AND MARK YOUR NAME AND ID ON THE SCANTRON.**

(b) **WRITE THE COLOR OF THE EXAM ON THE TOP OF THE SCANTRON, IN NUMBER 2 PENCIL.**

(c) **WRITE YOUR NAME ON ALL SIDES OF THE CHEAT SHEET, TOP RIGHT HAND CORNER.**

(d) **DO NOT DETACH ANY PAGES FROM THIS EXAM. EXAM MUST STAY STAPLED DURING THE WHOLE EXAM.**

(e) **ALL YOUR BELONGINGSMUST BE UNDER THE CHAIR.**

(f) **ONLY ID, NUMBER 2 PENCIL AND PEN, ERASER, SCIENTIFIC CALCULATOR, SCANTRONAND CHEAT SHEET ALLOWED IN THE EXAM.**

(g) **PUT DOWN THE TABLES ON YOUR RIGHT AND LEFT. ALL ITEMS MUST BE ON YOUR DESK.**

Other important Instructions given to you before the exam and posted in CCLE. Points lost for not following directions OR 0 POINTS IN THE EXAM, and further consequences.

- **Closed books, closed notes. Material covered is up to last day of lecture and TA session before the exam. B**
- **Students without a cheat sheet must let the professor know that do not have one before the exam and sit on the front row of the exam room.**
- **You must be silent in the exam room throughout the whole time that you are in the room, from the moment you enter until the you are outside the room.**
- **Only scientific calculator allowed for computations. You may not use your phone or any other electronic device as calculator. Graphics calculators are not allowed. No exceptions.**
- **Phones and other electronic devices must have been placed inside your backpack before you entered the room and not accessed again until you are out of the room. While in the classroom, they must be in your backpack and your backpack on the floor under the chair. If you do not have a backpack, you must put the items on the front desk. Iwatches and other devices are not allowed either. Phones or devices in pockets will lead to big loss of points in the exam. It is not worth the risk.**

- **Answer for multiple choice questions will be marked in scantron AND the exam. Work will not be read. Failure to mark your name, ID or some answers will result in point deduction from the exam grade. This is the second time you do a test. So no more grace period on this. Work for the "show work" problems will be graded.**

- **The scantron, together with the cheat sheet will be inserted inside the exam before you turn in the exam.**

- **Left handed students must sit in a seat for left-handed students. The professor will tell students where to sit. Please, let the professor know that you are left handed once seated and she will indicate where to move.**

- **ID must be ready to show BEFORE and at all times during the exam. NO ID, no exam.**

- **This midterm must show your individual work. Talking to others during the midterm, not adhering to the above, sharing information or breaking any other aspect of the student code of conduct at UCLA will not be tolerated and will be referred to the Dean of Students office. You can not exchange papers or information. All your things must be on the floor. You may not use the empty seats next to you to put things. Put the tables down. Honor code applies.**

- **Cheat sheet can have only formulas and definitions, no solved problems, no examples of any kind, no proofs, no numerical examples, no intermediate steps and no drawings or graphs of any kind.YOUR NAME MUST BE ON CHEAT SHEET AT ALL TIMES. Be ready to show your cheat sheet when the instructor requests it. The cheat sheet must be written all in English. Cheat sheets that do not comply will result in lower grade in the exam. You may have two sides of a 11 by 8 sheet If you do not have a cheat sheet, you must tell the professor at the beginning of the exam and sit on the front row of the room.**

- **The examples of distributions that you are allowed to have on the exam are attached to the exam. Those are the only examples you may have.**

- **You may not speak to each other in the exam room. Wait until you are out of the room.**

MULTIPLE CHOICE QUESTIONS. ONLY ONE ANSWER IS CORRECT. CHOICE MUST BE MARKED ON THE SCANTRON, AND ALSO HERE ON THE EXAM. ONLY THE SCANTRON WILL BE GRADED. NO MARKS ON SCANTRON OR MORE THAN ONE MARK WILL RESULT IN 0 POINTS FOR THE QUESTION NOT MARKED, EVEN IF IT IS MARKED ON THE EXAM. You may use the space near the question for scratch work, but scratch work will not be read.

