Mid-term Exam 2: PHYSICS 1C (Spring 2021)

Time: 2:00PM – 3:50PM, May 13, 2021, Instructor: Prof. Zhongbo Kang

Student Name: Liz + Fangi

Student I.D. Number: ______________________________

Note:

- The total time is 1 hour and 50 minutes. However, the exam time is **1 hour and 20 minutes**, and the remaining 30 minutes are used ***only*** for scanning and uploading your solution to gradescope. Everyone has to **STOP** working on the exam when we are at the 1 hour and 20-minute mark.

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- The exam is open book/open notes. Key physical equations are provided. You can use a calculator.

Score Sheet (total 50 points): ________________________

Problem 1 (8 points): ______________________________

Problem 2 (10 points): ______________________________

Problem 3 (12 points): ______________________________

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Formula Sheet

From mid-term 1:

$\vec{F} = q\vec{v} \times \vec{B}$ (magnetic force on a moving charged particle)

$\Phi_B = \int B \cos\phi \, dA = \int B_\perp \, dA = \int \vec{B} \cdot d\vec{A}$ (magnetic flux through a surface)

$\oint \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for magnetism)

$R = \dfrac{mv}{|q|B}$ (radius of a circular orbit in a magnetic field)

$\vec{F} = I\vec{l} \times \vec{B}$ (magnetic force on a straight wire segment)

$d\vec{F} = I d\vec{l} \times \vec{B}$ (magnetic force on an infinitesimal wire section)

$\tau = IBA \sin\phi$ (magnitude of magnetic torque on a current loop)

$\vec{\tau} = \vec{\mu} \times \vec{B}$ (vector magnetic torque on a current loop)

$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\phi$ (potential energy for a magnetic dipole)

$\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{q\vec{v} \times \hat{r}}{r^2}$ (magnetic field due to a point charge with constant velocity)

$d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{I d\vec{l} \times \hat{r}}{r^2}$ (magnetic field due to an infinitesimal current element)

$B = \dfrac{\mu_0 I}{2\pi r}$ (magnetic field near a long, straight, current-carrying conductor)

$\dfrac{F}{L} = \dfrac{\mu_0 I I'}{2\pi r}$ (two long, parallel, current-carrying conductors) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ (Ampere's law)

New for mid-term 2:

$\varepsilon = -\dfrac{d\Phi_B}{dt}$ (Faraday's law of induction)

$\varepsilon = vBL$ (motional emf; length and velocity perpendicular to uniform $\vec{B}$)

$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ (motional emf; closed conducting loop)

$\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ (stationary integration path)

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \dfrac{d\Phi_E}{dt} \right)_{\text{encl}}$ (Ampere's law for a stationary integration path)

$\varepsilon_2 = -M \dfrac{di_1}{dt}$ and $\varepsilon_1 = -M \dfrac{di_2}{dt}$ (mutually induced emfs)

$M = \dfrac{N_2 \Phi_{B2}}{i_1} = \dfrac{N_1 \Phi_{B1}}{i_2}$ (mutual inductance) $\varepsilon = -L \dfrac{di}{dt}$ (self-induced emf)

$L = \dfrac{N\Phi_B}{i}$ (self-inductance) $U = L\int_0^I i \, di = \dfrac{1}{2} L I^2$ (energy stored in an inductor)

$u = \dfrac{B^2}{2\mu_0}$ (magnetic energy density in vacuum) $\tau = \dfrac{L}{R}$ (time constant for an R-L circuit)

$\omega = \sqrt{\dfrac{1}{LC}}$ (angular frequency of oscillation in an L-C circuit)

$\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$ (underdamped oscillations in L-R-C series circuit)

$i = I\cos\omega t$ (sinusoidal alternating current) $I_{rav} = \dfrac{2}{\pi} I$ $I_{rms} = \dfrac{1}{\sqrt{2}} I$ $V_{rms} = \dfrac{1}{\sqrt{2}} V$

$V_R = I\,R$ $V_L = IX_L$ $V_C = I\,X_C$ $V = I\,Z$

$Z = \sqrt{R^2 + [\omega L - (1/\omega C)]^2}$ (impedance of an L-R-C series circuit)

$\tan\phi = \dfrac{\omega L - 1/\omega C}{R}$ (phase angle of an L-R-C series circuit)

$P_{av} = \frac{1}{2} VI\cos\phi = V_{rms} I_{rms}\cos\phi$ (average power into a general ac circuit)

$\omega_0 = \dfrac{1}{\sqrt{LC}}$ (L-R-C series circuit at resonance) $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ (terminal voltages in a transformer)

