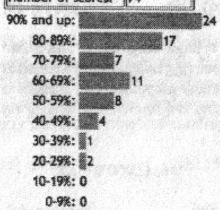
Stat 100A/Sanchez- Midterm 2
Winter 2010-February 11th, Kinsey Pavilion 1220B

NAME____________________

STUDENT ID (Must be ready to show ID at all times) ________________

Summary of Scores:	
High:	15 (100 %)
Low:	3 (20 %)
Median:	12 (80 %)
Mean:	11.5 (77 %)
Standard Deviation:	3
Number of scores:	74



- **CLOSED BOOKS/CLOSED NOTES**
- **PEN OR PENCIL**
- **PLEASE ANSWER ALL QUESTIONS IN THE SPACE PROVIDED ONLY**
- **ONLY SCIENTIFIC CALCULATOR IS ALLOWED**
- **MUST EXPLAIN ANSWER FOR PARTIAL CREDIT.**
- **CHEAT SHEET (TWO SIDES OF 11X8) CAN ONLY HAVE DEFINITIONS AND FORMULAS. NO GRAPHS OR DRAWINGS OR SOLVED PROBLEMS OR NUMERICAL EXAMPLES ARE ALLOWED IN IT. NO DRAWINGS. NO PROOFS ALLOWED EITHER. IF YOU DON'T COMPLY WITH THE ABOVE, YOU WILL NOT BE ABLE TO USE YOUR CHEAT SHEET**
- **PHONES OR ANY OTHER ELECTRONIC DEVICE ARE NOT ALLOWED.**
- **DO ALL SCRATCH WORK ON THE BACK OF THE PAGES**

I.- MULTIPLE CHOICE QUESTIONS. Please, circle one answer. If you think the answer is not given, write your answer. Scratch work for these questions will not be read.

1.- The time that it takes a driver to react to the break light on a decelerated vehicle is critical in avoiding rear-end collisions. Someone suggests that reaction time for an in-traffic response to a break signal from standard break lights can be modeled with a normal distribution having parameters m=1.25 seconds and s=0.46 seconds. In the long run, what proportion of reaction times will be between 1.00 seconds and 1.75 seconds?

(a) 0.321
(b) 0.567
(c) 0.432
(d) 0.930

Answer: (B)
X=reaction time
X is N(m =1.25, s=0.46)
P(1 < x < 1.75) = P(-0.54 < z < 1.09) = 0.8621-0.2946 = 0.5675

2. – It is known that 45% of a College's student loan applications are approved. If 500 applications are chosen at random, what is the probability that less than 200 are approved? Show work.

(a) 0.543 (b) 0.987 (c) 0.1 (d) 0.0125

X= # of applications out of 500 approved.
E(X) = np = 500 (0.45) =225
Var(X) = np(1-p)= 500((0.45)(0.55)=123.75
The standard deviation of X is 11.124
Check np > 10 np(1-p) > 10
P(X < 200) = P(Z < -2.247) = 0.0125

3.- .- Suppose that X is a normal random variable with E(X)= 3. If P[X> 5] =0.3, approximately, what is the Variance of X?

(a) 45.67 (b) 134 (c) 14.13 (d) 0.7

$$3 = P\left\{\frac{X-3}{\sigma} > \frac{5-3}{\sigma}\right\} = P\{Z > 2/\sigma\}$$ *where Z is a standard normal. But from the normal table*

$P\{Z < 0.53\} \approx .70$ *and so*

$.53 \approx 2/\sigma$ or $\sigma \approx 3.77$

That is, the variance is approximately $(3.76)^2 = 14.1376$

4.- A recruiting firm finds that 30 percent of the applicants for a certain industrial job have received advanced training in computer programming. Applicants are interviewed sequentially and selected at random from the pool. Suppose that the first applicant with advanced training is offered the position, and the applicant accepts. If each interview costs $30, find the expected value and variance of the total cost of interviewing until the job is filled. Show work.

(a) E(cost) = 30; Var(cost) = 150
(b) E(cost) = 100; Var(cost) =1000
(c) E(cost) =100; Var(cost)=7000
(d) E(cost) = 300; Var(cost)= 6000

X is geometric(p=0.3)
Cost = 30X
Expected Cost=30 (1/0.30) =100
Var(cost)=900 [(1-0.3)/0.3²]=7000

5.- People enter a gambling casino at a rate of 1 for every minute. What is the probability that no one enters between 12:00 and 12:05?

(a) 0.00673 (b) 0.082 (c) 0.0028 (d) 0.2424

$e^{-5} = 0.00673794$

6.- A type of capacitor has resistances that vary according to a normal distribution with a mean of 800 megohms and a standard deviation of 200 megohms. A certain application specifies capacitors with resistances of between 900 and 1000 meghoms. If two capacitors are randomly chosen from a lot of capacitors of this type, what is the probability that both will satisfy the specifications?