Question 1. Let X be the cosine of the angle at which electrons are emitted in muon decay. X is a random variable with the following density function

$$f(x) = \frac{1+\alpha x}{2}, \qquad -1 < x < 1$$

The α is a parameter, a constant. Your results will depend on α. The cumulative distribution function of X is

(a) $F(x) = \frac{1}{4}X^2, \quad -1 < x < 1$

(b) $F(x) = \frac{1}{2} + \frac{1}{2}x + \alpha\left(\frac{x^2-1}{4}\right), \quad -1 < x < 1$

(c) $F(x) = \frac{1}{2}x + \frac{\alpha}{4}x^2, \quad -1 < x < 1$

(d) $F(x) = 1 - e^{-\alpha x}, \quad -1 < x < 1$

(e) $F(x) = \frac{1}{\alpha}, \quad -1 < x < 1$

$F(x) = \int_{-\infty}^{\infty} f(x)\,dx$
$= \int_{-1}^{x} \left(\frac{1}{2} + \frac{\alpha}{2}x\right) dx, \quad -1 < x < 1$
$= \left[\frac{1}{2}x + \frac{\alpha}{4}x^2\right]_{-1}^{x}$
$= \left[\left(\frac{1}{2}x + \frac{\alpha}{4}x^2\right) - \left(-\frac{1}{2} + \frac{\alpha}{4}\right)\right]$
$= \frac{1}{2} + \frac{1}{2}x + \alpha\left(\frac{x^2-1}{4}\right)$

Question 2. The wait time for service at a Starbucks, X, has the following moment generating function:

$$M_X(t) = (1-100t)^{-1} = \frac{1}{1-100t} = \frac{0.01}{0.01-t}$$

where X is measured in seconds. For the distribution represented by this moment generating function, if s and t are constants, we can expect that

(a) $P(X > s+t) = P(X > s) + P(X > t)$

(b) $P(X > s+t) = P(X > s)P(X > t)$

(c) $E(X^2) = Var(X) - (E(X))^2$

(d) $E(X) = P(X > s+t)$

(e) $Var(X) = 100$

Question 3. The Ph level, a measure of acidity, is important in studies of acid rain. For a certain Florida lake, baseline measurements of acidity are made so that any changes caused by acid rain can be noted. The pH for water samples from the lake is a random variable X, with probability density function (pdf)

$$f(x) = \frac{3}{8}(7-x)^2, \qquad 5 \le x \le 7$$

The probability that the Ph of a water sample from this lake will be less than 5.5, given that it is known to be less than 6 is

(a) 0.3393

(b) 0.5618

(c) 0.6607

(d) 0.0034

(e) 0.991

Question 4. The cumulative distribution function of a random variable is given by

$$F(x) = 1 - (1-p)^x, \qquad x = 1, 2, 3, \ldots\ldots\ldots$$

The density function of this random variable is

(a) The normal density function with expected value p and variance p(1-p)

(b) The binomial distribution with parameters n=1 and p.

(c) The hypergeometric distribution with k =p

(d) The negative binomial distribution with parameters r=k and p

(e) The geometric distribution with parameter p This one

Question 5. Let X be the length of human pregnancies from conception to birth. X varies according to a Normal distribution, with a mean $\mu = 266$ days and standard deviation $\sigma = 16$ days. The median length of pregnancies is

(a) 133 days

(b) 216 days

(c) 282 days

(d) 266 days

(e) 300 days

Question 6. The amount of time that a lightbulb works before burning itself out is exponentially distributed with expected value 10 hours. Suppose that 10 persons enter a room in which a lighbulb is burning. If each of this persons desire to work for five hours, then what is the probability that they will be able to complete their work without the bulb burning out?

(a) 0.6065

(b) 0.000008894

(c) 0.006734

(d) 0.3934

(e) 1

Question 7. You arrive at a bus stop at 10:00 AM, knowing that the bus will arrive at some time uniformly distributed between 10:00 AM and 10:30 AM. If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

(a) 2/3

(b) 1/4

(c) 2/4

(d) 1/3

(e) 3/4

Question 8. Suppose X is a random variable for which the p.d.f. is as follows:

$$f(x) = e^{-x}, \qquad x \geq 0$$

The third moment, i.e., the $E(X^3)$ for this random variable is

(a) 2.89

(b) 6

(c) 1

(d) 21.5

(e) -3.67102

Question 9. The weekly downtime Y(in hours) for a certain industrial machine has approximately a gamma distribution with $\alpha = 3$ and $\lambda = 2$. The loss L (in dollars) to the industrial operation as a result of the downtime is given by

$$L = 30Y + 2Y^2$$

What is the expected weekly loss due to downtime?

