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MULTIPLE CHOICE QUESTIONS. ONLY ONE ANSWER IS CORRECT. CHOICE MUST BE MARKED ON THE SCANTRON, AND ALSO HERE ON THE EXAM. ONLY THE SCANTRON WILL BE GRADED. NO MARKS ON SCANTRON OR MORE THAN ONE MARK WILL RESULT IN 0 POINTS FOR THE QUESTION NOT MARKED, EVEN IF IT IS MARKED ON THE EXAM. You may use the space near the question for scratch work, but scratch work will not be read.

Question 1. The proportion of air pollution particles in the air of a urban area has the following probability density function

$$f(y) = 1, \quad 0 \le y \le 1$$

Let Y_1, Y_2, Y_3, Y_4 be the proportion in four independent urban areas, each of them with the density function given above. The moment generating function of $W = Y_1 + Y_2 + Y_3 + Y_4$ is

(a) $\frac{(e^t-1)^4}{t^4}$

(b) $e^3 t$

(c) $e^{3t+1/2t^2}$

(d) $\frac{(e^t-1)^3}{t^3}$

(e) $e^{-4} + 0.75^{10}$

$\int_0^1 e^{tx} = \frac{1}{t} e^{tx}\Big|_0^1 = \frac{1}{t}e^t - \frac{1}{t} = \left(\frac{e^t-1}{t}\right)^4$

$W = \sum^4 Y = \prod_{i=1}^{4} e^{\frac{t-1}{t}} = \frac{e^{t-1}}{t}\left(\frac{e^{t-1}}{t}\right)\left(\frac{e^{t-1}}{t}\right)\left(\frac{e^{t-1}}{t}\right) = \frac{(e^t-1)^4}{t^4}$

Question 2. The amount of distilled water dispensed by a certain machine has a normal distribution with $\mu = 30$ ounces and standard deviation $\sigma = 3$ ounces. What container size will ensure that overflow occurs only 0.01 ~~percent~~ of the time?

$\mu = 30 \quad \sigma = 3$

(a) 25.34

(b) 36.99

(c) 16.5

(d) 34.66

(e) 51.19

$z = 3.49 \qquad z = 2.33$

$30 + 3(3.49) = 2.73$

$P(z < 2.33) = .9901$

$1 - .9901 = 0.0099$

Question 3. The scores of students taking the SAT are normally distributed. What is the probability that the score is within one standard deviation of its expected value? Choose the closest value.

(a) Can not be answered without further information

(b) 0.68

(c) 0.9451

(d) 0.3110

(e) 0.1345

Question 4. A sugar refinery has three processing plants, all of which receive raw sugar in bulk. The amount of sugar that one plant can process in one day can be modeled as having an exponential distribution with expected value equal to 5 tons for each of the three plants.

If the three plants operate independently, find the probability that exactly two of the three plants will process more than 4 tons on a given day.

$\mu = 5$ $\quad p = .20e^{-.20x}$

$\mu = \frac{1}{\lambda} = .20$

$\int_0^4 .20e^{-.20x}$

$= 1 - e^{-.20x}\Big|^4$

$1 - e^{-.8}$

$p = 0.449$

~~Bin(3, 0.449)~~

$\binom{3}{2}(0.449)^2(1-0.449)$

$= 0.3335$

(a) 0.4493

(b) 0.3335

(c) 0.7121

(d) 0.2566

(e) 0.9157

Question 5. The median age of residents of the United States is 31 years. If a survey of 400 randomly selected United States residents is taken, find the approximate probability that more than 300 of them will be under 31 years of age.

$p = .5$ $\quad E(x) = 400(.5) = 200$

$Var(x) = 100 \quad \sigma = 10$

$P(x > 300) = P(Z > \frac{300-200}{10}) = 1 - 1 = 0$

(a) approximately 0

(b) 0.5

(c) 0.471

(d) 0.8413

(e) approximately 1

Question 6. The number of times that a person contracts a cold in a give year is a Poisson random variable with parameter $\lambda = 6$. Suppose that a new wonder drug (based on large quantitites of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 3$ for 75% of the population (that is, the drug is beneficial for that 75% of the population because it reduces the average number of colds). For the other 25 percent of the population the drug has no appreciable effect on colds (that is, their λ is still 6). If an individual tries the drug for a year and has 2 colds in that time, how likely is it that the drug is beneficial for him or her?