(a) 0.1498 (b) 0.5 (c) 0.0224 (d) 0.61

First find the prob that one capacitor satisfies specifications. Let Y=resistance. $P(900 < Y < 1000) = P(0.5 < Z < 1) = 0.1498$.
Define now X = number out of sample of 2 Bernoulli trials that meets the specifications. Then X is binomial with parameters n=2 and p=0.1498.
$P(X=2) = 0.1498^2 = 0.0224$

7.-The number of bacteria colonies of a certain type in samples of polluted water has a Poisson with a mean of 2 per cubic centimeter. If four 1-cubic centimeter samples of this water are independently selected, find the probability that at least one sample will contain one or more bacteria colonies.

(a) 0.1353 (b) 0.8646 (c) 0.999 (d) 0.4321

Answer: Let Y= number of bacterial colonies in a-cubic centimeter sample. Y is Poisson with lambda =2.

First compute the probability that at least one sample contains one or more bacteria colonies using the Poisson distribution.

$$P(Y \geq 1) = 1 - P(Y < 1) = 1 - P(Y = 0) = 1 - e^{-2} = 0.8646$$

Now define X=number of samples, out of 4, that contain one or more bacterial colonies.
This is a Binomial, with n=4 and p=1-e⁻²=0.8646

So we want

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \binom{4}{0}(1-e^{-2})^0(1-(1-e^{-2}))^4$$

$$= (e^{-2})^4 = 1 - 0.1353^4 = 0.999$$

8.-Ten software packages available to solve a linear programming problem have been ranked from 1 to 10 (best to worst). An engineering firm selects three of these packages for purchase, without looking at the ratings. Let Y denote the number of packages purchased by the firm that are ranked 6 or higher. The random variable Y is more likely to be…

(a) Binomial (b) Negative Binomial (c) Hypergeometric (d) Geometric

Answer: hypergeometric. This is a hypergeometric distribution. N=10, n=3, ranked 6 or higher are 5 of the 10 packages, so k=5, N-k = 5. Use the formula for the hypergeometric . Y can only take the values 0, 1, 2, or 3.

$$P(Y = y) = \frac{\binom{k}{y}\binom{N-k}{n-y}}{\binom{N}{n}}$$

Using this formula, we can find the whole probability mass function.

X	P(X=x)
0	0.0833
1	0.4166
2	0.4166
3	0.0833

9.- A firm sells four items randomly selected from a large lot that is known to contain 10% defectives. Let Y denote the number of defectives among the 4 sold. The purchaser of the items will return the defectives for repair, and the repair cost is given by

$$C = 3Y^2 + Y + 2$$

What is the expected repair cost?

(a) 3.96 (b) 0.36 (c) 5.67 (d) 100

Y is binomial (n=4, p=0.1)   E(Y) = 0.4, Var(Y)=0.36

$E(C) = 3E(Y^2) + E(Y) + 2 = 3[Var(Y) + [E(Y)]^2] + E(Y) + 2$

$= 3.96$

E (C) =3.96

10.-The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with density function

$f(x) = 1.5(1-x^2) \quad 0 \le x \le 1$

What is the probability that the amount of gravel sold in a given week is smaller than 0.4 tons? Show work.

(a) 0.432   (b) 0.375   (c) 0.244   (d) 0.568

Answer: $P(X < 0.4) = \int_0^{0.4} 1.5(1-x^2)dx = 0.568$

11.-In deciding how many customer service representatives to hire and in planning their schedules, a firm that markets electronic typewriters studies repair times for the machines. One such study revealed that repair times have an approximately exponential distribution, with a mean of 22 minutes. In planning schedules, how much time should the firm allow for each repair to ensure that the chance of any one repair time's exceeding this allowed time is only 0.01?

(a) 101.3   (b) 190.3   (c) 365.3   (d) 0.365

Answer:
$P(X > k) = e^{-k/22} = 0.010$

k= -22 ln(0.01) = 101.3

II.- SHORT ANSWER. WORK IS GRADED. MUST SHOW WORK FOR FULL CREDIT.

12.-Let X represent a discrete random variable that takes any of the values and probabilities given below

X	-2	5	6
P(X)	0.4	0.4	

(a) The P(X=6) is: ______________

Answer: 0.2

(b) The probability mass function for the random variable g(X), where

$g(X) = 3X^2 - 2$, is (write in a table format like that above)

g(X)			
P(X)			

Answer:

g(X)	10	73	106
P(X)	0.4	0.4	0.2

(c) What is the Expected value of g(X), i.e, E(g(X))? Show work.

Answer: (10)(0.4)+73(0.4)+106(0.2)= 54.4

13.- The cumulative distribution function of a random variable X is $F(x) = \frac{x-2}{5}$. What is the density function and the possible values of the random variable? Show work. What is the family name of this random variable?