May 24, 2018

Physics 1CH Midterm #2

Problem 1: Short Answer (40 points total):

a) True or False. Since a standing wave does not travel, it is not truly a wave and does not satisfy the wave equation. Explain your answer.

False. standing wave oscillates and its amplitude with respect to the horizontal axis shows a sinusoidal pattern.
It satisfies wave equation $\psi(x,t) = Ae^{i(kx-\omega t)}$

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b) You are in a store examining sunglasses displayed in a glass case. The salesperson claims that the sunglasses have Polaroid filters. You suspect that the sunglasses are just tinted plastic. You ask to see a couple of the sunglasses. Name two ways you could find out for sure (in the store). Explain your answer.

① Take 2 sunglasses and overlap them. rotate one of them and observe if the transmitted light intensity changes with respect to the rotation angle. If not. it is plastic.

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② take 3 sunglasses. put 2 of them perfectly perpendicular to each other, insert the third one at an angle $0° < \theta < 90°$, see if the insertion of third one changes the intensity of transmitted light. If not. it is plastic

+1

↑ Some thing really

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$\frac{4\pi n_f}{\lambda_0} d\cos\theta_t$.

$\frac{n_f}{\lambda_0} = \frac{1}{\lambda_f}$ $\frac{4\pi}{\lambda_f} d\cos t - \pi$.

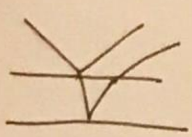
$(2m+1)\frac{\lambda_f}{4} \cdot \cancel{\frac{4\pi}{\lambda_f}}$ $\frac{4\pi}{\lambda_f}$ $2(m+1)\pi - \pi = 2m+1$.

Problem 1 (continued):

c) Imagine a soap bubble formed in air. As the bubble is just about to pop (i.e. as its thickness goes to zero), will the reflected light appear bright, dark, or neither of these? Explain your answer.

reflected light: ~~maximum brightness.~~ $\cancel{d\cos\theta_t = (2m+1)\frac{\lambda_f}{4}}$

when it goes to 0, the path difference traveled by the 2 beams is nearly 0 since $d \to 0$. then the two light beams will be $\delta = \frac{4\pi n_f}{\lambda_0} d\cos\theta_t - \pi = -\pi$ out of phase.



reflected light appear dark as the 2 beams destructively interfere.

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d) A student is working with a double-slit interference experiment. Instead of using light of a single wavelength, a light source having wavelengths of 400 nm, 500 nm and 600 nm is used. Describe the interference pattern you would see on the screen, if any. You can assume that the light hitting the two slits is coherent and has the same linear polarization.

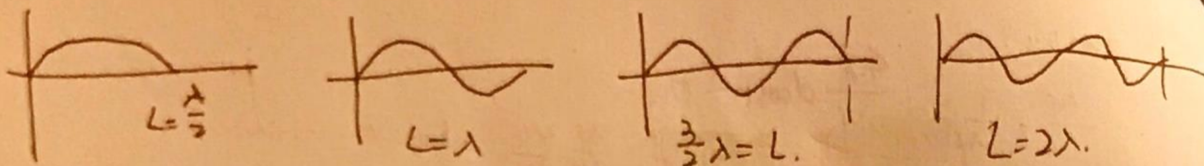
~~for interference pattern, $\Delta y = \frac{s}{a}\lambda$. It is dependent on wavelength.~~

~~there will be 3 overlapping interference pattern, each with different spacing $\Delta y = \frac{s}{a}\lambda_i$. However, due to~~

$\emptyset$

For addition of waves with different frequency, they will produce a wave with new frequency. so the interference pattern will be one interference pattern with spacing $\Delta y = \frac{s}{a}\lambda$. where λ is the wavelength of the composite light

NO



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Problem 2: (30 points total)

A road tunnel leading straight through a mountain greatly amplifies tones at frequencies of 135 Hz and 138 Hz.

a) Find the shortest length the tunnel can be. Explain why you know that this is indeeed the shortest length. Note that the speed of sound is given on page 1.

$V = 340 m/s.$

It is essentially a pipe with 2 open ends.

$v = \lambda f$

$\frac{n}{2}\lambda_1 = L.$ $\frac{m}{2}\lambda_2 = L.$ $\frac{n}{2}\lambda_1 = \frac{m}{2}\lambda_2.$ $\lambda_1 = \frac{340 m/s}{135 Hz} = 2.52 m.$

$\lambda_2 = \frac{340 m/s}{135 Hz} = 2.46 m.$

find shortest length = find minimum integer n, m that satisfies equation.

$1.252 n = 1.232 m.$

$\frac{n}{m} = \frac{1.232 m}{1.252 m} = \frac{135 Hz}{138 Hz} = \frac{45}{46}.$

does not explain

smallest $\frac{n}{m} = \frac{45}{46} = \frac{90}{92} = \frac{180}{184}$ etc. −5 −10

$L = 1.252 m \times n = 45 \times \frac{340 m}{135 Hz} = 115.33 m.$ X −5

Problem 2 (continued)

b) Suppose one end of the tunnel is completely closed off due to a rock slide. What would be the lowest frequency tone the tunnel would greatly amplify now? Would either of the tones of 135 Hz or 138 Hz be amplified in this case?

