Stat 100A -Intro Probability
Midterm 1

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PLEASE DO THE FOLLOWING BEFORE YOU START IF YOU FORGOT TO DO IT BEFORE YOU CAME IN.

- PLACE ALL YOUR ELECTRONIC DEVICES (PHONE, IPAD, IWATCHES, ETC) INSIDE YOUR BACKPACK AND PLACE YOUR BACKPACK UNDER THE SEAT (UNDER THE SEAT, NOT IN FRONT OF YOUR FEET). NO TALKING OR ACCESS TO ELECTRONIC DEVICES UNTIL OUT OF THE ROOM, EVEN IF YOU DO NOT HAVE THE EXAM OR HAVE RETURNED THE EXAM.
- WRITE YOUR NAME and ID, AND BUBBLE THEM, ON THE SCANTRON. USE NO.2 PENCIL.
- Write the COLOR of your exam in pencil on the top part of the scantron.
- Write your name and ID on the top right hand corner of your cheat sheet.
- Write your name and ID on this page.
- Have your UCLA ID on your desk throughout the whole exam. ID must have your name as it appears in the class roster.
- Put your backpack under your seat.
- Put down the tables next to you, unless the instructor indicates otherwise.
- Honor code applies. The exam must reflect your own work. Looking at other exams, talking or sharing anything with others during the exam will result in an F in the class and a trip to the Dean of students office.
- You must sit in the seat indicated by the professor.

DO NOT DETACH ANY PAGES FROM THIS EXAM. ALL PAGES OF THE EXAM MUST STAY STAPLED THE WHOLE TIME

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MULTIPLE CHOICE QUESTIONS. (ONLY THE SCANTRON WILL BE GRADED). ONLY ONE ANSWER IS CORRECT. CHOICE MUST BE MARKED ON THE SCANTRON, AND ALSO HERE ON THE EXAM. NO MARKS ON SCANTRON OR MORE THAN ONE MARK WILL RESULT IN 0 POINTS FOR THE QUESTION NOT MARKED, EVEN IF IT IS MARKED ON THE EXAM. You may use the space near the question , and other pages of this exam marked as "may use for scratch work" for scratch work, but scratch work will not be read. Do not tear off any page.

Question 1. If you roll two fair six sided dice and you had to bet on a sum of 8 or 9, which one would you choose (assuming you prefer the strategy that gives you the highest chance of winning)?

{"8"} = (2,6), (3,5), (4,4), (5,3) (6,2)

(a) 9 because the probability of 9 is 0.11111 and the probability of 8 is 0.08333

(b) 8 because the probability of 9 is 0.0555 and the probability of 8 is 0.1111 {"9"} = (3,6), (4,5), (5,4), (6,3)

(c) Both 8 and 9 have the same probability so would not choose any

(d) 8 because the probability of 8 is 0.13888 and the probability of 9 is 0.11111

Question 2. As an international student coming to the United States, there are three different student visas that a foreign student could be issued: F1 visa, J1 Visa or M1 visa. Let's say that 80% of the visas issued to students are F1, 15% are J1 and 5% are M1. Suppose an experiment consists of drawing three students at random from the very large population of foreign students to observe their visa status. The number of possible outcomes in the sample space of this experiment is

(A) = .8
(B) = .15
(C) = .05

F1 = A, J1 = B, M1 = C

(a) The same as the number of samples (or 3-tuples) that one could draw from an urn containing three balls, one for F1, other for J1 and other for M1, with replacement

(b) The same as the number of samples (or 3-tuples) that one could draw from an urn containing three balls, one for F1, other for J1 and other for M1, without replacement.

Question 3. If prior to rolling two six sided dice your model for the experiment consisting of rolling just one dice is as given in table 1, what is the probability that you will assign to obtaining a sum of 4?

{"4"} = (1,3), (2,2), (3,1)

dice number	1	2	3	4	5	6
probability.	0.4.	0.2	0.1	0.1	0.1	0.1

Table 1: Model for the roll of one dice

(.4)(.1) + (.2)(.2) + (.4)(.1)

.04 + .04 + .04 = .12

(a) 0.9

(b) 0.0833

(c) 0.12

(d) 0.535

Question 4. Let A, B and C be events in the sample space, $P(A) = 0.62$, $P(B) = 0.44$, $P(C) = 0.48$, $P(AB) = 0.2$, $P(AC) = 0.32$, $P(BC) = 0.18$, $P(ABC) = 0.04$. Calculate the following probability:

