Stat 100A -Intro Probability
Midterm 1

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Question 1. A box with 15 VLSI (Very Large Scale Integrated) chips contains five defective ones. A random sample of three chips is drawn. D= Defective W = not Defective

(a) List all the simple outcomes of the sample space. Define your notation before you do.

S = {DDD, DDW, DWD, WDD, DWW, WWD, WDW, WWW}

(b) Find the probability of each of the simple outcomes of the sample space. Show work. N = # of simple outcomes

P(simple outcome) = $\frac{1}{N}$ = $\boxed{\frac{1}{8}}$

simple outcome = any one of events listed in sample space.

−1

(c) Let Y be a random variable representing the number of components in the random sample of 3 that are defective. Write a table with three columns. The first column will contain the possible values of Y. The second the probability of each value of Y, showing how you find it, and the third will list, for each value of Y, the elements of the event corresponding to that value of Y.

Possible value of Y	Probability of value	Elements of event corresponding to value of Y
1	3/8	DWW, WWD, WDW
2	3/8	DDW, DWD, WDD
3	1/8	DDD
0	1/8	WWW

P(Y=1) = 3(P(simple outcome)) = 3(1/8) = $\frac{3}{8}$ P(Y=3) = 1(P(simple outcome)) = 1/8

P(Y=2) = 3(P(simple outcome)) = 3(1/8) = $\frac{3}{8}$ P(Y=0) = 1(P(simple outcome)) = 1/8

won't punish twic

(d) What is the probability of the event A="more than one are defective"? Show work. How would you ask the same question using the random variable Y?

−0.2

A = {DDD, DDY, DYD, YDD}

P(Y>1) = P(A) = $\frac{1}{8}+\frac{3}{8}=\boxed{\frac{1}{2}}$

Question 2. Consider the system of components connected as in the accompanying picture in Figure 1. The subsystem consisting of components 1 and 2 has components 1 and 2 connected in parallel. so that the subsystem works iff either 1 or 2 works; The subsystem consisting of components 3 and 4 has components 3 and 4 connected in series. That subsystem works iff both 3 and 4 work. Let A_i denote the event that component i works, ($i = 1, 2, 3, 4$).

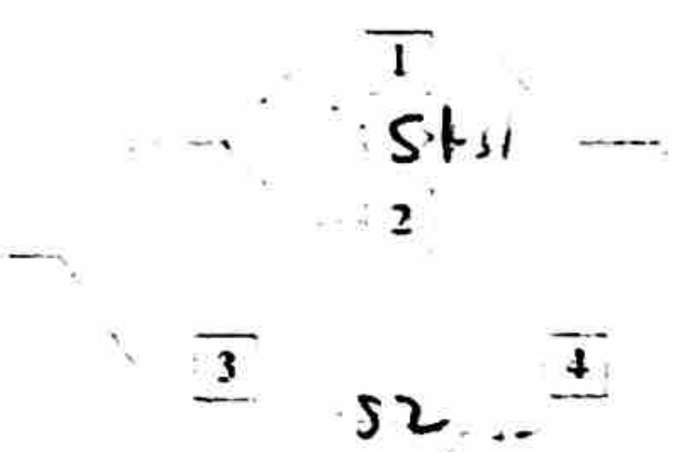


Figure 1:

(a) If components work independently of one another and the reliability of A_i, ($i = 1, 2, 3, 4$) is 0.8, calculate the reliability of the system. Show work.

$R = P(S1 \text{ or } S2 \text{ work}) = 1 - P(S1 \text{ and } S2 \text{ don't work}) = 1 - P(S1 \text{ doesn't work})\, P(S2 \text{ doesn't work})$

$= 1 - P(1 \text{ and } 2 \text{ don't work})\, P(3 \text{ and } 4 \text{ works}) = 1 - ((0.2)(0.2))(1 - (0.8)(0.8)) = \boxed{0.9856}$

(b) What does the event $(A_1 \cup A_2) \cap (A_3 \cap A_4)$ denote?

-0.8

A_1 or A_2 works and A_3 and A_4 works. It denotes the cases in which the system works, because both subsystems work.

(c) What will happen to the reliability of the system as the number of components of the series subsystem increases? Assume each additional component has the same reliability of 0.8. Show work to support your answer.

$R = 1 - P(1 \text{ and } 2 \text{ don't work})P(3 \text{ and } 4 \text{ works})$

$= 1 - (0.2)^2(1 - 0.8^n)$

$R(n) = 1 - (0.2)^2(1 - 0.8^n)$

$R(2) = 0.9856 \quad R(30) = 0.960$

$\boxed{\text{R decreases as n increases}}$

(d) The subsystem with components 1 and 2 could perhaps add more parallel components so that the reliability of this subsystem alone is at least 99%. What is the minimum number of parallel components that this subsystem should have to guarantee that? Show work.

