EVERYTHING, EXCEPT THE RED LETTERS (RED WHEN VIEWED ONLINE)) IN THIS BOX MUST APPEAR ON A FRONT, PAGE 1, OF YOUR SUBMITTED EXAM. YOU MAY WRITE EVERYTHING BY HAND ON PAGE 1 OF YOUR EXAM, EXACTLY AS IT LOOKS LIKE HERE. ONLY WHAT IS IN THIS BOX MUST BE ON YOUR PAGE 1 OF YOUR EXAM. YOU MAY ALSO PRINT, COMPLETE AND SCAN THIS PAGE IF YOU HAVE A PRINTER.

LAST NAME: −−−−−−−−−−−−−− FIRST NAME: −−−−−−−−−−−−−−− ID: −−−−−−−−−−−−

Write a statement like this below and add your full signature (in English) below it. Do not type your name, that is not an acceptable signature. The statement must be exactly like this, with your name on it and signature. Do not cut the phrases or simplify.

I, −−(your name here)−−−−−−−−−sign to confirm that this exam reflects my work and only my work, that I have not consulted with anyone or anything except the class material posted in CCLE, the textbook, and Cognella active learning and that I have taken the time specified in the instructions or very close to that time to complete the exam from the moment that I first looked at it until it was in Gradescope. I also confirm that I have adhered to UCLA Student Conduct Code `https://deanofstudents.ucla.edu/individual-student-code` and that I have and will not share this exam with anything or anyone.

YOUR SIGNATURE (in English, and hand written-typing your name is not an acceptable signature):− − −−−−−−−−−−−−−−−−−−−−−−

INSTRUCTIONS

(points deducted for not following these instructions and those posted in the final exam folder)

(1) The exam must be submitted to the Gradescope link for the exam **before 8 AM on 9/11/2021 Los Angeles Time. No excuses will be accepted for you falling sleep or waiting until the last minute or any other excuse. Start very early to do it to prevent all potential problems. I will not even open files sent by email or other form.**

(2) **This is a four hour exam from the moment you click on the pdf file or the gradescope submission link for the first time, until it is showing submitted in gradescope. The time at which you access it will be logged in by CCLE. If you click on either the pdf or the gradescope links,or the Q&A link, you must start and do the exam right away. "Accidental clicks" are clicks, and the clock starts for you with that click, if that happens to you.** The four hours includes the time it takes you to scan and submit the exam. You may choose any four hours between 8:00AM on 9/10/2021 and 8 AM on 9/11/2021 to do your exam, but you must submit before the deadline. No excuses will be accepted because you wait until the last minute to look at, download and (or) submit your exam. Start early to prevent problems. You may choose any four hours between 8:00 AM on 9/10/2021 and 8:00 AM on 9/11/2021 to do the exam.

(3) You must work on your own.During the 24 hours of the exam, no group work allowed, no consulting with anyone or anything allowed. No sharing of information allowed. You may use all the material in our CCLE course web site.

(4) Do not contact the TA or anybody else regarding class material or anything regarding the exam and the course during the time window allowed for the exam, even if you have finished your exam. Contact between students in the class during the exam time window is not allowed, even if you finished your exam. You may contact only Dr. Sanchez, but first check the Q& A file and the instructions given on these pages. If the answer to what you asked is there, or you should not have asked the question, you will not receive an answer from Dr. Sanchez by email. You must contact during regular working hours, Los Angeles time. So time yourself accordingly.

(5) There is a file called Q&A, where I will be posting anything worthwhile that may benefit the whole class, such as a typo found. You will look at that file before you email me (Dr. Sanchez). Read the instructions of the exam well to avoid wasting time asking questions that are answered in the instructions.

Part I. MULTIPLE CHOICE ANSWERS. WRITE YOUR ANSWER FOR QUESTIONS 1-20 ON A TABLE EXACTLY LIKE THE ONE GIVEN BELOW, WITH 4 COLUMNS AND 5 QUESTIONS PER COLUMN. YOU MAY WRITE A TABLE LIKE THIS BY HAND ON YOUR PAPER OR SUBMIT THIS PAGE. IF YOU SUBMIT A DIFFERENT FORMAT FOR THE TABLE POINTS WILL BE DEDUCTED.

SHOWING WORK IS NOT REQUIRED FOR THE MULTIPLE CHOICE QUESTIONS.

