21F-STATS100A-3 MIDTERM, Must submit before 7 AM on 11/1/2021

LEONARD CHEN

TOTAL POINTS

37 / 40

QUESTION 1

1 Front page with signature 3 / 3

✓ - **0 pts** Correct

- **1 pts** You must enter the url for UCLA student conduct code
- **1 pts** First sentence missing
- **3 pts** No first page turned in

QUESTION 2

2 MC Q 1-5 5 / 5

✓ - **0 pts** Correct

- **1 pts** MC1
- **1 pts** MC2
- **1 pts** MC3
- **1 pts** MC4
- **1 pts** MC5

QUESTION 3

3 MC Q 6-10 5 / 5

✓ - **0 pts** Correct

- **1 pts** MC6
- **1 pts** MC7
- **1 pts** MC8
- **1 pts** MC9
- **1 pts** MC10

QUESTION 4

4 MC Q 11-15 5 / 5

✓ - **0 pts** Correct

- **1 pts** MC11
- **1 pts** MC12
- **1 pts** MC13
- **1 pts** Mc14
- **1 pts** Mc15

QUESTION 5

5 MC Q 16-20 5 / 5

✓ - **0 pts** Correct

- **1 pts** MC16
- **1 pts** MC17
- **1 pts** MC18
- **1 pts** MC19
- **1 pts** MC20

QUESTION 6

6 Question 21 1.5 / 3

- **0 pts** Correct

✓ - **0.5 pts** **Clearly define the random variable**

✓ - **0.5 pts** **Check conditions for normal approximation on mean**

✓ - **0.5 pts** **Check conditions for normal approximation: independence and constant probability**

- **0.5 pts** Mention normal approximation for binomial distribution
- **0.5 pts** Show work to calculate the parameters of normal distribution
- **0.5 pts** Calculate the final probability using normal approximation
- **3 pts** Missing

QUESTION 7

7 Question 22 3 / 3

✓ - **0 pts** Correct

- **0.5 pts** Clearly define the random variable as Bernoulli(p)
- **0.5 pts** Write expectation of bernoulli
- **0.5 pts** Write variance of bernoulli
- **0.5 pts** Correctly recall formula Var[X] = E[X^2] -

E[X]^2

- **- 1 pts** Show correct work for final computation
- **- 3 pts** Missing

QUESTION 8

8 Q 23 **2.5 / 3**

- **- 0 pts** Correct
- **- 0.5 pts** Clearly define the random variable of interest
- ✓ **- 0.5 pts Assume independence of trials**
- **- 0.5 pts** Conclude that R.V follows geometric distribution
- **- 0.5 pts** Recall expectation of geometric r.v. correctly
- **- 1 pts** Carry out the final computation with the given information
- **- 3 pts** Missing

QUESTION 9

9 Question 24 **5 / 6**

- **- 0 pts** Correct
- **- 1 pts** Sample space
- **- 1 pts** PMF table has incorrect probabilities
- **- 1 pts** Calculating expected value of distance
- ✓ **- 1 pts Calculating expectation of squared distance**
- **- 1 pts** PMF table has incorrect events
- **- 1 pts** PMF table has incorrect values of the distance
- **- 2 pts** PMF table has significant deficiency
- **- 3 pts** Missing

QUESTION 10

10 Question 25 **2 / 2**

- ✓ **- 0 pts Correct**
- **- 0.4 pts** Clearly define the random variable
- **- 0.4 pts** Deduce that random variable is an exponential random variable with parameter 15.
- **- 1.2 pts** Show work for the final calculation
- **- 3 pts** Missing

Chen Leonard 005583342

I, Leonard Chen sign to confirm that this exam reflects my work and only my work, that I have not consulted with anyone or anything except the class material posted in CCLE, the textbook, and Cognella active learning and that I have taken the time specified in the instructions or very close to that time to complete the exam from the moment that I first looked at it until it was in Gradescope. I also confirm that I have adhered to the UCLA student conduct code https://deanofstudents.ucla.edu/student-conduct-code and that I have and will not share this exam with anything or anyone.

