. PLACE YOUR UCLA ID ON YOUR DESK. KEEP THERE THROUGHOUT THE EXAM.

- DO NOT DETACH ANY PAGES FROM THIS EXAM. EXAM MUST STAY STAPLED DURING THE WHOLE EXAM.

Other important Instructions–Read. Points lost for not following directions.

- SILENCE AT ALL TIMES IN THE EXAM ROOM. Wait until you are out to talk and access your phones and belongings in the backpack.
- Closed books, closed notes.
- Only scientific calculator allowed for computations. NO GRAPHICS CALCULATORS ALLOWED. You may not use your phone or any other electronic device as calculator. Graphics calculators are not allowed. No exceptions. You get 0 points in the exam.
- Phones and other electronic devices must be disconnected before you enter the classroom and not turned on again until you are out of the room. While in the classroom, they must be in your backpack and your backpack on the floor. Phones in pockets will lead to 0 points in the exam. It is not worth the risk.
- Left-handed students will sit in a seat for left-handed students. The professor will tell students where to sit. Please, let the professor know that you are left handed ahead of time and she will move you.
- ID must be ready to show BEFORE and at all times during the exam. NO ID, no exam.
- This midterm must show your individual work. Talking to others during the midterm, not adhering to the above, sharing information or breaking any other aspect of the student code of conduct at UCLA will not be tolerated. You can not exchange papers or information. All your things must be on the floor. You may not use the empty seats next to you to put things. Close the tables. Honor code applies.
- No cheat sheet allowed in this exam.
- Failure to follow instructions given here will result in loss of points in the exam in a first warning and more severe consequences on a second warning. Honor code applies. Familiarize yourself with student code of conduct by visiting the links provided in the course syllabus.

In all that follows, you must define your events and give notation for them when not given in the problem, you must indicate the event and the probabilities you are computing, must show work and provide the final answer in decimal form, without rounding.

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Question 1. Observers noted that 40 percent of the vehicles crossing a certain toll bridge are commercial trucks. Four vehicles will cross the bridge in the next minute. Determine the probability that more than 2 of the vehicles are commercial trucks. Select one answer and show work. T : # of commercial trucks

(a) 0.25

(b) 0.1792

(c) 0.561

(d) 0.6115

(e) 0.84

$$P(T>2) = 1-(P(T=0)+P(T=1)+P(T=2))$$

$$= 1 - 0.6^4 - 4(0.6)^3(0.4) - 6(0.6)^2(0.4)^2$$

$$\frac{4!}{2!2!}$$

$$\approx 0.1792$$

Question 2. A survey of 500 students taking one or more courses in algebra, physics and statistics during one semester revealed the following numbers of students in the indicated subjects.

Algebra 329; physics 186; statistics 295; algebra and physics 83; algebra and statistics 217; physics and statistics 63.

A P S

How many students were taking all three subjects? Select one answer and show algebraic work.

(a) 25

(b) 6

(c) 15

(d) 9

(e) 53

$$P(A\cup P\cup S) = P(A)+P(P)+P(S) - P(AP) - P(PS) - P(AS) + P(APS)$$

$$P(APS) = -(P(A)+P(P)+P(S)-P(AP)-P(PS)-P(AS)) + P(A\cup P\cup S)$$

$$P(APS) = 500 - 329 - 186 - 295 + 83 + 217 + 63 = \boxed{53}$$

Question 3. Prove algebraically, using probability results seen in class, that

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^c \cap F \cap G) - P(E \cap F^c \cap G) - P(E \cap F \cap G^c) - 2P(E \cap F \cap G)$$

$$P(E\cup F\cup G) = P(E) + P(F) + P(G) - P(EF) - P(FG) - P(EG) + P(EFG)$$

distribute P(EFG)

$$= P(E) + P(F) + P(G) - P(EF) + P(EFG) - P(FG) + P(EFG) - \cancel{P(EG)} + P(EFG) - 2P(EFG)$$

?

$$= P(E) + P(F) + P(G) - \cancel{P(E\cap F\cap G^c) - P(E^c\cap F\cap G) - P(E\cap F^c\cap G) - 2P(EFG)}$$

reorder

$$P(E\cup F\cup G) = P(E) + P(F) + P(G) - P(E^c\cap F\cap G) - P(E\cap F^c\cap G) - P(E\cap F\cap G^c) - 2P(E\cap F\cap G)$$

QED

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Question 4. After doing diligent work, a police detective arrested a suspect who is one of only three possible perpetrators of a jewel theft. To strengthen her case, the detective has the suspect undergo a lie-detector test. The suspect, of course, says he had nothing to do with the theft. If the test shows that the suspect is lying, the detective concludes that the subject is, indeed, guilty. However, it is known that the test is accurate only 80 percent of the time, meaning that for 20 percent of the times a suspect is telling the truth, the test will conclude that the suspect is lying, and for 20 percent of the times a suspect is lying, the test will conclude that he or she is telling the truth.

If the lie-detector test leads the detective to conclude that the subject is guilty, what is the probability that the subject is indeed guilty? Show work. Define your notation and events clearly.