① $L = 113.33\,m$. open-closed tube

~~lowest~~ Lowest freq → longest λ.

1st harmonic $L = \frac{\lambda}{4}$. $\lambda = 4L = 453.33\,m$.

$f = \frac{v}{\lambda} = 0.75\ Hz$. −2

② . $\lambda = \frac{4L}{2n-1}$ $L = \frac{(2n-1)\lambda}{4}$

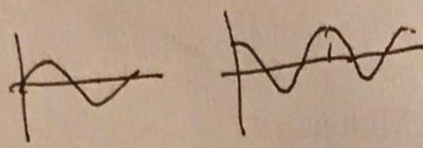
for $f = 135\ Hz$, $\lambda = 2.52\,m$. solve for $n = 90.5$ ✓

not an integer. not amplified ✓

−2

$f = 138\ Hz$. $\lambda = 1.232\,m$. $n = 92.5$ ✓

not an integer. not amplified. ✓



Problem 3: Fourier Techniques (30 points total)

Consider a function defined by:

$\cos' = -\sin$

$\sin' = \cos.$

$$\psi(x) = h,\ 0 < x < L$$

$$\psi(x) = 0,\ \text{~~elsewhere,~~}\ x=0 \,.\, x=L.$$

for h a constant.

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a) Determine the Fourier *sine series* expansion of $\psi(x)$ in the interval $(0,L)$. (Note: there is not a trivial solution to this problem).

$\cancel{\cos(n\pi) \to \cos\left(\frac{2\pi n x}{L}\right) \ \sin(n\pi) \to \sin\left(\frac{2\pi n x}{L}\right)}$

apply standing sine wave solution

since the graph is best modified by sine wave.

$\cancel{a_0 = \frac{2}{L}\int_0^L h \cos\frac{2\pi n x}{L}\,dx = \frac{2}{L}\int_0^L h\,dx = \frac{2}{L}\cdot hL = 2h}$

$$a_n = \frac{2}{L}\int_0^L h\cos\frac{2\pi n x}{L}\,dx = \frac{2}{L}h\left(\frac{L}{2\pi n}\sin\left(\frac{2\pi n x}{L}\right)\right)\Big|_0^L.$$

$$= \frac{2}{L}h\left(\frac{L}{2\pi n}(\sin 2\pi n - \sin 0)\right) = 0.$$

Standing wave solution

$$b_n = \frac{2}{L}\int_0^L h\sin\left(\frac{\pi n x}{L}\right) = \frac{2}{L}h\left(\frac{L}{2\pi n}\left(-\cos\frac{\pi n x}{L}\right)\right)\Big|_0^L.$$

$$= \frac{2}{L}h\left(\frac{L}{\pi n}\left(-\cos n\pi - (-\cos 0)\right)\right)\Big|_0^L$$

$$= \frac{2}{L}h\cdot\frac{L}{\pi n}\cdot 2.$$

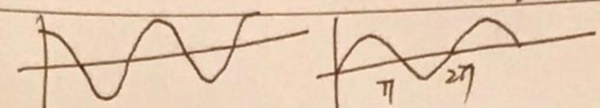
$$= \frac{4h}{n\pi}.$$

fourier sine series $= \sum_{n=1}^{\infty}\frac{4h}{n\pi}\cdot\sin\left(\frac{n\pi x}{L}\right)$

only odds.

-3.

Problem 3 (continued)



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b) For $h = \frac{\pi}{4}$ m, evaluate $\psi(\frac{L}{2})$ using what you found in part a), and thus determine a series expansion for π. Estimate π using the first [illegible] terms of the expansion.

$$\psi(\tfrac{L}{2}) = \sum_{n=1}^{\infty} \frac{4h}{n\pi} \cdot \sin\left(\frac{n\pi x}{L}\right)$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi \cdot \frac{L}{2}}{L}\right) = \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n}{2}\pi\right)$$

first 5 terms $= \sin(\frac{\pi}{2}) + \frac{1}{2}\cancel{\sin\pi}^{\,0} + \frac{1}{3}\sin(\frac{3}{2}\pi) + \frac{1}{4}\cancel{\sin(2\pi)}^{\,0} + \frac{1}{5}\sin(\frac{5}{2}\pi)$
$+ \frac{1}{6}\cancel{\sin(3\pi)}^{\,0} + \frac{1}{7}\sin(\frac{7}{2}\pi) + \frac{1}{8}\cancel{\sin(4\pi)}^{\,0} + \frac{1}{9}\sin(\frac{9}{2}\pi)$

$$= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} = 0.8349 = \frac{\pi}{4}$$

$\pi \simeq 4 \times 0.8349 = 3.34.$ ✓