

**Stat 100A/Sanchez- Midterm 1
Winter 2010-January 21st, Kinsey Pavilion 1220B KEY**

Notice that exam (B) has the questions in different order...

NAME _____

STUDENT ID (Must be ready to show ID at all times) _____

Summary of Scores:	
High:	12 (100 %)
Low:	3.8 (32 %)
Median:	9.2 (77 %)
Mean:	8.9 (74 %)
Standard Deviation:	2
Number of scores:	76

90% and up:	18
80-89%:	14
70-79%:	16
60-69%:	16
50-59%:	5
40-49%:	4
30-39%:	3
20-29%:	0
10-19%:	0
0-9%:	0

PART I Multiple Choice questions. Circle the correct answer. You do not need to show work in this part.

1.-An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability decreases to 0.2 for a non-accident prone person. We know that 30 percent of the population is accident prone. Suppose that a new policy holder has an accident within a year of purchasing the policy. Knowing that, what is the probability that she is accident prone?

- (A) 0.124 (B) 0.461 (C) 0.7 (D) 0.269

Answer: B

Let F = being accident prone

$$P(F|A) = \frac{P(A|F)P(F)}{P(A|F)P(F) + P(A|F^c)P(F^c)} = \frac{(0.4)(0.3)}{(0.4)(0.3) + 0.2(0.7)} = 0.4615385$$

2.- A manufacturing company has two retail outlets. It is known that 30% of all potential customers buy products from outlet 1 alone, 50% buy from outlet 2 alone, 10% buy from both 1 and 2, and 10% buy from neither. Let A denote the event that a potential customer, randomly chosen, buys from outlet 1, and let B denote the event that the customer buys from outlet 2. The prob($A^c \cap B^c$) is

- (A) 0.1 (B) 0.6 (C) 0.5 (D) 0.2

Answer: A

The event ($A^c \cap B^c$) is the same as the event ($A \cup B$)^c

3.-An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

The probability that a claim submitted to the insurance company includes operating room charges is

- (A) 0.1 (B) 0.4 (C) 0.75 (D) 0.25

Answer: (B)

O=event of operating room charges E=event of emergency room charges

Then

$$0.85 = P(O \cup E) = P(O) + P(E) - P(O \cap E) = P(O) + P(E) - P(O)P(E) \text{ (Independence)}$$

$$\text{Since } P(E) = 0.25 = 1 - P(E), \text{ it follows } P(E) = 0.75$$

$$\text{So } 0.85 = P(O) + 0.75 - P(O)(0.75)$$

$$P(O)(1 - 0.75) = 0.1$$

$$P(O) = 0.4$$

4. An auto insurance company has 10,000 policyholders. Each policyholder is classified as (i) young or old; (ii) male or female; and (iii) married or single. Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males.

How many of the company's policyholders are young, female and single?

- (A) 880 (B) 750 (C) 467 (D) 600

Answer: (A)

5. An insurance company examines its pool of auto insurance customers and gathers the following information:

- (i) all customers insure at least one car
- (ii) 64% of the customers insure more than one car
- (iii) 20% of customers insure a sport car
- (iv) Of those customers who insure more than one car, 15% insure a sport car

What is the probability that a randomly selected customer insures exactly one car, and that car is not a sport car?

- (A) 0.104 (B) 0.19 (C) 0.256 (D) 0.29 (E) 0.096

Answer: C

$$0.15 = \frac{P(\text{sports and } > 1 \text{ car})}{P(> 1 \text{ car})} = \frac{P(\text{sports and } > 1 \text{ car})}{0.64}$$

$$P(\text{sports and } > 1 \text{ car}) = 0.64(0.15) = 0.096$$

$$P(\text{sports and 1 car}) = 0.2 - 0.096 = 0.104$$

$$0.36 - 0.104 = 0.256$$

PART II

You must show work in this part

6. An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

A driver chosen at random was involved in at least one collision in the past year. What is the probability that the driver is a young adult driver?

Answer:

$$P(\text{at least one collision}) = P(\text{CT})P(\text{T}) + P(\text{CIY})P(\text{Y}) + P(\text{CIM})P(\text{M}) + P(\text{CIS})P(\text{S}) = 0.15(0.08) + 0.08(0.16) + 0.04(0.45) + 0.05(0.31) = 0.0583 \quad \text{Law of total probability}$$

$$\text{Want } P(\text{Y} | \text{C}) = \frac{P(\text{CIY})P(\text{Y})}{P(\text{C})} = \frac{0.08(0.16)}{0.0583} = 0.0128 / 0.0583 = 0.2195$$

7. A coin weighted so that $P(\text{H}) = 2/3$ and $P(\text{T}) = 1/3$ is tossed three times. Write down the outcomes in the sample space S and the probability of each of the individual outcomes.

Answer:

S = { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT } are as follows (write the probabilities next to each outcome)

$$S = \text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}$$

$$P = (2/3)^3 \quad (2/3)^2(1/3) \quad (2/3)^2(1/3) \quad (2/3)(1/3)^2 \quad (2/3)^2(1/3) \quad (2/3)(1/3)^2 \quad (1/3)^2(2/3) \quad (1/3)^3$$

8. A system with 3 components is such that the system fails if at least one of the individual components fail. The probability that a component fails is 0.04. Let G be the event that the system fails.

(a) Write down explicitly the outcomes in the sample space and the outcomes in the event G.

Answer:

F=component fails, W=component works

S={FFF,FFW,FWF,FWW,WFF,WFW,WWF,WWW}

G=all but WWW

(b) How many outcomes are there in G? How many in the sample space?

Answer:

7 in G, 8 in S.

(c) What is the probability that the system fails? Show work.

Answer:

$$P(\text{G}) = 1 - 0.96^3 = 1 - 0.884736 = 0.115264$$

9. A series of 10 jobs arrive at a computing center with 10 processors. Assume that each of the jobs is equally likely to go through any of the processors. Find the probability that all processors are occupied. Show work and explain.

$$\text{Answer: } \frac{10!}{10^{10}}$$