19.6 / 20

Stat 100A -Intro Probability
Midterm 2 (1) Answer sheet
Only this sheet will be graded.

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✓ **Problem 1.** People enter a gambling casino at a rate of 1 for every two minutes. What is the probability that no one enters between 12:00 and 12:05? Show work. $\lambda = 2.5$, Poisson X = # of people entering

$$P(X=0) = \frac{2.5^0 e^{-2.5}}{0!} = e^{-2.5} = \boxed{.08208}$$

Problem 2. With the recent emphasis on solar energy, solar radiation has been carefully monitored at various sites in Florida. Among typical July days in Tampa, 30 percent have total radiation of at most 5 calories, 60 percent have total radiation of at most 6 calories, and 100 percent have total radiation of at most 8 calories. A solar collector for a hot water system is to be run for 6 days. Find the probability that 3 days will produce no more than 5 calories each, 1 day will produce between 5 and 6 calories, and 2 days will produce between 6 and 8 calories. What assumptions must be true for your answer to be correct? Show work. ~~Assume~~

multinomial $\begin{cases} R_5 = .3 & R_6 = .6 & R_8 = 1 \\ X_5 = 3 & X_6 = 1 & X_8 = 2 \end{cases}$

Assume the solar collectors always produce the max calories of radiation

-.4 $\binom{6}{3,1,2}(.3)^3(.6)^1(1)^2 = \boxed{.972}$ ✗ ✗

Problem 3. A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable with probability density function

$$f(x) = \begin{cases} 5(1-x)^4 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

What need the capacity of the tank be so that the probability of the supplys being exhausted in a single week is 0.01? Show work.

$$\text{Prob} = .01 = 5\int_c^1 (1-x)^4\,dx = -(1-x)^5\Big|_c^1 = 0 + (1-c)^5 = .01 \rightarrow 1-c = \sqrt[5]{.01} \rightarrow c = 1-\sqrt[5]{.01}$$

✓ $\boxed{c = 0.60189}$

✓ **Problem 4.** You arrive at a bus stop at 10 oclock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. What is the probability that you will have to wait longer than 10 minutes? Show work.

X = waiting time

$$P(X>10) = \int_{10}^{30} \frac{1}{\beta-\alpha} \rightarrow \frac{30-10}{30-0} = \frac{20}{30} = \frac{2}{3} = \boxed{0.6667}$$

✓ **Problem 5.** According to the Census Bureau, 15.3% of Californians live below poverty level. A random sample of 1000 Californians is taken. What is the probability that less than 450 people in the sample live below the poverty level? Show work. Binomial with $n=1000$, $p=.153$, $np > 10$ $n(1-p) > 10$

$\rightarrow$ normal approx: $N(\mu = (1000)(.153) = 153, \sigma^2 = (1000)(.153)(1-.153) = 129.591)$

$$P\left(Z < \frac{450-153}{\sqrt{129.591}}\right) \rightarrow P(Z < 26.089) \approx \boxed{1}$$

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Problem 6. A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let X denote the disease state of a patient, and let Y denote the outcome of the diagnostic test. The joint probability function of X and Y is given by P(X=0,Y=0) = 0.800, P(X=1,Y=0) = 0.050, P(X=0,Y=1) = 0.025, P(X=1,Y=1) = 0.125. Calculate variance of the outcome of the diagnostic test for those with the disease. Show work.

X

	0	1	
0	.8	.05	.85
1	.025	.125	.15
	.825	.175	

Y	$P(y \mid x=1)$
0	$\frac{.05}{.175} = .28571$
1	$\frac{.125}{.175} = .714285$

$E(Y \mid X=1) = (0 \cdot .28571) + (1 \cdot .714285) = .714285$

$Var(Y) = (0 - .714285)^2 .28571 + (1 - .714285)^2 .714285$

$= .14577 + .058309 = \boxed{.20407}$

Problem 7. An incoming lot of silicon wafers is to be inspected for defectives by an engineer in a microchip manufacturing plant. Suppose that, in a tray containing twenty wafers, 8 are defective. Two wafers are to be selected randomly for inspection. What is the probability that at least one is defective? Show work.

$P(\text{at least } 1) = 1 - P(\text{none}) = 1 - \frac{\binom{8}{0}\binom{12}{2}}{\binom{20}{2}} = 1 - \frac{66}{190} = \boxed{0.6526}$

Problem 8. Daily sales records for a car dealership show that it will sell 0, 1, 2, or 3 cars, with probabilities as listed

Number of cars (X)	0	1	2	3
Probability (P(X))	0.5	0.3	0.15	0.05

The price of a car is \$3000, and Total revenue is given by $T = 3000X$. What is the expected revenue and the standard deviation of revenue? Show work.

$E(T) = 3000\,E(x)$

$E(x) = (0 \cdot .5) + (1 \cdot .3) + (2 \cdot .15) + (3 \cdot .05) = .3 + .3 + .15 = 0.75 \times 3000 = \boxed{\$2250 = \mu}$

$Var(x) = (0 - .75)^2 .5 + (1 - .75)^2 .3 + (2 - .75)^2 .15 + (3 - .75)^2 .05 =$

$= .28125 + .01875 + .234375 + .253125 = .7875 \times (3000)^2$

$\rightarrow \sigma = \sqrt{Var(x)} = \boxed{2662.23 = \sigma}$

Problem 9. Wires manufactured for a certain computer system are specified to have a resistance of between 0.12 and 0.14 Ohms. The actual measured resistances of the wires produced by Company A have a normal probability distribution, with expected value 0.13 Ohms and standard deviation of 0.005 Ohms. If four independent such wires are used in a single system and all are selected from company A, what is the probability that all four will meet specifications? Show work. $\mu = .13$ $\sigma = .005$, normal

$P(.12 < x < .14) = P\left(z < \frac{.14 - .13}{.005}\right) - P\left(z < \frac{.12 - .13}{.005}\right) = P(z < 2) - P(z < -2)$

$= .9772 - .0228 = .9544$

$P(\text{all } 4) = (.9544)^4 = \boxed{.8297}$

Problem 10. Let *X*, *Y* have the joint pdf

$$f(x, y) = \begin{cases} 6y & 0 \le y \le x \le 1 \\ 0 & elsewhere \end{cases}$$

Find $E(y \mid x = 0.5)$. Show work.

$f(y \mid x) = \frac{f(x,y)}{f(x)} \rightarrow f(x) = \int_0^x 6y\,dy = 3x^2 \quad 0 < x < 1 \rightarrow f(y \mid x) = \frac{6y}{3x^2} = \frac{2y}{x^2} \quad 0 \le y \le x$

$f(y \mid x = \tfrac{1}{2}) = \frac{2y}{(\frac{1}{2})^2} = 8y \quad 0 < y < \tfrac{1}{2}$

$E(y \mid x = .5) = \int_0^{1/2} 8y^2\,dy = \frac{1}{3} = \boxed{0.6667}$