1 Front page with signature 3 / 3

- ✓ **- 0 pts** Correct
- **- 1 pts** You must enter the url for UCLA student conduct code
- **- 1 pts** First sentence missing
- **- 3 pts** No first page turned in

Question(Q)	Answer	Q	Answer	Q	Answer	Q	Answer
Q 1	B	Q 6	C	Q 11	B	Q 16	A
Q 2	D	Q 7	A	Q 12	C	Q 17	D
Q 3	A	Q 8	D	Q 13	B	Q 18	E
Q 4	C	Q 9	C	Q 14	A	Q 19	C
Q 5	C	Q 10	D	Q 15	A	Q 20	A

2 MC Q 1-5 **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** MC1
- **- 1 pts** MC2
- **- 1 pts** MC3
- **- 1 pts** MC4
- **- 1 pts** MC5

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Q 4	C	Q 9	C	Q 14	A	Q 19	C
Q 5	C	Q 10	D	Q 15	A	Q 20	A

3 MC Q 6-10 **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** MC6
- **- 1 pts** MC7
- **- 1 pts** MC8
- **- 1 pts** MC9
- **- 1 pts** MC10

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4 MC Q 11-15 5 / 5

✓ **- 0 pts** Correct

- 1 pts MC11

- 1 pts MC12

- 1 pts MC13

- 1 pts Mc14

- 1 pts Mc15

Question(Q)	Answer	Q	Answer	Q	Answer	Q	Answer
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5 MC Q 16-20 **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** MC16
- **- 1 pts** MC17
- **- 1 pts** MC18
- **- 1 pts** MC19
- **- 1 pts** MC20

21. The student selects their songs at random, so we can say each choice is with replacement and independent from the last. Thus, using the binomial Random Variable, and considering a classic rock song as a "success", our sample population is 100, the chance of being successful is 0.15, and we want 71 successes.

Thus, $P(71 \text{ classic rock songs}) = \binom{n}{x} p^x (1-p)^{n-k}$

$$= \binom{100}{71} 0.15^{71}\ 0.85^{29}$$

$$\approx 3.5435 \cdot 10^{-36}$$

As we can see this probability is absurdly small, and $P(72), P(73) \ldots$ will only be smaller. Thus, we can say, $P(> 70 \text{ classic rock songs}) \approx 0$.

We can also use the normal approximation of the binomial, as n is high and p is not too low. We would have an $\mu = np = 100 \cdot 0.15 = 15$, and $\sigma^2 = np(1-p) = 12.75$, $\sigma = 3.571$.

Using the Normal Applet, the probability that $X > 70$ is 0, which makes sense, as 70 is about 15 standard deviations from the mean. Again, we see that $P(> 70 \text{ classic rock songs}) \approx 0$.

6 Question 21 **1.5 / 3**

- **0 pts** Correct
- ✓ **- 0.5 pts** Clearly define the random variable
- ✓ **- 0.5 pts** Check conditions for normal approximation on mean
- ✓ **- 0.5 pts** Check conditions for normal approximation: independence and constant probability
- **- 0.5 pts** Mention normal approximation for binomial distribution
- **- 0.5 pts** Show work to calculate the parameters of normal distribution
- **- 0.5 pts** Calculate the final probability using normal approximation
- **- 3 pts** Missing

22. Let us call the probability that Netflix sub status is 1 p, and thus, the probability that Netflix sub status is 0 is 1-p, as 0,1 make up the whole set. 4

Thus, $P(1) = p$, $P(0) = 1-p$

We know that $E(X^2) = \sum_x x^2 P(X=x)$, so in our case,

$$E(X^2) = 1^2 \cdot P(1) + 0^2 \cdot P(0)$$
$$= 1 \cdot p + 0 \cdot (1-p)$$
$$= p$$

7 Question 22 **3 / 3**

- ✓ **- 0 pts** Correct
- **- 0.5 pts** Clearly define the random variable as Bernoulli(p)
- **- 0.5 pts** Write expectation of bernoulli
- **- 0.5 pts** Write variance of bernoulli
- **- 0.5 pts** Correctly recall formula Var[X] = E[X^2] - E[X]^2
- **- 1 pts** Show correct work for final computation
- **- 3 pts** Missing

23. The data scientist is following a geometric pattern by leaving / stopping his observation after seeing a "success". Thus, we can use a geometric random variable to reverse engineer the probability of winning the game.

