For the next three problems, let X be the total number of face cards (K, Q, or J) you have in your hole cards, and let Y be the number of spades you have in your hole cards. Thus X could be 0, 1, or 2, and Y could also be 0, 1, or 2. Let $Z = XY$.

1. What is E(X)?
a. 0.213. b. 0.462. c. 0.671. d. 1.00. e. None of the above.

2. What is E(Z)?
a. 0.123. b. 0.175. c. 0.199. d. 0.231. e. None of the above.

3. What is $cov(X,Y)$?
a. -0.112. b. 0. c. 0.112. d. 0.204. e. None of the above.

4. Suppose $X = 0$ with probability 1/4, $X = 1$ with probability 1/2, and $X = 2$ with probability 1/4. What is the moment generating function of X?
a. $1/4 + 3e^t/4$. b. $1/2 + 3e^{2t}/4$. c. $1/4 + 2e^t + 3e^t/4$. d. $1/4 + e^t/2 + e^{2t}/4$. e. None of the above.

5. What is the probability that you will have a straight flush on the turn? (Hint: be careful not doublecount outcomes like the case where you have 10♥ 8♥ and the board is 5♥ 6♥ 7♥ 9♥.)
a. 0.000874%. b. 0.000906%. c. 0.000974%. d. 0.000991%. e. None of the above.

6. Suppose you play 400,000 hands, and X is the number of those hands where you have a straight flush on the turn. What is the expected value of X?
a. 0.0912. b. 0.906. c. 3.62. d. 7.84. e. None of the above.

7. Suppose X and Y are bivariate normal with mean 0 and variance 1, and cov(X,Y) = 0.4. What is cov(5X+Y, 4X-Y)?
a. -4.4. b. 4.8 c. 18.6. d. 22.3 e. None of the above.

8. What is the probability that you will have a flush or straight flush on the river?
a. 1.02%. b. 2.04%. c. 2.55%. d. 3.06%. e. None of the above.

9. Let X be the number of hands until the 5th time you have a flush or straight flush on the river. What is the SD of X?
a. 55.3. b. 62.3. c. 71.9. d. 92.5. e. None of the above.

10. Out of 10 players in a given hand, what is the expected number of players who are dealt at least one ace?
a. 1.49 b. 2.25. c. 2.56. d. 2.77. e. None of the above.

11. Let A be the event that your hole cards consist of a king and a queen, and let B be the event that both your hole cards are different colors, i.e. one is red and the other is black. Are A and B independent?
a. No. b. Yes. c. Cannot be determined from the information given.

12. You have 10♣ 10♠, your opponent has K♣ K♥, and the flop is Q♠ 10♥ 4♣. The pot is $30. The turn is 5♣, you bet $20, and your opponent calls. How much expected profit did you gain due to skill on the turn?

a. $10.0. b. $15.0. c) $18.2. d. $19.4. e. None of the above. $40 \times P(\text{win}) - 20$

For the next two problems, let $X = N(0, 0.5^2)$. Let $\varepsilon = N(0, 0.3^2)$ where ε is independent of X, and let $Y = 10 + 0.1\,X + \varepsilon$.

13. What is E(Y|X)?

a. 10. b. $10 + 0.1\,X$. c. 10.3. d. $10 + .1\,X + 0.3$. e. None of the above.

14. What is cov(X,Y)?

a. 0. b. 0.01. c. 0.02. d) 0.025. e. None of the above.

15. If (X,Y) are bivariate normal with E(X) = 10, var(X) = 16, E(Y) = 12, var(Y) = 25, and ρ = 0.7, what is the distribution of Y given X = 14?

a) $N(15.5, 3.57^2)$. b. $N(17.5, 2.09^2)$. c. $N(15.5, 2.09^2)$. d. $N(15.5, 4.02^2)$. e. None of the above.

$E(10 + 0.1X + \varepsilon | X)$

$0.1\,\text{var}(X)$

$\beta_2 = \frac{\rho\sigma_y}{\sigma_x}$

$= 0.875$

$12 = \beta_1 + 0.875\,E(X) + E(\varepsilon)$

$\beta_1 = 3.25$

$25 = 0.875^2\,\text{var}(X) + \text{var}(\varepsilon)$

$\text{var}(\varepsilon) = 12.75$

$\text{sd}(\varepsilon) = 3.57$

$Y = 3.25 + 0.875(14) + \varepsilon$

$= 15.5 + \varepsilon$