EVERYTHING IN THIS BOX MUST (WORD BY WORD, WITH NAME AND ID ADDED) APPEAR ON A FRONT PAGE 1 OF YOUR SUBMITTED EXAM. YOU MAY WRITE EVERYTHING BY HAND ON PAGE 1 OF YOUR EXAM. DO NOT WRITE ANYTHING ELSE ON THAT PAGE.

LAST NAME: – – – – – – – – – – – – – **FIRST NAME:** – – – – – – – – – – – – – – **ID:** – – – – – – – – – – –

Write a statement like this below and add your full signature (in English) below it.

I, ––(your name here)– – – – – – – – –sign to confirm that this exam reflects my work and only my work, that I have not consulted with anyone or anything except the class material posted in CCLE, the textbook, and Cognella active learning and that I have taken the time specified in the instructions or very close to that time to complete the exam from the moment that I first looked at it until it was in Gradescope. I also confirm that I have adhered to Section 102.01 or 102.02 of the UCLA Student Conduct Code and that I have and will not share this exam with anything or anyone.

YOUR SIGNATURE (in English):– –

INSTRUCTIONS

(points deducted for not following these instructions and those posted in the midterm folder)

(1) The exam must be submitted to Gradescope link for the exam **before 5 PM on 12/13/2020 Los Angeles Time.**

(2) **This is a three hour exam, but you are allowed four hours from the moment you click on it for the first time (which will be logged in by CCLE) until the time it appears as submitted in gradescope (which will also be logged by CCLE).** So the four hours includes the time it takes you to scan and submit the exam. You can choose any four hours during a 48 hour window to do your exam, but you must submit before the deadline. No excuses will be accepted because you wait until the last minute to look at, download and (or) submit your exam. Start early to prevent problems. If you "accidentally" click on the exam, you will have to do it then. Do not contact the professor saying you clicked accidentally, because the answer is in this paragraph.

(3) You must work on your own. No group work allowed, no consulting with anyone or anything allowed. No sharing of information allowed. You may use all the material in our CCLE course web site and the required textbook, including Cognella active learning materials that come with the textbook.

(4) You may only talk to Dr. Sanchez regarding the exam while doing the exam and during the 48 hours of the exam. Do not contact the TA or anybody else regarding class material or anything regarding the exam and the course during the time window allowed for the exam.

(5) There is a file called Q&A, where I will be posting anything worthwhile that may benefit the whole class, such as a typo found, for example. You will look at that file before you email me (Dr. Sanchez). Read the instructions of the exam well to avoid wasting time asking questions that are answered in the instructions.

(6) The exam has two parts.

- Part I (20 points) is 20 multiple choice questions (pages 3-9). *Answer on the table given on page 3 or a table exactly like that written by hand. Required.* You must write the letter of the chosen answer or answers (A, B, C,...etc). Read MC instructions on page 3. If your table does not have 4 columns and is like the one given it will not be graded.
- Part II (20 points) problems where you must show work. Work is 80% or more of your grade. Make sure you read ALL the instructions carefully.

(7) **Uploading to (Gradescope)**: You will scan and upload to the gradescope link in the midterm folder a pdf file called lastname-uclaID-final.pdf, containing, in this order, and telling gradescope the page where things are (scan your work using the adobe scan app downloaded to your phone, as in homework):

- (i)The front page (you may either print the one on page 1 of the exam or make your own on your notebook. The page will have ONLY everything that appears in the framed (boxed) part of the top of page of the exam (name, ID, statement saying what the paragraph says and full signature) exactly as it is in that box, not your version of it. Your name and ID must be included. This will be page 1 of your pdf file submission. There will be points deduction if this page is not submitted or if it contains more than the information requested. Indicate to gradescope where that page is.
- (ii) the page with the table containing the multiple choice answer(s) and only the letter chosen. Work is not required. You must use the table on page 3, printed, or make one of your own exactly like that one, drawn by hand. This will be page 2 of your pdf submission. Warning: if your table is not like that one on page 3 (i.e., with 4 columns) points will be deducted from your exam. This page may also contain any comments you may have regarding a multiple choice question for which you think there is no answer among the options given. You do not need to contact the professor; just select what you think is the closest answer and write your comment, showing work for only that question, work that justifies your answer. You must indicate to gradescope the page where the table with the multiple choice answers appears.
- The remaining pages of your submitted exam will contain the complete work and answers for the questions that require work, in the order given. You must indicate to gradescope the page where we will find the answer to each question, one by one. But you do not need to copy all the question. Copy only what the professor says you must copy exactly on your answer. It is not necessary to have each question in a separate page but points will be deducted if you do not indicate the page number where the answer is when you submit to gradescope, even if it is in the same page as another question.