Per the scientists observations, the expected value $E(Y)$ of this geometric random variable is 10. We know $E(Y) = 1/p$ in a geometric random variable model, thus $p = 1/10$.

So, he should publish that there is a 10% chance to hit the 100 mark.

$P(\text{hit } 100) = 0.10$.

8 Q 23 **2.5 / 3**

- **- 0 pts** Correct
- **- 0.5 pts** Clearly define the random variable of interest
- ✓ **- 0.5 pts** **Assume independence of trials**
- **- 0.5 pts** Conclude that R.V follows geometric distribution
- **- 0.5 pts** Recall expectation of geometric r.v. correctly
- **- 1 pts** Carry out the final computation with the given information
- **- 3 pts** Missing

24. We denote a left jump as a L, and right as a R. So, LLR would refer to 2 left jumps then a right jump. We have that $P(R) = 2/5$, $P(L) = 3/5$.

So, S consists of: $S: \{LLL, LLR, LRL, RLL, LRR, RLR, RRL, RRR\}$

We also have some macrostates, depending on end position: Its sample space is $S' = \{-3, -1, 1, 3\}$.
Due to the Jumps being independent and S being disjoint, we can product rule and summation rule to find probabilities for S and S'.

$P(A \cup B) = P(A)P(B)$ if A, B independent

$P(LLL) = P(L)^3 = 0.216$ $\quad P(LLR) = P(LRL) = P(RRL) = P(L)^2 P(R) = 0.144$

$P(RRR) = P(R)^3 = 0.064$ $\quad P(RRL) = P(RLR) = P(LRR) = P(L)\,P(R)^2 = 0.096$

$P(-3) = P(LLL) = 0.216$ $\quad P(-1) = P(LLR) + P(LRL) + P(RRL) = 3 \cdot P(L)^2 P(R) = 0.432$

$P(3) = P(RRR) = 0.064$ $\quad P(1) = P(RRL) + P(RLR) + P(LRR) = 3 \cdot P(L) \cdot P(R)^2 = 0.288$

$P(A \cup B) = P(A) + P(B)$ if A, B disjoint

Thus:

End Distance	Jumps	Probability
-3	$\{LLL\}$	$P(-3) = \left(\frac{3}{5}\right)^3 = 0.216$
-1	$\{LLR, LRL, RLL\}$	$P(-1) = 3 \cdot \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right) = 0.432$
1	$\{LRR, RLR, RRL\}$	$P(1) = 3 \cdot \left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^2 = 0.288$
3	$\{RRR\}$	$P(3) = \left(\frac{2}{5}\right)^3 = 0.064$

Thus, $E(\text{end Distance}) = \sum_x x \cdot P(X = x)$

$$= -3 \cdot P(-3) - 1 \cdot P(-1) + 1 \cdot P(1) + 3 \cdot P(3)$$

$$= -3 \cdot 0.216 - 1 \cdot 0.432 + 1 \cdot 0.288 + 3 \cdot 0.064$$

$$= -0.6 \text{ inches}$$

9 Question 24 **5 / 6**

- **- 0 pts** Correct
- **- 1 pts** Sample space
- **- 1 pts** PMF table has incorrect probabilities
- **- 1 pts** Calculating expected value of distance
- ✓ **- 1 pts** **Calculating expectation of squared distance**
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25. The probability density function for an exponentially distributed function is $f(x) = \lambda e^{-\lambda x}$, where $\lambda = 1/u$.

So, in our case $\lambda = 1/15$, as we expect to wait 15 minutes.

We want the probability $f(x) \geq 5$.

So, we can take the integral of the probability density function from 5 to inf to get the probability $f(x) \geq 5$, or $F(5)$

$$\int_5^\infty \frac{1}{15} e^{-\frac{1}{15}x} \, dx$$

$$= -e^{-\frac{1}{15}x} \Big|_5^\infty$$

$$= -e^{-\infty} + e^{-1/3} = e^{-1/3} = 0.71653$$

10 Question 25 **2 / 2**

- ✓ **- 0 pts** Correct
- **- 0.4 pts** Clearly define the random variable
- **- 0.4 pts** Deduce that random variable is an exponential random variable with parameter 15.
- **- 1.2 pts** Show work for the final calculation
- **- 3 pts** Missing