$P(A \cup C) = P(A) + P(C) - P(AC)$

$$P(AC \cup BC)$$

$P(AC \cup BC) = P(AC) + P(BC) - P(AC \cap BC)$

$= P(AC) + P(BC) - P(ABC)$

$= .32 + .18 - .04 = .46$

(a) 0.04

(b) 0.5

(c) 0.32

(d) 0.46

* **Question 5.** A mail spam filter is not perfect. When a spam email arrives, 99.7% of the time the spam is detected and prevented from entering your Inbox. For emails that are not spam, 95% of the time the filter correctly thinks they are not spam and the emails go to your Inbox. It is estimated that 4% of the emails are spam. What proportion of emails that are declared spam by the filter are really spam?

A = spam, $P(A) = .04$, B = not spam, $P(B) = .96$

C = correct, D = incorrect

$P(A|C)$?

$P(C|A) = .997$

$P(C|B) = .95$

$P(A|C)\,P(C) = P(C|A)\,P(A)$

$P(A|C) = \dfrac{P(C|A)P(A)}{P(C)}$

$P(A|C) = \dfrac{(.997)(.04)}{(.997)(.04) + (.95)(.96)}$

$P(C) = P(A \cap C) + P(B \cap C) = P(C|A)P(A) + P(C|B)P(B) = (.997)(.95) + (.95)(.96) = .0471 + .912$

.952

(a) 0.4538

(b) 0.547

(c) 0.31

(d) 0.9326

Question 6. Suppose that key elections for the position of president of a country are held. There are three voting districts, I,II, III. The winner of the election has to have won the majority of votes in 2 of the three voting districts. Assume that there are two candidates, A, B for the position. Define events W='A wins the election' and T="The first two districts vote for the same candidate.'

In the event $(W \cup T)^c$, who wins the election?

win in 2/3

$W = \{AAB, ABA, BAA\}$

$T = \{AAB, BBA\}$

$W \cup T = \{AAB, ABA, BAA, BBA\}$

(a) A

(b) B

(c) neither, that set is empty

$P(C) = P(A \cap C) + P(B \cap C) = (.997)(.04) + (.05)(.96)$

$= .04 + .05 = \frac{.04}{.09}$

(d) A or B win.

Question 7. As genetic theory shows, there is very close to an even chance that both children in a two-child family will be of the same sex. Here are two scenarios.

(i) 15 couples have two children each. In 10 or more of these families, it will turn out that both children are of the same sex.

(ii) 30 couples have two children each. In 20 or more of these families, it will turn out that both children are of the same sex.

According to the laws of probability, which possibility is more likely to be observed?

(a) (i) is more likely to be observed than (ii)

(b) (ii) is more likely to be observed.

15 — 10 same sex, 5 ~~not~~ diff

30 — 20, 10

Question 8. Consider a population where the probability of having HIV is 0.0000143. The probability that a person with HIV tests positive in this population is 0.993. What is the probability that a person randomly chosen from this population tests positive for HIV and actually has HIV?

(a) 0.993

(b) 0.0000141999

(c) 0.5681

(d) 0.3439

$A = HIV$, $P(A) = .0000143$

T = tests positive

$P(T|A) = .993$

$P(T|A) = \frac{P(A \cap T)}{P(A)} \rightarrow P(A \cap T) = P(T|A)P(A)$

$= (.993)(.0000143)$

Question 9. A service station has both self-service (A) and full service (B) islands. On each island, there is a single regular unleaded pump with two hoses. We observe the number of hoses being used on the self-service island at a particular time, and the number of hoses being used on the full-service island in use at that time. The joint probabilities appear in table 2.

What is the probability that in two random visits that you pay during a week, at each visit you see 1 hose in use in the self service island and 2 in the full service island?

(a) 0.588

(b) 0.40

(c) 0.12

(d) 0.0036

$\underbrace{\text{A}}$ hose, $\underbrace{\text{B}}$ hose

$P(A_1 \cap B_2) = .06$

$(P(A_1 \cap B_2))^2 = .0036$

		Number of full service hoses in use		
		0	1	2
Number of self-service hoses in use	0	0.10	0.04	0.02
	1	0.08	0.20	0.06
	2	0.06	0.14	0.30

Table 2: Self-service and full service pumps use $P(A_1 \cap B_2) = .06$

Question 10. A diagnostic test for the presence of a disease has two possible outcomes: 1 when the test says that disease is present and 0 when the test says that disease is not present. On the other hand, a patient could actually have the disease (s) or not have the disease (f). We select a patient at random from the large population. The joint probabilities for the possible outcomes of this experiment are given in the following table.

