$\frac{3}{\binom{52}{3}}$ or $\frac{1}{\binom{52}{1}}$ $\frac{\binom{13}{2}\binom{39}{1}}{\binom{52}{3}}$

P(win before) = any card not a 6.

$\frac{43}{44}$ $P=0$ $+P\cdot$

B 1. What is the probability that the flop contains exactly two diamonds?
a. 10.0%. b. 13.8%. c. 14.5%. d. 19.0%. e. None of the above.

D 2. What is the probability that the flop contains exactly two diamonds, given your two hole cards are the ace and king of diamonds?
a. 8.55%. b. 9.01%. c. 9.54%. d. 10.9%. e. None of the above.

$\frac{\binom{11}{2}\binom{39}{1}}{\binom{50}{3}}$

P(2 diamonds | A◇K◇)

D 3. Suppose a flop is said to "have a flush draw possibility" if it contains exactly two cards of the same suit, plus another card of a different suit. For instance, A♥ K♥ 10♠ would have a flush draw possibility, but A♥ K♦ 10♠ would not, and also A♥ K♥ 10♥ would not. What is the probability that the flop has a flush draw possibility?
a. 48.2%. b. 49.1%. c. 51.2%. d. 55.1%. e. None of the above.

$\frac{\binom{13}{2}\binom{4}{1}\binom{13}{1}\binom{3}{1}}{\binom{52}{3}}$

A 4. What is the probability that the 3 cards on the flop are all of the same suit?
a. 5.18%. b. 5.56%. c. 6.02%. d. 6.54%. e. None of the above.

$\frac{\binom{13}{3}\binom{4}{1}}{\binom{52}{3}}$

B 5. In the hand from High Stakes Poker between Daniel Negreanu and Gus Hansen, Hansen had 5♦ 5♣ and Negreanu had 6♠ 6♥. The board was 9♣ 6♦ 5♥ 5♠ 8♠. After the betting on the turn, the size of the pot was $111,700. After the 8 of spades was revealed on the river, Hansen checked, Negreanu bet $65,000, Hansen raised $167,000 more to a total of $232,000, and Negreanu called. The total pot was $575,700. Given only their cards and the board, how much did Hansen gain due to luck and skill on the river?
a. 2788 due to luck and 232,000 due to skill. b. 2539 due to luck and 232,000 due to skill.
c. 2788 due to luck and 245,000 due to skill. d. 2539 due to luck and 245,000 due to skill.
e. 2737 due to luck and 245,000 due to skill. f. None of the above.

100% (111,700) − P(win) before × (111,700)

2 × 232000 × 100% − 232,000

D 6. Let X = the number of black cards on the flop. What is E(X)?
a. 0.500. b. 0.875. c. 1.24. d. 1.50. e. None of the above.

$E(X) = 0 \times P(X=0) + 1 \times P(X=1) + 2 \times P(X=2) + 3(P(X=3))$

D 7. Let X = the number of clubs in your hand, so X must be 0, 1, or 2. What is $E(X^2)$?
a. 0.500. b. 0.512. c. 0.548. d. 0.618. e. None of the above.

$\frac{\binom{13}{1}\binom{39}{1}}{\binom{52}{2}}$

$E(X^2) = 0^2 \times P(X=0) + 1^2 \times P(X=1) + 2^2 \times P(X=2)$

B 8. Suppose you play 10 hands of poker. Let X = the number of these hands where you are dealt two suited cards, i.e. where your two cards are both of the same suit. What is SD(X)?
a. 1.09. b. 1.34. c. 1.76. d. 1.92. e. None of the above.

$\sqrt{npq}$

$\frac{\binom{13}{2}}{\binom{52}{2}}$

A 9. Let B = the event that the 3 cards on the flop are all spades. Let C = the event that the 3 cards on the flop are all black. Find P(B|C). = P(B and C) / P(C)
a. 11.0%. b. 12.2%. c. 12.8%. d. 13.3%. e. None of the above.