(a) 0.75

(b) 100

(c) 51

(d) 68

(e) 46

Question 10. Cynthia's Mail order Company provides free examination of its products for seven days. If not completely satisfied, a customer can return the product within that period and get a full refund. According to past records of the company, an average of 2 of every 10 products sold by this company are returned for a refund. Using the Poisson probability distribution formula, find the probability that exactly 6 of the 40 products sold by this company on a given day will be returned for a refund.

(a) 0.0003

(b) 0.1221

(c) 0.2

(d) 0.3456

(e) 0.51

Question 11. The cumulative distribution function of a random variable X is

$$F(x) = 1 - e^{(-x/\beta)^\alpha}, \qquad x \geq 0$$

where α, β are the parameters. The density function of X is

(a) $f(x) = \frac{1}{\sqrt{2\pi\beta^2}} e^{-\frac{1}{2\beta^2}(x-\alpha)^2}$

(b) $f(x) = \alpha e^{-\alpha x}$

(c) The binomial distribution

$f(x) = \frac{dF}{dx} = -e^{\left(-\frac{x}{\beta}\right)^\alpha} \cdot \frac{\alpha x^{\alpha-1}}{(-\beta)^\alpha} = (-1)^{\alpha+1} \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{\left(-\frac{x}{\beta}\right)^\alpha}$

(d) The Gamma density

(e) $f(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{(-\frac{x}{\beta})^\alpha}$

Question 12. The scores of students taking the SAT are normally distributed with mean 500 and standard deviation 100. Mary is at the 90th percentile. What was her score?

(a) 500

(b) 1.29

(c) 629

(d) 770

(e) 100

Question 13. A sugar refinery has three processing plants, all of which receive raw sugar in bulk. The amount of sugar that one plant can process in one day can be modeled as having an exponential distribution with expected value equal to 5 tons for each of the three plants.

If the three plants operate independently, find the probability that exactly two of the three plants will process more than 4 tons on a given day.

(a) 0.4493

(b) 0.3335

(c) 0.7121

(d) 0.2566

(e) 0.9157

Question 14. A salesperson for a large pharmaceutical company makes 3 calls per year on a drugstore, with the chance of a sale each time being 80%. Let X denote the total number of sales in a year. What is the probability of at least 2 sales?

(a) 0.008

(b) 1.7320

(c) 0.60

(d) 0.488

(e) 0.896

Question 15. A large stockpile of used pumps contains 20% that are currently unusable and need to be repaired. A repairman is sent to the stockpile with three repair kits. He selects pumps at random and tests them one at a time. If a pump works, he goes on to the next one. If a pump doesn‘t work, he uses one of his repair kits on it. Suppose that it takes 10 minutes to test whether a pump works, and 20 minutes to repair a pump that does not work. Find the expected value and standard deviation of the total time it takes the repairman to use up his three kits.

(a) $\mu = 450, \sigma = 42.43641$

(b) $\mu = 6000, \sigma = 210$

(c) $\mu = 180, \sigma = 20.784$

(d) $\mu = 1001, \sigma = 10$

(e) $\mu = 210, \sigma = 77.4596$

II. YOU MUST SHOW WORK IN THE NEXT PROBLEM FOR FULL CREDIT. THE WORK OF THIS PROBLEM WILL BE READ.

Problem 1. Suppose that X is a Gamma random variable with parameters α and λ. Define $Y = \frac{X}{n}$.

(a) Show how to find the Expected value μ_Y and the standard deviation σ_Y.

$\mu_Y = E\left(\frac{X}{n}\right)$ $\quad$ $\sigma_Y^2 = Var\left(\frac{X}{n}\right)$

$\mu_Y = \frac{1}{n}\mu_X$ $\quad$ $\sigma_Y^2 = \frac{1}{n^2}\sigma_X^2$

$\mu_Y = \frac{1}{n}\cdot\frac{\alpha}{\lambda}$ $\quad$ $\sigma_Y^2 = \frac{1}{n^2}\cdot\frac{\alpha}{\lambda^2}$