$V_1 I_1 = V_2 I_2$ (terminal voltages and currents in a transformer)

$E = cB$ (electromagnetic wave in vacuum) $c = \dfrac{1}{\sqrt{\epsilon_0\mu_0}}$

$\vec{S} = \dfrac{1}{\mu_0}\vec{E} \times \vec{B}$ (Poynting vector in vacuum)

$I = S_{av} = \dfrac{E_{max}B_{max}}{2\mu_0} = \dfrac{E_{max}{}^2}{2\mu_0 c} = \dfrac{1}{2}\sqrt{\dfrac{\epsilon_0}{\mu_0}} E_{max}{}^2 = \dfrac{1}{2}\epsilon_0 c E_{max}{}^2$

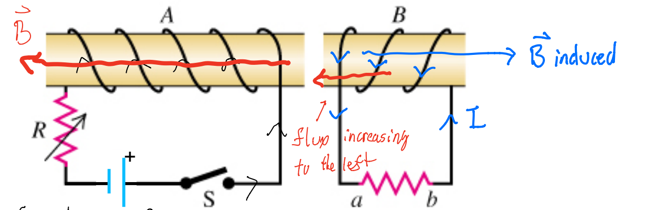
(intensity of a sinusoidal electromagnetic wave in vacuum)

$\dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{S}{c} = \dfrac{EB}{\mu_0 c}$ (flow rate of electromagnetic momentum)

Two constants for your reference: $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$

Problem 1 (8 pts): please ***be very careful*** in writing down your answers for these two questions. They are graded by the final answers ONLY, no partial credits for any intermediate steps.

(a) (2 pts) In the figure, the switch S is open originally. Now let us close it, please determine the direction of the current in the resistor ab. Your choice: ___a___



Current starts flowing counter clockwise. RHR for solenoids tells us that the magnetic field in solenoid A is pointing to the left. The solenoid gradually allows more current to flow through it, causing $\vec{B}$ to increase to the left.

As the mag. field in solenoid A increases to the left, solenoid B experiences an increase in magnetic flux to the left. To counteract this, a magnetic field is induced in solenoid B pointing in the opposite direction, so $\vec{B}_{induced}$ points to the right. Using RHR for solenoids, we see that the current must flow from a to b.

a. From point a to b

b. From point b to a

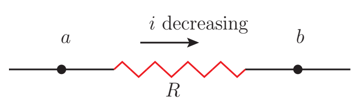
c. Cannot be determined, not enough information

(b) (4 pts) In the following four situations, please determine the sign of potential difference V_{ab} between point a and b? For each of them, choose your answers from:

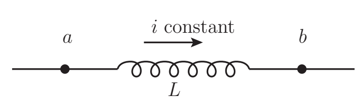
a. $V_{ab} > 0$ b. $V_{ab} < 0$ c. $V_{ab} = 0$ d. V_{ab} cannot be determined

$V = IR$

No dependence on $\frac{dI}{dt}$



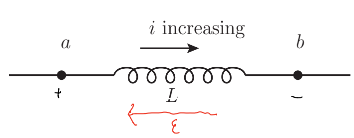
Your choice: ___a___



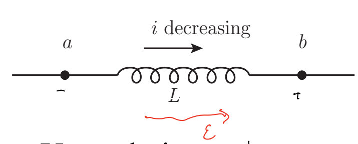
Your choice: ___c___

$V = L\frac{dI}{dt} = L \cdot 0 = 0$

$V_{ab} = L\frac{dI}{dt} > 0$



Your choice: ___a___

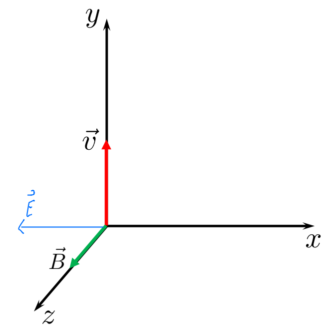


Your choice: ___b___

$V_{ab} = L\frac{dI}{dt} < 0$

(c) (2 pts) An electromagnetic plane wave propagates in the vacuum, the direction of propagation velocity v and the B field are shown in the figure, please determine the direction of E field at the origin.

Your choice: ___C___



a. $+y$ b. $-y$ c. $-x$ d. $+x$ e. $-z$ f. $+z$

g. not enough information, cannot be determined

$\vec{v}$ is in the $+\hat{y}$ direction

$\vec{B}$ is in the $+\hat{z}$ direction

We know $\vec{S} = \vec{E} \times \vec{B}$, and

$\vec{S}$ points in the direction of propagation ($\hat{y}$).