(8) Only gradescope is allowed as a form of submission. Allow yourself enough time to be able to upload on time. I will not accept emails with files or any other form of submission. No late exams will be allowed. You are given two days in order for this not to happen. Plan accordingly to avoid an F for the class.

(9) Any indication that you did not follow these instructions or Sections 102.01 or 102.02 of the UCLA Student Code of Conduct at

`https://www.deanofstudents.ucla.edu/portals/16/documents/uclacodeofconduct_rev030416.pdf`

or indication that you shared information or work with others will result in an F in the class and this exam, and a virtual visit to the Dean of Students office.

(10) Indication that this exam has been posted or shared with anyone will result in a visit to the Dean of Students Office. The exam can not be shared with anyone, during or after the exam is completed. It is copyrighted.

Part I. MULTIPLE CHOICE ANSWERS. WRITE YOUR ANSWER FOR QUESTIONS 1-20 ON A TABLE EXACTLY LIKE THE ONE GIVEN BELOW. YOU MAY WRITE A TABLE LIKE THIS BY HAND ON YOUR PAPER.

The work you do to solve the MC will not be graded. Only this table or a table exactly like this written by hand (if you use your own paper) with the letter corresponding to what you think is the answer will be graded for this part.

Since this is not a scantron exam (meaning it will be graded manually), if by any chance you think that the options given do not contain the answer, look at the Q&A file. If nothing is there, select the closest answer to what you think is correct and then write a small comment indicating your concern below the table, indicating question number and concern, as well as the complete work for only that question. Your work must show why you have the concern. I will look at it when grading. This strategy will save you time.

THERE IS A BOX FOR EACH QUESTION. WRITE YOUR ANSWER FOR EACH QUESTION INSIDE THE BOX WHERE THE QUESTION NUMBER IS. THE FORMAT OF THE BOX YOU WRITE (IF YOU WRITE IT BY HAND ON YOUR NOTEBOOK PAGES) MUST BE EXACTLY LIKE THIS (4 columns).

Question(Q) Answer	Q Answer	Q Answer	Q Answer
Q 1	Q 6	Q 11	Q 16
Q 2	Q 7	Q 12	Q 17
Q 3	Q 8	Q 13	Q 18
Q 4	Q 9	Q 14	Q 19
Q 5	Q 10	Q 15	Q 20

PART I. Multiple choice questions. *Showing work is not required. Write all your answers to the multiple choice questions in the table on page 3 or a hand-written version of the table that you have created on your own paper, a table identical to the one on page 3.* **There is only one answer to each question**

NOTE: Since this is not a scantron exam (meaning it will be graded manually), if by any chance you think that none of the choices given is the answer, look at the Q&A file. If nothing is there, select the closest answer to what you think is correct and then write a comment and your complete work for only that question below the table, on the same page where your table is. I will look at it when grading. This strategy will save you time.

Question 1. Consider an automobile accident on a city street in which car I stops suddenly and is hit from behind by car II. Suppose the three persons, whom we call A,B,and C, witness the accident. Suppose the probability that each witness has correctly observed that car I stopped suddenly is estimated by having the witnesses observe a number of contrived incidents about which each is then questioned. Assume that it is found that A has probability 0.9 of stating that car I stopped suddenly, B has probability 0.8 of stating that car I stopped suddenly, and C has probability 0.7 of stating that car I stopped suddenly. Let A, B, and C denote, respectively, the events that persons A, B,and C will state that car I stopped suddenly. Assuming that A, B, and C are independent events, what is the probability that at least two of them will state that car I stopped suddenly?

(a) 0.398

(b) 0.010

(c) 0.902

(d) 0.

Question 2. Too Many Burger Orders! A popular burger joint receives orders distributed $X \sim \text{Poisson}(\lambda = \frac{5}{3})$, where X denotes "number of orders per minute". The kitchen becomes overwhelmed if more than 120 orders are placed within an hour. What is the probability that the kitchen becomes overwhelmed in a given hour?

(a) 0.026

(b) 0.010

(c) 0.500

(d) 0.250

Question 3. Suppose that the height, in inches, of a randomly chosen 25-year-old person is a normal random variable with parameters $\mu = 71$ and $\sigma^2 = 6.25$. What percentage of persons in the 6-footer club are over 6 feet, 5 inches?