-0.5

$R(n) = 1 - (0.2)^n(1 - 0.8^2) = 0.99$

$\frac{0.01}{(1 - 0.8^2)} = 0.2^n$

$\log_{0.2} \frac{0.01}{1 - 0.8^2} = n$

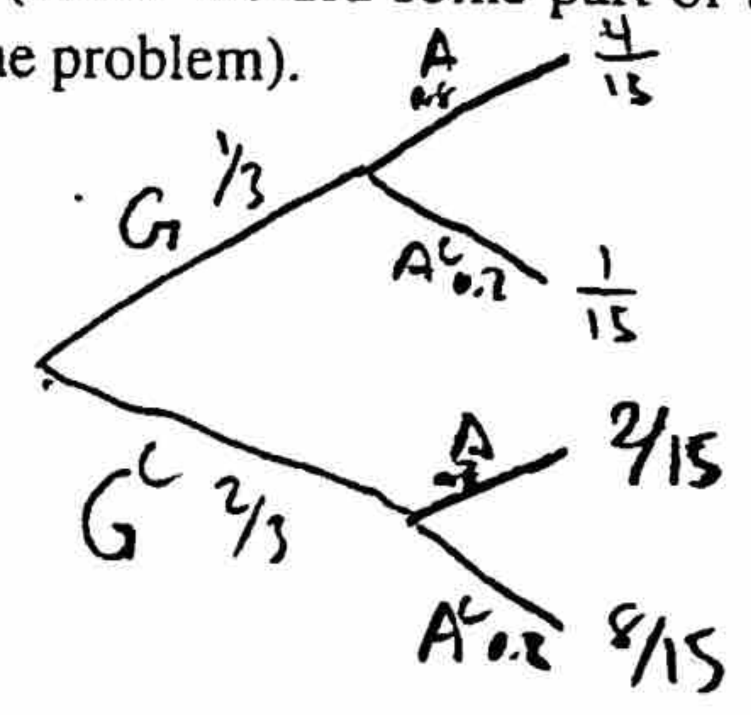
$n = 2.226 \approx \boxed{3}$

Question 3. After doing diligent work, a police detective arrested a suspect who is one of only three possible perpetrators of a jewel theft. To strengthen her case, the detective has the suspect undergo a lie-detector test. The suspect, of course, says he had nothing to do with the theft. If the test shows that the suspect is lying, the detective concludes that the subject is, indeed, guilty. However, it is known that the test is accurate only 80 percent of the time, meaning that for 20 percent of the times a suspect is telling the truth, the test will conclude that the suspect is lying, and for 20 percent of the times a suspect is lying, the test will conclude that he or she is telling the truth.

If the lie-detector test leads the detective to conclude that the subject is guilty, what is the probability that the subject is indeed guilty? Show work. Define your notation and events clearly.

(Note: we did some part of this problem in a simulation in class. Now it is time for you to use theory for the same problem).

G = Guilty
G^c = Not guilty
A = Indicates guilty
A^c = Indicates not guilty



$$P(G\mid A) = \frac{P(G\cap A)}{P(A)} = \frac{P(G\cap A)}{P(G\cap A)+P(G^c\cap A)} = \frac{4/15}{4/15+2/15} = \boxed{\frac{2}{3}}$$

Question 4. If P(E)=0.9 and P(F) = 0.8, use probability results seen in class, to show algebraically that

$$P(E \cap F) \geq 0.7$$

1.5

~~If E and F are independent, $P(E)P(F)=P(E\cap F)$~~
~~$=0.9\times 0.8$~~
~~$=0.72$~~

Prove general case.

Independent events will produce minimum intersection, so $\Rightarrow P(E\cap F) \geq \cancel{0.72} \geq 0.7$ □

Question 5. Prove algebraically, using probability results seen in class, that

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^c \cap F \cap G) - P(E \cap F^c \cap G) - P(E \cap F \cap G^c) - 2P(E \cap F \cap G)$$

$P(E\cup F\cup G) = P(E)+P(F)+P(G) - P(E\cap F) - P(E\cap G) - P(F\cap G) + P(E\cap F\cap G)$

$= P(E)+P(F)+P(G) - (P(E\cap F\cap G)+P(E\cap F\cap G^c)) - (P(E\cap F\cap G)+P(F^c\cap E\cap G)) - (P(E\cap F\cap G)+P(E^c\cap F\cap G))$
$+ P(E\cap F\cap G)$

$= P(E)+P(F)+P(G) + P(E\cap F\cap G) - 3P(E\cap F\cap G) - P(E^c\cap F\cap G) - P(E\cap F^c\cap G) - P(E\cap F\cap G^c)$

$= P(E)+P(F)+P(G) - P(E^c\cap F\cap G) - P(E\cap F^c\cap G) - P(E\cap F\cap G^c) - 2P(E\cap F\cap G)$

□

Question 6. An inspector for a pharmaceutical firm is inspecting a box containing five pills, denoted by a,b,c,d,e. Underfilled pills, that is pills with a lesser amount of medication than they should contain, pose an unwanted risk to the patient consuming them. Unknown to the inspector, pills a,b,c, contain the proper amount of medication while pills d and e are underfilled. The inspector selects two pills at random without replacement. What is the probability that at least one of the pills selected by the inspector contains the proper level of medication? Select one answer and show work.

P = prop.
W = impro.

P(at least one pill is proper) $= P(\{PW, WP, PP\}) = \left(\frac{3}{5}\cdot\frac{2}{4}\right)+\left(\frac{2}{5}\cdot\frac{3}{4}\right)+\left(\frac{3}{5}\cdot\frac{2}{4}\right)$

$= 0.9$

(a) 0.84

(b) 0.9

(c) 0.6

(d) 0.5

(e) 0.0001

Question 7. In a certain community, 40 percent of the families own a dog, and 30 percent of the families that own a dog also own a cat. In addition, 25 percent of the families in the community own a cat. What is the probability that a randomly selected family owns both a dog and a cat? Select one answer and show work.

$P(A) = 0.4 \quad P(B) = 0.25 \quad P(B|A) = 0.3$

$P(A\cap B) = P(B|A)P(A)$

$= 0.4 \times 0.3$

$= 0.12$

$P(B|A) = \frac{P(B\cap A)}{P(A)}$

(a) 0.0792

(b) 0.3

(c) 0.075

(d) 0.4

(e) 0.12