$.75 \quad p = \frac{e^{-3}3^{x}}{3!} = 0.168$

$.25 \quad p = \frac{e^{-6}6^{x}}{3!} = 0.011154$

$\frac{P(0.168)}{0.168 + 0.01154} = 0.9377$

(a) 0.2518

(b) 0.75

(c) 0.9377

(d) 0.11

(e) 0.8886

Question 7. Let X be the time (in seconds) that Alice waits for a traffic light to turn green, and let Y be the time in seconds (at a different intersection) that Bob waits for a traffic light to turn green. Suppose that X is exponential with expected value 1/4 and Y is exponential with expected value 1/5. The two random variables are independent. What is the probability that Alice waits less than 2 seconds and Bob waits more than 1 second.

$E(X) = .25$ $E(Y) = .2$ $\lambda_1 = 4$ $\lambda_2 = 5$

(a) 0.340027

(b) 0.0067

(c) 0.32

(d) 0.5911

(e) 0.997

$P(X<2) = \int_0^2 4e^{-4x}\,dx = -e^{-4x}\Big|_0^2 = .99966$

$1 - P(Y<1) = 0.006737947$

$0.99966(0.006737947) = 0.0067$

Question 8. The variation of a certain electrical current source X (in milliamps) can be modelled by the pdf

$$f(x) = 1.25 - 0.25x, \qquad 2 \le x \le 4$$

If this current passes through a 220-Ω resistor, the resulting power (in microwatts) is given by the expression

$$g(x) = \texttt{current}^2(\texttt{resistance}) = 220X^2$$

What is the expected power?

$E(X^2) = \int x^2(1.25 - 0.25x) = \int 1.25x^2 - 0.25x^3$

$= \left(\frac{1.25}{3}x^3 - \frac{0.25}{4}x^4\right]_2^4 = 8.333$

$220(8.333) = 1833.3$

(a) 1666.668

(b) 1765.696

(c) 1833.3

(d) 2031

(e) 789.131

Question 9. Let X be a continuous random variable with the following density function.

$$f(x) = \frac{1}{2}x, \qquad 0 \le x \le 2$$

The Interquartile range is:

$F(x) = \frac{1}{4}x^2\Big|_0^x = \frac{1}{4}x^2 = .75$

$\frac{1}{4}c^2 = .75$

$c^2 = 3$ $c = \sqrt{3}$

(a) 1.1

(b) 2.821

$\frac{1}{4}x^2 = .25$

$x^2 = 1$ $x = 1$

$\sqrt{3} - 1 = .732$

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(c) 1.7320

(d) 0.67

(e) 0.7320

Question 10. Consider the volumes of soda remaining in 100 cans of soda that are nearly empty. Let $X_1, \ldots\ldots, X_{100}$ denote the volumes (in ounces) of cans one through one hundred, respectively. Suppose that the volumes X_i are independent, and that each X_i is uniformly distributed between 0 and 2. What is the expected total amount of soda in the cans? What is the standard deviation?

(a) $\mu_{\sum_{i=1}^{n} x_i} = 1$ and $\sigma_{\sum_{i=1}^{n} x_i} = 57.73$

(b) $\mu_{\sum_{i=1}^{n} x_i} = 120$ and $\sigma_{\sum_{i=1}^{n} x_i} = 10.241$

(c) $\mu_{\sum_{i=1}^{n} x_i} = 5.773$ and $\sigma_{\sum_{i=1}^{n} x_i} = 100$

(d) $\mu_{\sum_{i=1}^{n} x_i} = 100$ and $\sigma_{\sum_{i=1}^{n} x_i} = 5.773$

(e) $\mu_{\sum_{i=1}^{n} x_i} = 201$ and $\sigma_{\sum_{i=1}^{n} x_i} = 12.11$

$100\left(\frac{a+b}{2}\right) = 100\left(\frac{2}{2}\right) = 100$

$Var(x) \frac{(b-a)^2}{12} = \frac{4}{12} = \frac{1}{3}(100)^2 = 3333.33$

$E(X) = 1$ $\sigma_{100x} = 0.5773(\sqrt{100}) = 5.733$

$Var(X_1) = \frac{4}{12} = \frac{1}{3}$ $\sigma_{X_1} = 0.5773$

Question 11. Homes in three different countries, A, B, and C, have seen their values decrease as a consequence of the recession. Since the homes are in different countries, it is reasonable to assume that the values lost in these countries (random variables J, K, L) are independent. The moment generating functions for the distribution of the loss in values of the countries (random variables J, K, L, respectively) are:

$$M_J(t) = (1-2t)^{-4} \quad M_K(t) = (1-2t)^{-5} \quad M_L(t) = (1-2t)^{-3}$$

The exact distribution of J+K+L is

gamma

$\left(\frac{2}{2-t}\right)^3$

$\left(\frac{\frac{1}{2}}{\frac{1}{2}-t}\right)^3$ $\lambda = 1/2$ $\alpha = 3$

(a) Gamma with $\mu = 1/2$ and $\sigma = 12$

(b) Normal with $\mu = 2, \sigma = 6$

(c) Exponential with $\mu = 0.5, \sigma = 0.5$

(d) Uniform(2, 12)