ONLY THIS TABLE MUST BE ON PAGE 2 OF YOUR EXAM

Question(Q) Answer	Q Answer	Q Answer	Q Answer
Q 1	Q 6	Q 11	Q 16
Q 2	Q 7	Q 12	Q 17
Q 3	Q 8	Q 13	Q 18
Q 4	Q 9	Q 14	Q 19
Q 5	Q 10	Q 15	Q 20

Table 1: There is a box for each question. Write your answer for each question inside the box where the question number is. The table you submit must be exactly like this, with 4 columns and 5 questions per column. Other formats will be deducted points. Only the table must be on this page.

Part I. Multiple Choice questions. Showing work is not required. Write all your answers to the multiple choice questions in the table on page 3 or a hand-written version of it. There is only one answer to each question. If you think the answer is not among the choices given, choose the option closest to your answer. The questions are not graded by machine or scantron. A human will grade them. Similarly, if you think there is more than one answer, write them on the table. The exam is intended to have just one answer, but accidents could happen.

Question 1. Consider 30 random variables each distributed as exponential with expected value 10 and independent from each other. The sum of these 30 random variables is denoted as $X_1 + X_2 + ... + X_{30}$. The probability that the sum of these 30 random variables is larger than 120 is close to (where z is N(0,1))

(a) $P\left(z > \frac{120(30)-10(30)}{\sqrt{30000}}\right)$

(b) $P\left(z > \frac{12(30)-10(30)}{\sqrt{30}\sqrt{30(10)}}\right)$

(c) $P\left(z > \frac{120-10(30)}{\sqrt{30}}\right)$

(d) $P\left(z > \frac{120-10(30)}{\sqrt{3000}}\right)$

(e) $P(z > 0)$.

Question 2. Let the random variable $X_i \quad i = 1, 2, ..., 100$, be distributed as Poisson with parameter $\lambda = 3$. Find the approximate probability that $\bar{X}$ is larger than 4.

(a) 0.5733

(b) approximately 0

(c) approximately 1

(d) 0.334

(e) 0.643

Question 3. There are eight similar chips in a bowl: three marked (0,0), two marked (1,0), two marked (0,1) and one marked (1,1). If X and Y represent those two coordinates, respectively, their joint distribution is

$$P(X, Y) = \frac{1}{8}(3 - X - Y), \qquad X = 0, 1 \quad Y = 0, 1$$

The P(X) formula is

(a) $P(x) = \frac{1}{8}(2x), \quad X = 0, 1$

(b) $P(x) = \frac{1}{8}, \quad X = 0, 1$

(c) $P(X = x) = \frac{1}{8}(5 - 2x), \quad X = 0, 1$

(d) $P(X = x) = \frac{1}{8}(2y), \quad X = 0, 1$

Question 4. The length of life X_i of fuse i, $\quad i = 1, 2, 3$, has the probability density

$$f(x_i) = \frac{e^{-\frac{x_i}{\theta}}}{\theta} \qquad x_i > 0$$

The parameter θ is a constant. Three such fuses operate independently. Find the joint density of their lengths of life.

(a)

$$f(x_1, x_2, x_3) = \frac{e^{-\frac{\Sigma_{i=1}^3 x_i}{\theta}}}{\theta^3}$$

(b)

$$f(x_1, x_2, x_3) = \frac{e^{-\frac{x_i^3}{\theta^3}}}{\theta}$$

(c)

$$f(x_1, x_2, x_3) = \frac{e^{-\frac{x_i}{\theta^3}}}{\theta^3}$$

(d)

$$f(x_1, x_2, x_3) = \frac{e^{-\frac{x_i^3}{\theta^3}}}{\theta^3}$$

(e)

$$f(x_1, x_2, x_3) = \frac{\prod_{i=1}^3 e^{-\frac{x_1^3}{\theta^3}}}{\theta}$$

Question 5. The joint probability mass function of two random variables, $P(X, Y)$, appears below

		Y	
		2	1
	1	1/8	2/8
X	2	2/8	1/8
	3	1/8	0
	4	0	1/8

The Covariance$(2 + 3X, 4 - 2Y)$ is equal to $E[((2 + 3X) - (2 + 3\mu_x))((4 - 2Y) - (4 - 2\mu_y))]$. Consequently, Covariance$(2 + 3X, 4 - 2Y)$ is equal to

(a) 0

(b) 25/9

(c) 2/9

(d) 1.5

(e) 2

Question 6. The joint probability mass function of two random variables, $P(X, Y)$, appears below

		Y	
		2	1
	1	1/8	2/8
X	2	2/8	1/8
	3	1/8	0
	4	0	1/8

The $Var(Y \mid X = 2)$ is

(a) 3

(b) 25/9

(c) 2/9

(d) 1.5

(e) 2

Question 7. An urn contains 10 red balls, 6 yellow balls and 2 blue balls. If we select 10 balls without replacement, What is the probability of having 4 red and 1 blue balls?