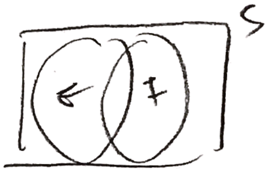
(Note: we did some part of this problem in a simulation in class. Now it is time for you to use theory for the same problem). G = subject is actually guilty   D = detect as guilty

$$P(G|D) = \frac{P(D|G)P(G)}{P(D|G)P(G) + P(D|G^c)P(G^c)}$$

$$= \frac{0.8 \cdot \frac{1}{3}}{0.8 \cdot \frac{1}{3} + 0.2 \cdot \frac{2}{3}} = \frac{0.266}{0.266 + 0.133} \approx \boxed{0.667}$$

Question 5. If P(E)=0.9 and P(F) = 0.8, use probability results seen in class, to show algebraically that

$$P(E \cap F) \geq 0.7$$



$P(E \cap F) = P(E) + P(F) - P(E \cup F)$

$P(E \cup F)$ at max can only be 1.
axiom 2: full sample space $S = 1$
$P(E \cup F)$: If they fill all of S, max is 1

$P(E \cap F) \geq P(E) + P(F) - P(S)$

$\geq 0.9 + 0.8 - 1$

$P(E \cap F) \geq 1.7 - 1$

$\boxed{P(E \cap F) \geq 0.7}$ ☒

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Question 6. A box with 15 VLSI (Very Large Scale Integrated) chips contains five defective ones. A random sample of three chips is drawn.

(a) List all the simple outcomes of the sample space. Define your notation before you do.

D= defective chips

sample space $S = \{DDD, DDD^c, DD^cD, DD^cD^c, D^cDD, D^cDD^c, D^cD^cD, D^cD^cD^c$

(b) Find the probability of each of the simple outcomes of the sample space. Show work.

$P(DDD) = \frac{5}{15}\cdot\frac{4}{14}\cdot\frac{3}{13} \approx \boxed{0.022}$

$P(DD^cD^c) = P(D^cD^cD) = P(D^cDD^c) = \frac{10}{15}\cdot\frac{9}{14}\cdot\frac{5}{13} \approx \boxed{0.165}$

$P(DDD^c) = P(D^cDD) = P(DD^cD) = \frac{10}{15}\cdot\frac{5}{14}\cdot\frac{4}{13} \approx \boxed{0.073}$

$P(D^cD^cD^c) = \frac{10}{15}\cdot\frac{9}{14}\cdot\frac{8}{13} \approx \boxed{0.264}$

(c) Let Y be a random variable representing the number of components in the random sample of 3 that are defective. Write a table with three columns. The first column will contain the possible values of Y. The second the probability of each value of Y, showing how you find it, and the third will list, for each value of Y, the elements of the event corresponding to that value of Y.

Y	P(Y)	{#D = Y}
0	0.264	$D^cD^cD^c$
1	0.495	$DD^cD^c, D^cDD^c, D^cD^cD$
2	0.219	DDD^c, DD^cD, D^cDD
3	0.022	DDD

(from part b above)
3(0.495) from part b
3(0.073) from part b
(gotten from part b)

(d) What is the probability of the event A="more than one are defective"? Show work. How would you ask the same question using the random variable Y? $\boxed{P(Y>1)\text{, what is the probability of } Y>1\text{?}}$

$P(A) = P(Y=2) + P(Y=3) = 0.219 + 0.022 = \boxed{0.241}$

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Question 7. Consider the system of components connected as in the accompanying picture in Figure 1. The subsystem consisting of components 1 and 2 has components 1 and 2 connected in parallel, so that the subsystem works iff either 1 or 2 works; The subsystem consisting of components 3 and 4 has components 3 and 4 connected in series. That subsystem works iff both 3 and 4 work. Let A_i denote the event that component i works, $(i = 1, 2, 3, 4)$.

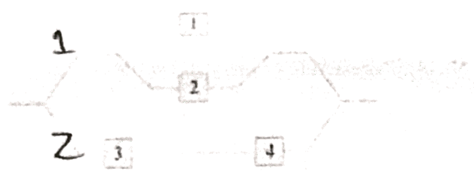

Figure 1:

S1 = subsystem 1 works S2 = subsystem 2 works

(a) If components work independently of one another and the reliability of A_i, $(i = 1, 2, 3, 4)$ is 0.8, calculate the reliability of the system. Show work.

$$\text{Reliability} = P(\text{working}) = 1 - P(S1^c) \cdot P(S2^c)$$
$$= 1 - (1 - P(S1))(1 - P(S2))$$
$$= 1 - (1 - (1 - P(A_1^c)P(A_2^c)))(1 - P(A_3)P(A_4))$$
$$= 1 - (1 - (1 - 0.2^2))(1 - 0.8^2)$$
$$= 1 - 0.04 \cdot 0.36 = \boxed{0.9856}$$

(b) What does the event $(A_1 \cup A_2) \cap (A_3 \cap A_4)$ denote?

Event that either component 1 or 2 is working AND both components 3 and 4 are working. i.e. $\boxed{\text{Event of both subsystems working}}$

(c) What will happen to the reliability of the system as the number of components of the series subsystem increases? Assume each additional component has the same reliability of 0.8. Show work to support your answer.

$\boxed{\text{reliability decreases}}$

$$\text{reliability} = P(\text{working}) = 1 - P(S1^c) \cdot P(S2^c)$$
$$= 1 - (0.04)(1 - 0.8^n)$$
$$= 0.96 + (0.04)(0.8)^n$$

As n increases, it decreases from 0.9856 to approach 0.96 reliability

(d) The subsystem with components 1 and 2 could perhaps add more parallel components so that the reliability of this subsystem alone is at least 99%. What is the minimum number of parallel components that this subsystem should have to guarantee that? Show work.

$$P(S1) = 1 - P(S1^c)$$
$$= 1 - 0.2^n > 0.99$$

$0.2^n < 0.01$

$\log_{0.2} 0.01 \approx 3$

$n = 3$

$\boxed{n = 3}$