Thus, the direction of $\vec{E}$ must have the property that $\hat{e}_E \times \hat{z} = \hat{y}$

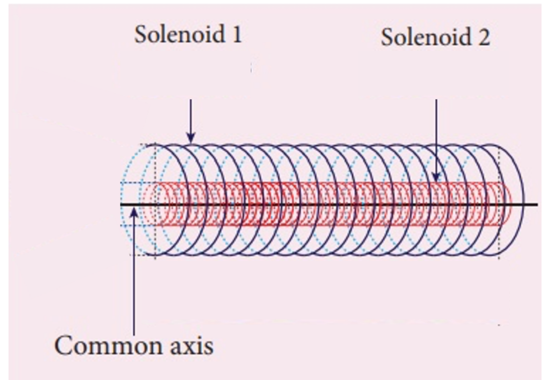
$\hat{e}_E$: Unit vector in direction of $\vec{E}$

We know that $-\hat{x} \times \hat{z} = \hat{y}$, so $\vec{E}$ must point in the $-\hat{x}$ direction.

Problem 2 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

As shown in the figure, the solenoid 1 is 50 cm long and has a cross-sectional diameter of 3.0 cm. This solenoid has a winding of 2000 turns per meter, and the electric current through the windings is changing at a rate $\frac{dI}{dt} = 3.0\ A/s$. A small coil (the red one called solenoid 2 in the figure) consisting of $N = 20$ turns has a cross-sectional diameter of 1.0 cm, and it is placed in the middle of the solenoid such that the plane of the coil is perpendicular to the central axis of the solenoid. Please determine the magnitude of the emf induced in this coil (i.e., solenoid 2).



Mutual Inductance!

$$M = \frac{N_2 \Phi_{B2}}{I_1}$$

Φ_{B2} := flux through Solenoid 2 due to the $\vec{B}$ field created by the current in the windings of Solenoid 1.

We know: $B_{solenoid} = \mu_0 n I$

And $\Phi_B = \vec{B} \cdot \vec{A}$

So, $B_1 = \mu_0 n_1 I_1$

$\Phi_{B2} = B_1 A_2 = \mu_0 n_1 I_1 A_2$

Then, $M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_2(\mu_0 n_1 \cancel{I_1} A_2)}{\cancel{I_1}}$

$M = \mu_0 n_1 N_2 A_2$

We also know:

$|\mathcal{E}_2| = \left| M \frac{dI_1}{dt} \right|$

So, $|\mathcal{E}_2| = \left| \mu_0 n_1 N_2 A_2 \frac{dI_1}{dt} \right|$

Now we just need to plug and chug!

$\mu_0 = 4\pi \times 10^{-7}\ \frac{T \cdot m}{A}$

$n_1 = 2000\ m^{-1}$

$N_2 = 20$

$A_2 = \pi(.005)^2\ m^2 = 7.85 \times 10^{-5}\ m^2$

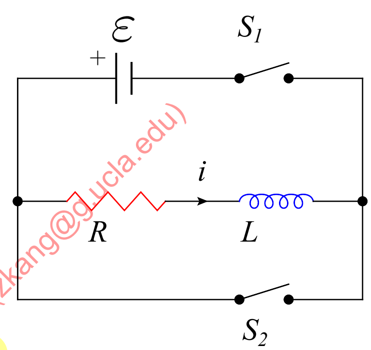
$\frac{dI}{dt} = 3.0\ A/s$

$\Rightarrow |\mathcal{E}_2| = (4\pi \times 10^{-7}\ \frac{T \cdot m}{\cancel{A}})(2000\ \cancel{m^{-1}})(20)(\pi(.005^2)\ m^2)(3\ \frac{\cancel{A}}{s}) = 1.18 \times 10^{-5}\ \frac{T \cdot m^2}{s} = 1.18 \times 10^{-5}\ \frac{V \cdot \cancel{s} \cdot \cancel{m^2}}{\cancel{m^2} \cdot \cancel{s}} = \boxed{1.18 \times 10^{-5}\ V}$

Problem 3 (12 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

For a *R*-*L* circuit as shown in the figure, the voltage $\varepsilon = 12.0$ V, the resistance $R = 100\ \Omega$, and the inductance $L = 0.01$ H. Suppose both switches are open to begin with, and then at some initial time $t = 0$, we close switch S_1 (leave S_2 still open). The current i is shown in the figure.