1 = disease present in test
0 = disease not present in test

$$
\begin{aligned}
P(s, 1) &= 0.800 \\
P(f, 1) &= 0.050 \\
P(s, 0) &= 0.025 \\
P(f, 0) &= 0.125
\end{aligned}
$$

s = has disease
f = no disease

What is the probability that someone for which the test is positive actually has the disease?

$P(s|1) = \frac{P(s \cap 1)}{P(1)}$

(a) 0.05882353

(b) 0.1311

(c) 0.9411765

(d) 0.71

$P(1) = P(s \cap 1) + P(f \cap 1)$

$P(1) = (.8) + (.05) = .85$

$P(s|1) = \frac{.8}{.85} = .94$

Question 11. A robotic tool is given a 1-year guarantee after purchase. The tool contains four components, each of which has a probability of 0.01 of failing during the warranty period. The tool fails if any one of the components fails. Assuming that the components fail independently, what is the probability that the tool will fail during the warranty period?

$P(1 \text{ works}) = .99$

$P(\text{fails}) = 1 - P(\text{all work})$

$= 1 - (.99)^4 = .0394$

(a) 0.96059

(b) 0.03940

(c) 0.88234

(d) 0.19001

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Question 12. Sunshine Studios' audition office has told actors to be to prepare excerpts from 10 scripts telling them that the day of the audition they will be asked a random selection of 5 of them. A hopeful actor has practiced only 7 of the scripts. What is the probability that the day of the audition, the hopeful actor will be able to do 4 of the scripts?

(a) 0.4166667

(b) 0.10

(c) 0.71201

(d) 0.54209

$$\frac{\binom{7}{4}\binom{3}{1}}{\binom{10}{5}} = \frac{\left(\frac{7!}{3!\,4!}\right)\left(\frac{3!}{2!\,1!}\right)}{\left(\frac{10!}{5!\,5!}\right)}$$

$$= \frac{(35)(3)}{252} = .4167$$

Question 14. This question is a proof you must do. You will use the axioms of probability to prove:
"Let A and B be two nonempty sets contained in a sample space S. None of these events is S. If event A is contained in event B, then $P(A) < P(B)$"
Please, mention at each step which axiom you are using in your proof.

According to Kolmogorov's Axiom, $P(S) = 1$

so $P(A)$ and $P(B) \leq 1$ since A and B are contained in S

According to the next Axiom, $P(A)$ and $P(B)$ must have a probability between 0 and 1

However, following from the Axiom which states $P(A)$ and $P(B) \leq P(S) = 1$ since A and B are contained in S,

It can be argued that $P(A) < P(B)$ since the entire event A is contained in the event B.

-1.8

0 8/3

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PART II. WORK MUST BE SHOWN IN THE FOLLOWING QUESTIONS TO GET FULL CREDIT. The work is 80% of the score for the question.

Question 13. A lie detector test is accurate 70% of the time, meaning that for 30% of the time a suspect is telling the truth, the test will conclude that the suspect is lying, and for 30% of the time a suspect is lying the test will conclude that the suspect is telling the truth. A police detective arrested a suspect who is one of only four possible perpetrators of a jewel theft. If the lie detector test result is positive (i.e. concludes the suspect is lying), what is the probability that the arrested suspect is actually guilty? Show work.

A = accurate, T = truth, L = lie, G = Guilty

B = inaccurate, B

$P(G) = (\frac{1}{4}) = .25$

$P(A) = .7$ $\quad P(A \mid L) = .70$ $\quad P(B \mid L) = .30$

$P(B \mid T) = .30$

$P(A \mid T) = .70$ X

$P(A) = P(A \mid L)P(L) + P(A \mid T)P(T) = (.7)(.25) + (.7)(.75) =$ $\quad P(A \cap L) = \frac{P(A \mid L)\,P(L)}{P(A)}$

$P(G) = P(L \mid A)$

$P(G) = \frac{P(A \mid L)\,P(L)}{P(A)}$

$A \cap L + A \cap T$

$\frac{(.25)(.7)}{(.25)(.7) + (.7)(.75)} = \frac{(.25)(.7)}{(.25)(.7) + (.75)(.7)} = \frac{.175}{.175 + .525}$

$\boxed{P(G) = \frac{.175}{.7} = .25}$

−0.4

See question on next page