Stat 100 -Intro Probability
Midterm 1

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DO NOT DETACH ANY PAGES FROM THIS EXAM. EXAM MUST STAY STAPLED DURING THE WHOLE EXAM.

UCLA ID: -

LAST NAME (Please, PRINT): - FIRST NAME: - - - - - - - - - -

Instructions

- Closed books, closed notes.
- Answer for multiple choice questions will be marked in scantron AND the exam. Work will not be read in multiple choice;Failure to mark your name, ID or some answers will result in point deduction from the exam grade.
- Left handed students will sit in a seat for left-handed students. The professor will tell students where to sit. Please, let the professor know that you are left handed ahead of time.
- ID must be ready to show at all times during the exam. NO ID, no exam.
- This midterm must show your individual work. Talking to others during the midterm, not adhering to the above, sharing information or breaking any other aspect of the student code of conduct at UCLA will not be tolerated. You can not exchange papers or information, and you can not use your phone or other electronic devices. All your things must be on the floor. You may not use the empty seats next to you to put things. Close the tables. Honor code applies. You may not talk or use your electronic devices from the moment the professor starts distributing the midterm until they have been collected from everybody.
- Only scientific calculator and one side of a 8×11 cheat sheet, and pen or pencil and exam and scantron can be on your desk. Everything else (disconnected) in your backpack and your backpack on the floor. Close the tables of the chair near you.
- Cheat sheet can have only formulas and definitions, no solved problems, no examples of any kind, no proofs, no numerical examples, no intermediate steps and no drawings or graphs of any kind.YOUR NAME MUST BE ON CHEAT SHEET AT ALL TIMES. Be ready to show your cheat sheet when the instructor requests it. The cheat sheet must be written all in English. Cheat sheets that do not comply will result in lower grade in the exam.
- In questions where you show work, you must use the same notation we have used in lecture and the textbook.
- Phones and all other electronic devices must be disconnected and in your backpack. Your backpack must be on the floor.

- Failure to follow instructions given here will result in loss of points in the exam in a first warning. Honor code applies. Familiarize yourself with student code of conduct by visiting the links provided in the course syllabus.

The exam contains

- Cover page (write your name and ID)
- 16 Multiple choice questions. Answers must be marked on exam and the scantron with your name and ID. Work is not required and will not be read.
- Question where you must show work. Work must be written in the space provided.
- One extra page for scratch work.
- The page with the distributions formulas.

DO NOT DETACH ANY PAGE FROM THIS EXAM.

MC score: 100%

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MULTIPLE CHOICE QUESTIONS. ONLY ONE ANSWER IS CORRECT. CHOICE MUST BE MARKED ON THE SCANTRON, AND ALSO HERE ON THE EXAM. ONLY THE SCANTRON WILL BE GRADED. NO MARKS ON SCANTRON OR MORE THAN ONE MARK WILL RESULT IN 0 POINTS FOR THE QUESTION NOT MARKED, EVEN IF IT IS MARKED ON THE EXAM. Work near the multiple choice question will not be read.

B P C

Problem 1. In a computer installation, 20% of the new programs are written in Cobol, 30% in Pascal and 50% in C. 20% of the Cobol programs compile on the first run, 70% of the Pascal programs compile on the first run and 80% of the C programs compile on the first run.

What is the overall percentage of programs that compile on the first run?

(a) 0.615

(b) 0.385

(c) 0.65

(d) 0.35

(e) 0.465

$$P(R) = P(R|B)P(B) + P(R|P)P(P) + P(R|C)P(C)$$
$$= (0.2)(0.2) + (0.7)(0.3) + (0.8)(0.5)$$
$$= 0.04 + 0.21 + 0.4$$

Problem 2. Let $S = (0, 1]$ and define $A_i = ((1/i), 1]$, $i = 1, 2, \ldots\ldots$ The intersection of these events is

(a) S, the whole sample space.

(b) The interval $[0, 0.5]$.

(c) The set with only 1 in it.

(d) The empty set.

(e) The interval $[0, 10]$.