C 10. Let X = the lowest number of the 3 cards on the flop, where J = 11, Q = 12, K = 13, and ace = 14. Let F denote the cumulative distribution function of X. What is F(10)? Hint: the 1 minus trick can be helpful here.
a. 89.3%. b. 91.2%. c. 97.5%. d. 98.2%. e. None of the above.

$\frac{\binom{13}{3}}{\binom{26}{3}}$

$F(x) = P(X \le 10)$

$= 1 - (P(X=11) + \rightarrow 3 \frac{\binom{13}{2}\binom{4}{1}}{\binom{52}{2}}$

$P(X=12) + \rightarrow 1$

$\binom{4}{3} \leftarrow \cancel{P(X=13)} +$

$\cancel{P(X=14)}) \rightarrow \binom{4}{3}$

$\frac{\binom{26}{1}\binom{26}{2}}{\binom{52}{3}}$ $\frac{\binom{26}{2}\binom{26}{1}}{\binom{52}{3}}$ 0.3824 $\frac{\binom{26}{3}}{\binom{52}{3}} \rightarrow 0.1176$

B 11. Let B = the event that all the cards on the flop are hearts. Let C = the event that the flop contains at least one ace. Are B and C independent?

a. Yes. b. No. c. Cannot be determined. d. None of the above.

B 12. Consider just the first 4 board cards. What is the probability that these 4 board cards are all of different suits?

a. 8.03%. b. 10.5%. c. 11.0%. d. 12.3%. e. None of the above.

D 13. A "paired board" means a board where at least 2 of the cards have the same number. For instance, K K Q 2 3, K K K Q 3, K K K K 3, and K K Q 3 3 would all be examples of paired boards. What is the probability of a paired board? Assume all 5 board cards are going to be seen regardless of the players' cards and actions.

a. 38.2%. b. 40.1%. c. 43.7%. d. 49.3%. e. None of the above.

D 14. Suppose in a given tournament every player started with 10,000 chips, and there were 2,000 players. After 7 hours, 500 players are left. Suppose you choose one of these 500 players at random and let X = the number of chips she has left. What does the Markov inequality tell you about $P(X \geq 100{,}000)$?

a. $P(X \geq 100{,}000) \leq 0.257$. b. $P(X \geq 100{,}000) \leq 0.353$. c. $P(X \geq 100{,}000) \leq 0.371$.

→ d. $P(X \geq 100{,}000) \leq 0.400$. e. $P(X \geq 100{,}000) \leq 0.429$. f. None of the above.

$P(C) = P(C|B)$

$P(C) = P(\text{at least one Ace}) = 1 - P(\text{no Ace}) = 1 - \frac{\binom{48}{3}}{\binom{52}{3}} = 0.217$

$P(\text{at least 1 ace} \mid \text{all hearts}) = 1 - P(\text{no ace} \mid \text{all hearts})$

$= 1 - \cancel{\frac{\binom{12}{3}}{\binom{13}{3}}} = 0.230$

$\frac{\binom{12}{2}}{\binom{13}{3}}$

$\frac{\binom{13}{1}^4}{}$

$P(\text{paired board}) = P(\text{at least 2 cards are the same value})$

$= 1 - P(\text{no card same value})$

$= 1 - \frac{\binom{13}{5}\binom{4}{1}^5}{\binom{52}{5}}$

$P(X \geq 100{,}000) \leq E(X)/100{,}000$

↓

$P(B) = P(B|C)$

$\frac{\binom{13}{3}}{\binom{52}{3}} = \frac{\quad}{} + 0 + 0$

given 1 ace

2 JQQ
2 JKK
2 JAA

3 JJJ
2 JJQ
3 JJK
2 JJA

JQA 1
JKA 1
JQK 1

$P(X \leq 10) = 1 - P(X > 10) = 1 - (P(X=11) + P(X=12) + P(X=13) + P(X=14))$

$\frac{4 \times \binom{4}{1}^3 + 12 \times \binom{4}{2}\binom{4}{1} + 4 \times \binom{4}{3}}{\binom{52}{3}}$

QAK 1

3 QQQ
2 QQK
2 QQA
2 QKK
2 QAA

3 KKK
2 KKA
2 KAA

3 AAA