$\sigma_Y = \frac{1}{n}\cdot\frac{\sqrt{\alpha}}{\lambda}$

✓ ✓

(b) Show how to find the $E(Y^2)$

$E(Y^2) = E\left(\frac{X^2}{n^2}\right)$

$= \frac{1}{n^2}E(X^2)$

$= \frac{1}{n^2}(\mu_X^2 + \sigma_X^2)$

$= \frac{1}{n^2}\left(\left(\frac{\alpha}{\lambda}\right)^2 + \frac{\alpha}{\lambda^2}\right)$

$= \frac{1}{n^2}\left(\frac{\alpha^2+\alpha}{\lambda^2}\right)$

✓

Problem 2. The failure of a circuit board interrupts work by a computing system until a new board is delivered. Delivery time is uniformly distributed over the interval of from one to 5 days. The cost C of this failure and interruption consists of a fixed cost c_o for the new part and a cost that increases proportionally to X^2, so that $C = c_o + c_1X^2$.

(a) What is the probability that the delivery time is 2 or more days?

$P(X\geq 2) = f(5) - f(2)$ $= \frac{4}{4} - \frac{1}{4} = \frac{3}{4} = 0.75$

$= \frac{5}{5} - \frac{2}{5}$

$= \frac{3}{5} = \cancel{0}.600$

3

1.3

(b) Find the expected cost of a single failure, in terms of c_o and c_1.

$E(c_0+c_1X^2) = c_0+c_1E(X^2)$

$= c_0+c_1(\mu_X^2+\sigma_X^2)$ $+1$ $= c_0+c_1\left(\left(\frac{1+5}{2}\right)^2+\frac{(5-1)^2}{12}\right) = c_0+c_1\cdot\frac{31}{3}$

$= c_0+c_1\left(\left(\frac{0+5}{2}\right)^2\cancel{+\frac{(5-0)^2}{12}}\right)$ wrong a, b $= c_0+10.\overline{33}\,c_1$

$= c_0+c_1\cdot\frac{25}{3}$

NOW THAT YOU ARE DONE, MAKE SURE YOU:

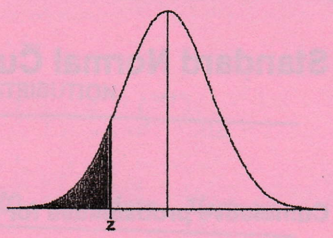
(a) **Have marked your name and ID in the scantron and indicated color of your exam.**

(b) **Have written your name and id on the first page of this exam and have signed with your signature.**

(c) **Have signed on the front page of the exam.**

Insert cheat sheet and scantron between the pages of the exam. Close the exam and wait until we collect it from you. Remain seated and in silence until every student has turned in the exam. Do not talk or use the phone until you are out of the room.

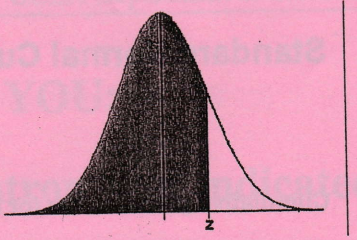
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

TABLE 7.1: DISCRETE PROBABILITY DISTRIBUTION

	Probability mass function, $p(x)$	Moment generating function, $M(t)$	Mean	Variance
Binomial with parameters n, p; $0 \leq p \leq 1$	$\binom{n}{x} p^x(1-p)^{n-x}$ $x = 0, 1, \ldots, n$	$(pe^t + 1 - p)^n$	np	$np(1-p)$
Poisson with parameter $\lambda > 0$	$e^{-\lambda}\frac{\lambda^x}{x!}$ $x = 0, 1, 2, \ldots$	$\exp\{\lambda(e^t - 1)\}$	λ	λ
Geometric with parameter $0 \leq p \leq 1$	$p(1-p)^{x-1}$ $x = 1, 2, \ldots$	$\frac{pe^t}{1-(1-p)e^t}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative binomial with parameters r, p; $0 \leq p \leq 1$	$\binom{n-1}{r-1} p^r(1-p)^{n-r}$ $n = r, r+1, \ldots$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$