S 0 1

A_1 (1, 1]

A_2 (0.5 1]

A_3 (0.33 1

Problem 3. Daily sales records for a car dealership show that it will sell 0, 1, 2, or 3 cars, with probabilities as listed

Number of cars (X)	0	1	2	3
Probability (P(X))	0.5	0.3	0.15	0.05

The expected value of X^3 is

(a) Impossible to compute without more information

(b) 0.75+1

(c) 2.85

(d) 5

(e) 3

(f) 0

$$E(X^3) = (0)^3 0.5 + (1)^3(0.3) + (2)^3(0.15) + (3)^3(0.05)$$
$$= 0.3 + 1.2 + 1.35$$
$$= 2.85$$

Problem 4. A coin which lands heads with probability p is tossed repeatedly. Assuming independence of the tosses, the probability that the same number of heads, k, appear in the first 8 tosses as in the next 5 tosses is

$P(H) = p$

$k = $ # heads

(a) $\binom{11}{k}p^{k-1}(1-p)^3.p$

(b) $\binom{9}{k}p^k(1-p)^k - 1$

P(k heads in first 8 tosses) $\cap P$(0 heads in 5 more tosses)

(c) $(1-p)^k p$

(d) $\sum_{k=0}^{5}\binom{8}{k}p^k(1-p)^{8-k}.\binom{5}{k}p^k(1-p)^{5-k}$ $\quad \left[\binom{8}{k}(p)^k(1-p)^{8-k}\right]\left[\binom{13}{k}(p)^k(1-p)^{13-k}\right]$

(e) 2/x

Problem 5. The Center for Disease Control says that about 30% of high school students smoke tobacco (down from a high of 38% in 1997). Suppose you randomly select 10 high school students to survey them on their attitude towards scenes of smoking in the movies. What is the probability that there are 2 smokers among the 10 people you choose?

(a) 0.000729 $\quad P(X=2) = \binom{10}{2}(0.3)^2(0.7)^8$

(b) 0.0504 $\quad = 45(0.3)^2(0.7)^8$

(c) 0.000504

(d) 0.2334 $\quad 0.2374$

(e) 0.002

Problem 6. Only 5% of male high school athletes go on to play at the college level. Of these, only 1.7% enter major league professional sports. About 40% of the athletes who compete in college and then reach the pros have a career of more than 3 years. What is the probability that a high school athlete competes in college and then goes on to have a pro career of more than 3 years?

(a) 0.005

(b) 0.00034

(c) 0.0215

(d) 0.017

(e) 0.5

Problem 7. The number of people arriving to an emergency room can be modeled by a Poisson process with a rate parameter λ of five per hour.

What is the probability that at least 4 people arrive during a particular hour?

(a) 0.265 $\quad \lambda = 5$

$P(x \geq 4) = 1 - P(x < 4)$

(b) 0.735

$= 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3)$

(c) 5

$= 1 - e^{-5}\frac{5^0}{0!} - e^{-5}\frac{5^1}{1!} - e^{-5}\frac{5^2}{2!} - e^{-5}\frac{5^3}{3!}$

(d) 0.8

$e^{-5} = A \quad = 1 - A - B - C - D$

$= 0.7349$

(e) 0.1

Problem 8. 20% of the applicants for a certain sales position are fluent in English and Spanish. Suppose that four jobs requiring fluency in English and Spanish are open. Find the probability that two unqualified applicants are interviewed before finding the fourth qualified applicant, if the applicants are interviewed sequentially and at random.

$$P(6) = \binom{6-1}{4-1}(0.2)^4(1-0.2)^{6-4}$$
$$= 10\,(0.2)^4\,(0.8)^2$$
$$= 0.01024$$

(a) 0.5

(b) 0.8

(c) 0.2

(d) 0.0124 Typo?

(e) 0.001

Problem 9. A computer student can repeat an examination until it is passed, but it is allowed to attempt the examination at most four times. The probability that the student passes the exam in each attempt is 0.6. Let E be the event that the student passes. The probability of E is

$$1 - P(E^c) = 1 - (0.4)^4$$
$$= 0.9744$$

(a) 0.443

(b) 0.974

(c) 0.612

(d) 0.78

(e) 0.0256

Problem 10. An oil exploration firm is to drill ten wells, each of which has probability 0.1 of successfully striking recoverable oil. It costs \$10,000 to drill each well so there is a total fixed cost of \$100,000. A successful well will bring in oil worth \$500,000. The expected value and standard deviation of the firms gain are closest to, respectively, (Choose one, work will not be read).