(e) Gamma with $\mu = 24, \sigma = 6.928$

$12/\frac{1}{2} = 24$

$12/\frac{1}{4} = 48$ ≈ 6.928

$\alpha = 12$ $\lambda = \frac{1}{2}$

Question 12. A large stockpile of used pumps contains 20% that are currently unusable and need to be repaired. A repairman is sent to the stockpile with three repair kits. He selects pumps at random and tests them one at a time. If a pump works, he goes on to the next one. If a pump doesn't work, he uses one of his repair kits on it. Suppose that it takes 10 minutes to test whether a pump works, and 20 minutes to repair a pump that does not work. Find the expected value and standard deviation of the total time it takes the repairman to use up his three kits.

(a) $\mu = 450, \sigma = 42.43641$

$\frac{r}{p} = \frac{3}{\frac{1}{5}} = 15$ $Var = 60$

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(b) $\mu = 6000, \sigma = 210$

(c) $\mu = 180, \sigma = 20.784$

(d) $\mu = 1001, \sigma = 10$

(e) $\mu = 210, \sigma = 77.4596$

Question 13. A bomb is to be dropped along a 1-mile-long line that stretches across a practice target zone. The target zone's center is at the midpoint of the line. The target will be destroyed if the bomb falls within $\frac{1}{20}$ mile on either side of the center. Find the probability that the target will be destroyed, given that the bomb falls at a random location along the line.

$-\frac{1}{20}$ $\frac{1}{20}$

(a) 0.1

(b) 0.2

(c) 0.3

(d) 0.04

(e) 0.61

uniform

$\frac{1}{6-9} = \frac{1}{10}$

$\int 1 \, dy$

$\frac{11}{20} - \frac{9}{20} = \frac{2}{20} = .1$ $\quad \frac{x-9}{6-9} = \frac{x}{1}$

Question 14. A student proposes the following as a density function for a random variable Y,

$$f(y) = e^{-y}y^3, \qquad y \geq 0.$$

Which of the following is true?

(a) The expected value of this random variable is 1

(b) $f(y) = 1, y \geq 0$.

(c) The moment generating function of this random variable is $\left(\frac{1}{1-t}\right)^4$

(d) This is an exponential random variable.

(e) This is not a density function.

$\int_0^\infty e^{-y} y^3$

$y^3 \quad e^{-y}$
$3y^2 \quad -e^{-y}$
$6y \quad e^{-y}$
$6 \quad -e^{-y}$
$\quad e^{-y}$

$1 \int -y^3 e^{-y} - 3y^2 e^{-y} - 6y e^{-y} - 6e^{-y} \Big|_0^\infty$

Question 15. The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with density function

$$f(x) = 1.5(1 - x^2), \qquad 0 \leq x \leq 1$$

The Cumulative distribution function of X is

$1.5\int_0^t (1-x^2)\,dx = 1.5\left[x - \frac{1}{3}x^3\right]_0^t$

(a) $F(x) = \frac{1.5-x}{3-x}, \qquad 0 \leq x \leq 1$

(b) $F(x) = 1 - e^{-x/2}, \qquad 0 \leq x \leq 1$

$= 1.5\left(t - \frac{1}{3}t^3\right) = 1.5t - .5t^3$
$= 1.5t\left(1 - \frac{1}{3}t^2\right)$

(c) $F(x) = 1/2, \quad 0 \leq x \leq 1$

(d) $F(x) = 1.5x(1 - x^2/3), \quad 0 \leq x \leq 1$

(e) $F(x) = df(x)/dx$

NOW THAT YOU ARE DONE, MAKE SURE YOU:

(a) Have marked your name and ID in the scantron and indicated color of your exam.

(b) Have written your name and id on the first page of this exam and have signed with your signature.

(c) Have signed on the front page of the exam.

Insert cheat sheet and scantron between the pages of the exam. Close the exam and wait until we collect it from you. Remain seated and in silence until every student has turned in the exam. Do not talk or use the phone until you are out of the room.