(a) 0.02379

(b) 0.36827

(c) 0.02239

(d) 0.00819

Question 4. Students visiting UCLA during Bruin Transfer day sign up for one free ice cream of their choice among three unique flavors offered by the hospitality booth. In past years, 10% of students chose strawberry, 40% chose chocolate, and 50% chose lemon ice cream. If next year is like past years, what is the probability that, among 5 visiting students, 1 chooses chocolate, 2 choose strawberry and 2 choose lemon ice cream

(a) 0.03

(b) 0.23

(c) 0.651

(d) 0.4

Question 5. Rakiya interns in a lab in a hospital and is in charge of assessing whether the hospital's pharmacy is doing a good job when it comes to putting the right amount of medication in Tetroprix pills. Tetroprix medicine comes in a box containing five pills, denoted by M,T,W,R,F. Underfilled pills, that is pills with a lesser amount of medication than they should contain, pose an unwanted risk to the patient consuming them. Unknown to Rakiya, pills M,T,W, contain the proper amount of medication while pills R and F are underfilled. Rakiya selects two pills at random without replacement. What is the probability that at least one of the pills selected by the Rakiya contains the proper level of medication?

(a) 0.84

(b) 0.9

(c) 0.6

(d) 0.5

(e) 0.0001

Question 6. In a small private liberal arts college, 40 percent of the students receive financial aid, and 30 percent of the students that receive financial aid also are first generation college students. In addition, 25 percent of the students in the college own a hybrid car. What is the probability that a randomly selected student receives financial aid and is first generation college student?

(a) 0.0792

(b) 0.3

(c) 0.075

(d) 0.4

(e) 0.12

Question 7. Let $S_n = X_1 + X_2 +, , , , , +X_n$, where $X_i, i = 1, 2, ..., n$ is an exponential random variable with parameter $\lambda = 2$. Which of the following must be true for $P(S_n > k)$ to to be reasonably well approximated by the Gaussian distribution?

(a) $np > 10$ and $n(1 - p) > 10$

(b) The pairwise correlation between any two random variables must be 0.

(c) $\mid S_n - 6n \mid > 0$

(d) $(S_n - 1.5n) > 0$

Question 8. Waiting time (W) to vote at the local voting station in election days is exponentially distributed, with expected value 60 minutes. If 100 unrelated persons will be waiting to vote in voting day this year, the moment generating function of the total waiting time, and the distribution of waiting time will be, respectively:

(a) $(1 - 60t)^{-1}$, and f(w) will be exponential with expected value 60

(b) $(1 - 6000t)^{-1}$, and f(w) will be exponential with expected value 6000.

(c) $(1 - 6000t)^{-100}$, and f(w) will be gamma with expected value 6000

(d) $(1 - 60t)^{-100}$, and f(w) will be gamma with expected value 6000.

Question 9. The cumulative distribution function F(x) of a discrete random variable X is given by F(0)=0.3, F(1) = 0.7, F(2)=0.9, and F(3) = 1. Then the value of the probability mass function P(X=1) is

(a) 0.8

(b) 0.3

(c) 0.4

(d) 0.2

Question 10. The probability distribution of X, the number of defective tires on randomly selected cars at a large repair shop, is given in the table below:

x	0	1	2	3	4
P(X=x)	0.54	0.16	0.06	0.04	0.2

Determine the probability that we we might have to observe 6 cars in order to find three cars with a number of defective tires that exceed the expected value of X.

(a) None of the above

(b) 0.3

(c) 0.1638

(d) 0.09261

(e) 0.7

Question 11. Which of the following statements is true?

(a) For any discrete random variable X and constants a and b, the variance $Var(aX + b) = (a + b)^2 Var(X)$.

(b) For any discrete random variable X and constants a and b, the expected value $E(aX + b) = (a + b)E(X)$

(c) If a constant c is added to each possible value of a discrete random variable X, then the expected value of X will be shifted by that same constant amount.

(d) If a constant c is added to each possible value of a discrete random variable X, then the variance of X will be shifted by that same constant amount.

Question 12. Which of the following experiments is a binomial experiment?:

(a) Survey 50 investors to see how many different stocks they own. The random variable represents the number of different stocks owned by each investor.

(b) Each week, a gambler plays blackjack at the local casino. The random variable is the number of times per week the player wins.

(c) Selecting five cards, one at a time without replacement, from a standard deck of cards. The random variable is the number of face cards obtained.

(d) Survey 150 college students to see whether they are enrolled as new students. The random variable represents the number of students enrolled as a new student.

Question 13. Consider the following expression, where a and b are constants, and X is a random variable .

$$(a + bX - (a + b\mu_x))^2$$

The expectation of that expression is:

(a) The covariance between X and $a + bX$

(b) The variance of (a +bX)

(c) The expectation of (a+bX)

(d) The conditional expectation of Y given X, where Y is another random variable.