$P(S) = 0.1$ X = successful wells

Cost = $-100\,000 + 500\,000X$

$E(X) = (0.1)(10) = 1$

$Var(X) = (10)(0.1)(0.9) = 0.9$

$E(C) = -100\,000 + 500\,000\,E(X)$
$= -100\,000 + 500\,000(1)$
$= 400{,}000$

$Var(C) = (2.5\times10^{11})\,Var(X)$
$\sqrt{\sigma^2} = 474341$

(a) \$500000, \$500000

(b) \$ 500000, \$ 0.9

(c) \$ 450000, \$474.34

(d) \$400000, \$474341.6

(e) \$100000, \$1001.34

Problem 11. When an emergency occurs, the response time (in hours) of the first police car is a random variable X with density

$\lambda = 0.2$

$$f(x) = 0.2e^{-0.2x} \quad x \geq 0$$

The probability that X is larger than 6 hours is $F(x) = 1 - e^{-\lambda x}$

$P(X>6) = 1 - P(X \leq 6) = 1 - (1 - e^{-0.2(6)})$
$= 0.30119$

(a) 0.698

(b) 0.301

(c) 0.5

(d) 0.2

(e) 0.871

Problem 12. Let x be a continuous random variable with the following density function.

$$f(x) = \frac{1}{2}x, \qquad 0 \le x \le 2$$

The 70th percentile is (circle one, work will not be read):

(a) 1.673

(b) 2.821

(c) 0.52

(d) 1.1

(e) 1.5

$0.7 = \int_0^c \frac{1}{2}x \, dx$

$0.7 = \frac{1}{4}x^2 \Big|_0^c$

$0.7 = \frac{1}{4}c^2$

$2.8 = c^2$ $\quad c = 1.673$

Problem 13. The cumulative distribution function of a random variable X is

?

$$F(x) = \frac{x-2}{5} \qquad 0 < x < 5$$

. Which of the following could be the density function of this random variable?

$\frac{x}{5} - \frac{2}{5}$

(a) $f(x) = 2x, \quad 0 < x < 5$

$\frac{d}{dx}F(x) = f(x)$

(b) Uniform (a=0, b=2/5)

$\frac{1}{5} = f(x)$

(c) Exponential with parameter $\lambda = \frac{2}{5}$

$\frac{d}{dx}\left(\frac{x-2}{5}\right) = f(x)$

(d) Uniform(a=0,b=5) $\quad f(x) = \frac{1}{5-0} = \frac{1}{5}$

$\frac{1}{5}\left(\frac{d}{dx}(x-2)\right) = f(x)$

(e) Poisson($\lambda = 3$)

$\frac{1}{5} = f(x)$

Problem 14. By definition, if X is a random variable,

$$Var(X) = E(X - E(X))^2 = \sum_x (X - E(X))^2 P(X)$$

Which of the following does NOT equal the variance of X?

(a)

$$\sum_x (X^2 + (E(X))^2 - 2XE(X))P(X)$$

(b)

$$\sum_x X^2P(X) + \sum_x (E(X))^2P(X) - \sum_x 2XE(X)P(X)$$

(c)

$$\sum_x X^2P(X) + \sum_x \mu^2P(X) - \mu\sum_x 2XP(X)$$

(d)

$E(X^2) + \mu^2 \quad -2\mu^2$

$$E(\mu^2) + (E(X))^2 - 2(E(X))^2$$

(e)

$$E(X^2) - \mu^2$$

$\sigma^2 = E(X^2) - \mu^2$

Problem 15. The moment generating function of X is given by

$$M_X(t) = \left(\frac{3}{4}e^t + \frac{1}{4}\right)^{10}$$

and the moment generating function of Y is

$$M_Y(t) = e^{2e^t-2}.$$

If X and Y are independent, the probability of the event $X + Y = 2$ is given by,

(a) 15

Let $Z = X+Y$

$P(Z=2) = P(X+Y=2)$

(b) 0.1353361

(c)

$$45e^{-2}(0.75)^2(0.25)^8 + 20e^{-2}(0.75)(0.25)^9 + 2e^{-2}(0.25)^{10}$$

(d)

$$45e^{-2}(0.75)^2(0.25)^8 + 20e^{-2}(0.75)(0.25)^9$$

(e)

$$e^{-2} + 0.75^{10}$$

Problem 16. A student proposes the following as a density function for a random variable Y,

$$f(y) = e^{-y}y^3, \qquad y \geq 0.$$

Which of the following is true?