(a) ≈ 0

(b) 0.04724

(c) 0.05759

(d) 0.1667

Question 8. The time in hours required to repair a machine is an exponentially distributed random variable with expected value of two hours. A company has four machines that need repairs. What is the probability that two of them require a repair time of less than two hours?

(a) 0.6321

(b) 0.3245

(c) 0.8910

(d) 0.05407931

Question 9. The response time at an online computer terminal follows, approximately, a gamma distribution, with expected value 4 seconds and variance of 8 seconds. Which of the following is the probability density function for the response times (all functions below have domain X from 0 to infinity?

(a) $f(x) = \frac{xe^{-1/2x}}{4}$

(b) $f(x) = \frac{e^{-1/2x}}{2}$

(c) $f(x) = \frac{xe^{-1/2x}}{2}$

(d) $f(x) = \frac{1}{2}e^{-x}$

Question 10. Wires manufactured for a certain computer system are specified to have a resistance of between 0.12 and 0.14 ohms. The actual measured resistances of the wires produced by company A have a normal probability density distribution, with expected value 0.13 ohms and standard deviation 0.005 ohms. If three independent such wires are used in a single system and all are selected from company A, what is the probability that they all will meet the specifications?

(a) approximately 1

(b) approximately 0.8696

(c) approximately 0.13

(d) approximately 0.005

Question 11. In 1000 flips of a fair coin, heads came up 560 times and tails 440 times. Are these results consistent with a fair coin?

(a) Not consistent with a fair coin

(b) Consistent with a fair coin

(c) The results could have happened just by chance

(d) There is no way to statistically determine the answer to this question.

Question 12. Personnel at an Engineering company use an online terminal to make routine engineering calculations. The time each engineer spends in a session at a terminal has an exponential density function with expected value of 36 minutes. Ninety percent of the sessions end in less than R minutes. What is R?

(a) 90

(b) 0.1888756

(c) -36

(d) 82.89306

Question 13. Suppose that the length of time X needed to conduct a periodic maintenance check on a pathology lab's microscope (known from previous experience) has expected value 6 minutes and variance equal to 12. Suppose that a new repairperson requires 20 minutes to check a particular microscope. Does this time required to perform a maintenance check seem out of line with prior experience? (assuming that the distribution of maintenance time has not changed from prior experience).

(a) No, 20 minutes to check a microscope seems consistent with prior experience, it happens very often.

(b) Yes, it is out of line. 20 minutes is not very frequent in this lab.

(c) Since we do not know the distribution of X, we can not tell

(d) The 20 minutes or more is very likely to happen by chance.

Question 14. The joint density function of a random variable is

$$f(x,y) = x + y, \qquad 0 \le x \le 1; \quad 0 \le y \le 1$$

Which of the following is true?

(a) The conditional density function of x given y equals the unconditional density of x.

(b) The two variables are independent

(c) The conditional density of x given y multiplied by the f(y) will give the joint density f(x,y)

(d) f(x)f(y) =f(x,y)

Question 15. We are stuck in the following calculation concerning two random variables X and Y. Can you tell us what it is equal to?

$$\int_y \int_x (x-\mu_x)^2 f(x,y)dxdy + \int_y \int_x (y-\mu_y)^2 f(x,y)dxdy$$

(a) $2E(X-\mu_x)^2$

(b) $2Var(Y)$

(c) $Var(X)f(X) + Var(Y)$

(d) $\int_x (x-\mu_x)^2 f(x)dx + \int_y (y-\mu_y)^2 f(y)dy$

Question 16. Oysters are grown on three marine farms for the purpose of pearl production. The first farm yields 20% of total production of oysters, the second yields 30%, and the third yields 50%. The share of the oyster shells containing pearls in the first farm is 5%, the second farm is 2%, and the third farm is 1%.