Question 14. A soft drink machine has a random supply Y at the beginning of a given day and dispenses a random amount X during the day (with measurements in gallons). It is not resupplied during the day. Hence $X \leq Y$. It has been observed that X and Y have joint density

$$f(x, y) = \frac{1}{2}, \qquad 0 \leq x \leq y \leq 2.$$

The expected random supply when the day's dispensed amount is 1 is:

(a) 1.5

(b) 0.4

(c) 2.75

(d) 1.89

Question 15. The number of ants per square meter x_i, has probability density

$$P(x_i) = \frac{e^{-\theta}\theta^{x_i}}{x_i!}, \qquad \texttt{for} \quad x_i \geq 0, \quad \theta \geq 0$$

The parameter θ is a constant.

7 square meters are randomly chosen. Find the joint density function of the number of ants in the 7 square meters, i.e $f(x_1, x_2, x_3, x_4, x_5)$

(a) $P(x_1, x_2, x_3, x_4, x_5) = \dfrac{e^{-\frac{2\sum_{i=1}^{7} x_i}{\theta}}}{\theta^2}$

(b) $P(x_1, x_2, x_3, x_4, x_5) = \frac{e^{-5\theta}\theta^{\sum_{i=1}^{6} x_i}}{\prod_{i=1}^{7} x_i!}$

(c) $P(x_1, x_2, x_3, x_4, x_5) = \dfrac{e^{-\frac{x_i}{5\theta}}}{5\theta}$

(d) $P(x_1, x_2, x_3, x_4, x_5) = \dfrac{e^{-\frac{x_5}{\theta}}}{\theta}$

(e) $P(x_1, x_2, x_3, x_4, x_5) = \frac{e^{-7\theta}\theta^{\sum_{i=1}^{7} x_i}}{\prod_{i=1}^{7} x_i!}$

Question 16. Let

$$f(x, y) = x + y, \qquad 0 \leq x \leq 1, \qquad 0 \leq y \leq 1$$

The conditional expectation of Y^2 when $X = \frac{1}{2}$ is

(a) 11/144

(b) 7/12

(c) 10/24

(d) 0

(e) 1

Question 17. The random variable X is a continuous variable with density function

$$f(x) = 0.75(1 - x^2), \qquad -1 \leq X \leq 1.$$

Consider 100 independent and identically distributed random variables with that density. That is,

$$X_i \sim f(x), \quad i = 1, 2, ..., 100$$

. What is the expected value of

$$\sum_{i=1}^{100} \left(\frac{1}{100}(x_i - 3) \right)$$

(a) 0

(b) -3

(c) 50

(d) 9/100

(e) 200/5

Question 18. The census bureau reported that 70% of all americans have hospitalization coverage by a private insurance plan. In a large hospital, there were 200 patients admitted one week. What is the probability that more than 65% have a private insurance plan?

(a) 0.7

(b) 0.458

(c) 0.21

(d) 300

(e) 0.93

Question 19. A city plans to create a new stadium. The manufacturer can give the information that the expected cost is 10 billion dollars, according to the distribution of costs of past projects. What is the probability that the cost is less than 20 billion dollars.

(a) more than 1/2

(b) less than 1/2

(c) more than 3/4

(d) less than 3/4

(e) 0

Question 20. Consider the number of months since a patient had the last medical examination. This is a random variable that varies across patients. At a given point in time, this distribution can be assumed to be uniform between 4 and 20 months. Consider 150 patients randomly chosen. What is the probability that the average number of months since the last examination is 12 or larger?

(a) 0.5

(b) 0.3

(c) 0.1

(d) 0.8

(e) 0

GO TO NEXT PAGE FOR WORK QUESTIONS IN PART II

PART II. SHOW WORK FOR THE FOLLOWING QUESTIONS.

For this part of the exam, you must show work to obtain full credit even if the question does not ask for it. The grading rubric will be as in the homework, 80 or 90% of the grade comes from work, including proper definition of your random variable(s), events, notation, thorough work, assumptions and final answer. When the results are numeric, please, do not leave your final result as a fraction. Calculate the value of the fraction, providing at least three decimals.

In the work, do not assume that we know what you are thinking and that we know the material. Explain as if the person reading it did not know the answer. Make sure you provide the WHY and show work.

Note: Some students print the exam and scan the printed version with their answers. For those students, I leave space. You do not need to print the exam, just copy what you are asked to put in your exam. You may write your answers on your notebook and scan that, as indicated in the instructions on the front pages of this exam.