(a) The expected value of this random variable is 1

$\int_0^\infty e^{-y}y^3dy \quad 0.5 = \int_0^c e^{-y}y^3\,dy$

(b) This is not a density function.

(c) $f(y) \leq 1, y \geq 0$.

(d) The moment generating function of this random variable is

$$\left(\frac{1}{1-t}\right)^4$$

$-4\left(\frac{1}{1-t}\right)^3$

$-4(1)^3 = -4$

(e) This is an exponential random variable.

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5.8/6

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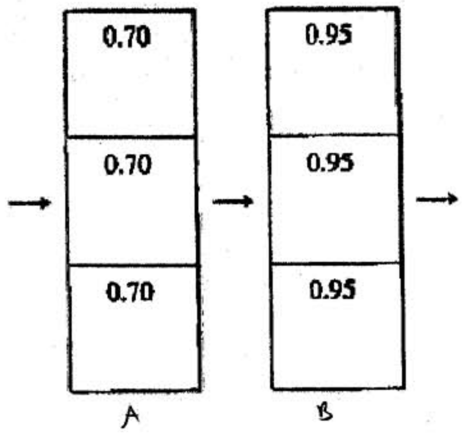
LAST NAME (Please, PRINT): - - - - - - - - - - - - - - - - - FIRST NAME: - - - - - - - - - -

PROBLEMS WHERE YOU MUST SHOW WORK. YOU MUST ENTER YOUR NAME AND ID ABOVE.

PLEASE, WRITE YOUR ANSWER HERE ~~AND ON THE SHEET HANDED OUT FOR THIS QUESTION. ONLY THE SHEET WILL BE GRADED.~~

Problem 17. The diagram in Figure **??** is a representation of a system that comprises two parallel systems in series. Calculate the reliability of the the whole system.



$P(A \text{ works}) = 1-(0.3)^3 = 0.973$

$P(B \text{ works}) = 1-(0.05)^3 = 0.999875$

$\text{Reliability} := (0.973)(0.999875) \approx \boxed{0.9729}$

Problem 18. Let X be the number of boys in a family with 3 kids, where the probability of being boy is 1/3. Answer the following questions:

(a) Write the sample space for the experiment having three kids.

B = is a boy

$S = \{BBB, BBB^c, BB^cB, BB^cB^c, B^cBB, B^cBB^c, B^cB^cB, B^cB^cB^c\}$

(b) Write the probability distribution of X in the form of a table.

X	$P(x)$
0	$\left(\frac{2}{3}\right)^3 = 0.2963$
1	$\binom{3}{1}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 = 0.4444$
2	$\binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) = 0.2222$
3	$\left(\frac{1}{3}\right)^3 = 0.0370$

(c) Compute the expected value and standard deviation of X.

$E(x) = \sum_x x P(x) = 0(0.2963) + 1(0.4444) + 2(0.2222) + 3(0.0370)$

$= \boxed{1}$

$Var(x) = E(x^2) - E(x) = 1.6667 - 1 = \boxed{0.6666}$ sd? -0.2

$E(x^2) = 1(0.4444) + 4(0.2222) + 9(0.0370) = 1.6667$

(d) Compute $E(2X^2 - 3X + 10)$. Show work.

$E(2x^2 - 3x + 10) = 2E(x^2) - 3E(x) + 10$

$= 2(1.6667) - 3(1) + 10$

$= \boxed{10.3333}$

(e) Write the moment generating function of the random variable X and show how to get the expected value of X from it.

$M_x(t) = \sum_x e^{tx} P(x)$

$= e^{t0}(0.2963) + e^{t1}(0.4444) + e^{t2}(0.2222) + e^{t3}(0.0370)$

$= \boxed{0.2963 + 0.4444e^t + 0.2222e^{2t} + 0.0370e^{3t}}$

$E(x) = \frac{d}{dx} M_x(t)\big|_{t=0} = 0.4444e^0 + 0.4444e^{2(0)} + 0.1111e^{3(0)}$

$= \boxed{1}$