Hypergeometric

TABLE 7.2: CONTINUOUS PROBABILITY DISTRIBUTION

	Probability mass function, $f(x)$	Moment generating function, $M(t)$	Mean	Variance
Uniform over (a, b)	$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential with parameter $\lambda > 0$	$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$	$\frac{\lambda}{\lambda - t}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Gamma with parameters $(s, \lambda), \lambda > 0$	$f(x) = \begin{cases} \frac{\lambda e^{-\lambda x}(\lambda x)^{s-1}}{\Gamma(s)} & x \geq 0 \\ 0 & x < 0 \end{cases}$	$\left(\frac{\lambda}{\lambda - t}\right)^s$	$\frac{s}{\lambda}$	$\frac{s}{\lambda^2}$
Normal with parameters (μ, σ^2)	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \quad -\infty < x < \infty$	$\exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$	μ	σ^2
Chisquare with n degrees of freedom	a gamma with parameters $\lambda = 1/2$ $\alpha = \frac{n}{2}$	$(1-2t)^{\frac{n}{2}}$	n	$2n$

STAT 100A MIDTERM 2 PINK KEY W17

exam SYSTEM II

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2	B (2)
3	C (3)
4	E (5)
5	D (4)
6	A (1), C (3) — A or C
7	D (4)
8	B (2)
9	C (3)
10	B (2)
11	E (5)
12	C (3)
13	B (2)
14	E (5)
15	E (5)

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Pink key

J. Sanchez
UCLA Department of Statistics

PINK KEY II. YOU MUST SHOW WORK IN THE NEXT PROBLEM FOR FULL CREDIT. THE WORK OF THIS PROBLEM WILL BE READ.

3 points **Problem 1.** Suppose that X is a Gamma random variable with parameters α and λ. Define $Y = \frac{X}{n}$.

(a) Show how to find the Expected value μ_Y and the standard deviation σ_Y.

$$X \sim Gamma(\alpha, \lambda) \Rightarrow E(x) = \frac{\alpha}{\lambda}, \quad Var(x) = \frac{\alpha}{\lambda^2}.$$

$$\mu_Y = E(Y) = E\left(\frac{1}{n}X\right) = \frac{1}{n}E(X) = \frac{1}{n} \cdot \frac{\alpha}{\lambda} = \frac{\alpha}{n\lambda} \, /\!/$$

$$\sigma_Y = \sqrt{V(Y)} = \sqrt{\frac{1}{n^2}\frac{\alpha}{\lambda^2}} = \frac{1}{n\cdot\lambda}\sqrt{\alpha}.$$

(b) Show how to find the $E(Y^2)$

$$E(Y^2) = Var(Y) + [E(Y)]^2.$$

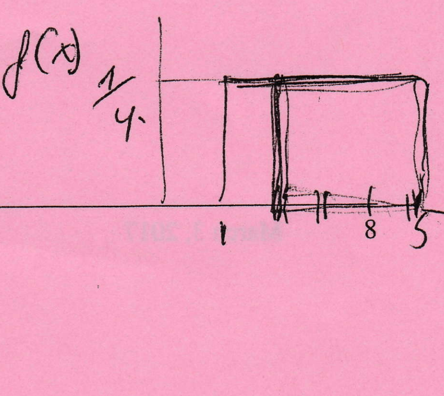
$$= \frac{\alpha}{n^2\lambda^2} + \left(\frac{\alpha}{n\lambda}\right)^2 = \frac{\alpha}{n^2\lambda^2} + \frac{\alpha^2}{n^2\lambda^2} = \frac{\alpha}{n^2\lambda^2}(1+\alpha)$$

4 pts **Problem 2.** The failure of a circuit board interrupts work by a computing system until a new board is delivered. Delivery time is uniformly distributed over the interval of from one to 5 days. The cost C of this failure and interruption consists of a fixed cost c_o for the new part and a cost that increases proportionally to X^2, so that $C = c_o + c_1X^2$.

(a) What is the probability that the delivery time is 2 or more days?

Delivery time $X \sim U(1,5)$.

$$f(x) = \frac{1}{4}, \quad 1 \le X \le 5$$



$$P(X>2) = \frac{3}{4}.$$

(b) Find the expected cost of a single failure, in terms of c_o and c_1.

$$E(C) = C_0 + C_1 E(X^2)$$

$$= C_0 + C_1 [Var(X) + [E(X)]^2]$$

$$= C_0 + C_1 \left[\frac{(5-1)^2}{12} + \left(\frac{5+1}{2}\right)^2 \right]$$

$$= C_0 + C_1 \left[\frac{16}{12} + \frac{36}{4} \right]$$

$$= C_0 + C_1 \left[\frac{124}{12} \right]$$

$$= C_0 + C_1 (10.33)$$