For the Gaussian probability calculations, whether you use the app or R to find the answers, you must convert the problem to one in terms of z, showing how you are calculating the z. You will get more partial credit this way. For example, if your probability final answer is wrong, perhaps some parts of your Z score calculation are right, and you would get partial credit. See notes in class for examples we have done.

If we see the value of the Z score you do not need to do screenshots of the app or your R command. That is another advantage of calculating the Z for you.

Question 21. The following was one of the multiple choice questions. Indicate, before you start showing work, again, the answer that you wrote in the MC section and, after that, show work to justify your answer.

In a small private liberal arts college, 40 percent of the students receive financial aid, and 30 percent of the students that receive financial aid also are first generation college students. In addition, 25 percent of the students in the college own a hybrid car. What is the probability that a randomly selected student receives financial aid and is first generation college student?

(a) 0.0792

(b) 0.3

(c) 0.075

(d) 0.4

(e) 0.12

Question 22. The following was one of the multiple choice questions. Select again the answer that you wrote in the MC section and write it before you prove your statement using summation operator throughout your prove and the appropriate definitions. Then prove that the same result would be true for a continuous random variable using integration operator throughout your proof and the appropriate definitions.

Which of the following statements is true?

(a) For any discrete random variable X and constants a and b, the variance $Var(aX + b) = (a + b)^2 Var(X)$.

(b) For any discrete random variable X and constants a and b, the expected value $E(aX + b) = (a + b)E(X)$

(c) If a constant c is added to each possible value of a discrete random variable X, then the expected value of X will be shifted by that same constant amount.

(d) If a constant c is added to each possible value of a discrete random variable X, then the variance of X will be shifted by that same constant amount.

I leave space here for those who might print the exam and scan the printed version. But you do not need to print the exam if you do not want to. You may write the answer on your notebook paper.

Question 23. Answer the following short question showing work.

A Gamma random variable with expected value n and variance equal 2n, where n is a natural number, has the following moment generating function:

$$M_x(t) = \left(\square - \square\right)^{\square}$$

.

Write inside each of the empty boxes what is supposed to be there to complete the expression for the moment generating function. If you do not print the exam, write the formula and boxes in your notebook, and write inside the boxes what goes inside the boxes. Your work and an explanation for what goes on the boxes should be written in space outside the boxes and below the completed formula.

(I write here space for those who print the exam. You do not have to print the exam. You may write the answers on your notebook pages. But if you print, show your work in this space)

Question 24. Answer the following short question showing work.

W is a random variable that has expected value μ_W and Variance σ^2_W. Consider a random sample of independent, identically distributed $W_1, W_2, ..., W_n$. Consider also constants $a_1, a_2, ..., a_n$. You are asked to complete what is in the boxes first, to indicate what the law of large numbers says about $\sum_{i=1}^{n} \frac{a_i w_i}{n}$, being specific about what each thing you write is equal to. Then prove what you are claiming with your answer:

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{a_i w_i}{n} - \square\right) = \square$$

.

Show complete and detailed work to justify your answer. If you do not print the exam, write the formula and boxes in your notebook, and write inside the boxes what goes inside the boxes. Your work and an explanation for what goes on the boxes should be written in space outside the boxes and below the completed formula.

(I write here space for those who print the exam. You do not have to print the exam. You may write the answers on your notebook pages. But if you print, show your work in this space)

Question 25. Consider discrete random variables X and Y, which have the following joint probability mass function:

$$P(X = x, Y = y) = \frac{1}{32}\left(x^2 + y^2\right), \quad x = 0, 1, 2, 3; \quad y = 0, 1$$

(a) Find the formula for the $P(X|Y = y)$ and the formula for $P(Y|X = x)$

(b) Calculate $P(X > 1 \mid Y = 1)$ using the formula for the conditional and summation operators.

(c) Calculate the correlation between X and Y showing work.

(d) Calculate the correlation between W=20X and R=5Y showing work.

(e) Write the joint probability mass function formula in terms of whatever functions you need from the marginal and conditional probability mass functions.

Question 26. The time that a randomly chosen passenger waits for a local train between downtown Los Angeles station and St Diego station is uniformly distributed between 5 and 80 minutes.

(a) Calculate the probability that a randomly chosen passenger's waiting time is between 10 and 30 minutes.

(b) Calculate the probability that the average waiting time of a random sample of 35 passengers is larger than 40 minutes.

(c) Calculate the probability that the total waiting time of 3 passengers is larger than 30 minutes.

(d) Show that the distribution of a sum of 3 uniform distributions like that in part (a) is not uniform.

(e) If there are two groups of 20 passengers each, one group waiting on the right-hand-side platform, and the other on the left-hand-side platform, what is the probability that difference between the average waiting time of the two is larger than 5 minutes?