Under the conditions presented above, a randomly chosen shell contains the pearl. What is the probability of the event that this shell is grown in the first farm?

(a) 0.021

(b) 0.2

(c) 0.5238

(d) 0.4762

Question 17. Oysters are grown on three marine farms for the purpose of pearl production. The first farm yields 20% of total production of oysters, the second yields 30%, and the third yields 50%. The share of the oyster shells containing pearls in the first farm is 5%, the second farm is 2%, and the third farm is 1%.

Under the conditions presented above, a randomly chosen shell contains the pearl. Where was the pearl most likely to have come from?

(a) the first farm

(b) the second farm

(c) the third farm

(d) equally likely to have come from either farm

Question 18. A parallel system is a system that works if at least one of the components work. A series system is a system that works if all components work. There are systems that are a mixture of parallel and series. The one in Figure 1 is one of them. It is a parallel system, but in each part of the parallel system there is a series system.

Calculate the reliability of this system assuming that the probability that each individual component works is 0.8.

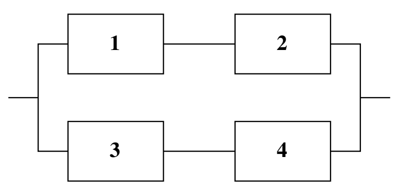

Figure 1: A system with 4 components

(a) 0.64

(b) 0.79

(c) 0.8704

(d) 0.9911

Question 19. (This problem is from Khilyuk, Chillingar, and Rieke 2005, page 58.) Consider an urban water-supply system. It can fail because of either the lack of water or damage of supplying pipes. On any given day, the supply system can be in one of the following two states: proper functioning (event A) or failure (event B). Reliability of the system can be defined as the probability of proper functioning on any given day P(A). Based on the information presented below, one needs to evaluate the reliability of the water supply system.

$$P(A) = reliability$$

Let W = event lack of water. Let D denote damage of supplying pipes.
$P(W) = 0.014$ and $P(D) = 0.030$.
It is known that $P(DW) = 0.011$.
Calculate $P(A)$.

(a) 0.033

(b) 0.967

(c) approximately 1

(d) 0.8911

GO TO NEXT PAGE FOR WORK QUESTIONS IN PART II

PART II. SHOW WORK FOR THE FOLLOWING QUESTIONS

For this part of the exam, you must show work to obtain full credit. The grading rubric will be as in the homework, 80 or 90% of the grade comes from work, and you must include proper definition of your random variable(s), explicitly say what it is you are calculating, events, notation, assumptions, distributions used if your calculation is based on a distribution and final answer. If calculations are based on a distribution and the distribution is not given, you must calculate and provide it in your work. When the results are numeric, please, do not leave your final result as a fraction. Calculate the value of the fraction, providing at least three decimals. Defining your random variables when they are not defined in the problem is required, don't forget that.

Question 20. There are eight similar chips in a bowl: three marked (0,0), two marked (1,0), two marked (0,1) and one marked (1,1). If X and Y represent those two coordinates, respectively, their joint distribution is

$$P(X,Y) = \frac{1}{8}(3 - X - Y), \qquad X = 0,1 \quad Y = 0,1$$

Show the work you did to obtain the P(X) formula that you obtained in the multiple choice questions.

Question 21. Suppose that the length of time X needed to conduct a periodic maintenance check on a pathology lab's microscope (known from previous experience) has expected value 6 minutes and variance equal to 12. Suppose that a new repairperson requires 20 minutes to check a particular microscope. Does this time required to perform a maintenance check seem out of line with prior experience (assuming that the distribution of maintenance time has not changed from prior experience). Show the work that resulted to your answer in the multiple choice question. Explain your conclusion.

Question 22. In 1000 flips of a fair coin, heads came up 560 times and tails 440 times. Are these results consistent with a fair coin? Show work and explain your conclusion.

Question 23. Using a partition of $A \cup B \cup C$) show that
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$
Define first what a partition is.

Question 24. (a) Suppose we are interested in estimating the length of words in the Gettysburgh address. Explain why random sampling is preferred to subjectively selecting a sample or words. Give at least two reasons given in lecture and explain the mathematical constructs that we use to study sampling in probability.

(b) What do conditional probability distributions and unconditional probability distributions have in common? Explain.

(c) What does a probability model tell us? Explain.