(a) (4 pts) Please obtain an expression of the current i as a function of time t. If one denotes the value of the current as I_0 when $t \to \infty$, please determine I_0. (*Note: Do not just copy the final expression from the notes or book, but including intermediate steps please.*)

(b) (4 pts) Please calculate the instantaneous power P_L in the inductor as a function of t. At what value of t, P_L is a maximum?

(c) (4 pts) When the current becomes I_0, one then resets our stopwatch to redefine the initial time, we open switch S_1 but close switch S_2 at $t = 0$. Obtain an expression of the current i as a function of time t.

(a)

$$V_R = iR, \quad V_L = L\frac{di}{dt}.$$

$$\varepsilon = V_R + V_L \Rightarrow \varepsilon = iR + L\frac{di}{dt}. \qquad (2\text{ pts})$$

$$\varepsilon - iR = L\frac{di}{dt} = -\frac{L}{R}\frac{d(-iR)}{dt}.$$

$$-\frac{R}{L}dt = \frac{d(\varepsilon - iR)}{\varepsilon - iR}.$$

Integrate both sides: $-\frac{R}{L}t = \ln(\varepsilon - iR)\Big|_0^i = \ln\frac{\varepsilon - iR}{\varepsilon}.$

$$e^{-\frac{R}{L}t} = \frac{\varepsilon - iR}{\varepsilon}.$$

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right).$$ (1 pt)

At $t=0$, $i=0$; $t \to \infty$, $i = \frac{\varepsilon}{R} = I_0$. (1 pt)

(b) $P_L = i V_L$

$= i L \frac{di}{dt}$

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right),$$

$$\frac{di}{dt} = \frac{\varepsilon}{R}\frac{R}{L} e^{-\frac{R}{L}t} = \frac{\varepsilon}{L} e^{-\frac{R}{L}t}.$$

$$\Rightarrow P_L = \frac{\varepsilon^2}{R}\left(1 - e^{-\frac{R}{L}t}\right) e^{-\frac{R}{L}t}.$$ (2 pts)

Maximum : $$\frac{dP_L}{dt} = \frac{\varepsilon^2}{R}\left[\frac{R}{L} e^{-\frac{2R}{L}t} - \frac{R}{L}\left(1 - e^{-\frac{R}{L}t}\right) e^{-\frac{R}{L}t}\right]$$

$$= \frac{\varepsilon^2}{L} e^{-\frac{R}{L}t}\left(2e^{-\frac{R}{L}t} - 1\right)$$ (1 pt)

Let $\frac{dP_L}{dt} = 0$, one obtains $2e^{-\frac{R}{L}t} = 1$.

$\frac{R}{L}t = \ln 2$, $t = \frac{L}{R}\ln 2$. (1 pt).

(c) $V_R + V_L = 0$. $iR + L\frac{di}{dt} = 0$ (2.5 pts)

$\frac{-iR}{L} = \frac{di}{dt}$. $\Rightarrow -\frac{R}{L}dt = \frac{di}{i}$

Integrate both sides: $i = I_0 e^{-\frac{R}{L}t}$, (1.5 pts).

$I_0 = \frac{\varepsilon}{R}$.

Problem 4 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

The electric field associated with an electromagnetic wave traveling in vacuum is given by

$\vec{E}(x,t) = \left(20\,\frac{\mathrm{V}}{\mathrm{m}}\right)\sin(kx+\omega t)\,\hat{\jmath}$, where $k = 10^{7}\,\mathrm{m}^{-1}$.

(a) (2 pts) What is the wavelength λ and period T of the wave?

(b) (4 pts) Please derive the expression for the wave corresponding to the magnetic field $\vec{B}$.

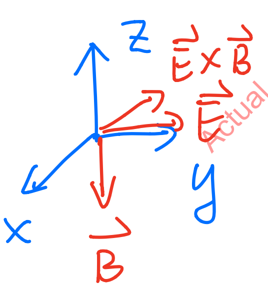
(c) (4 pts) Please derive the expression for the Poynting vector $\vec{S}$.

(a) $\lambda = \frac{2\pi}{k} = 2\pi \times 10^{-7}\ \mathrm{m} \doteq 628\ \mathrm{nm}$ (1 pt).

$T = \frac{1}{f} = \frac{\lambda}{c} = 2.09 \times 10^{-15}\ \mathrm{s}.$ (1 pt).

(b) $B_{max} = \frac{E_{max}}{c}$

$kx+\omega t \Rightarrow$ direction of propagation is $(-\hat{x})$.



$\vec{B}$ has direction $(-\hat{k})$.

$\vec{B} = \frac{E_{max}}{c}\sin(kx+\omega t)(-\hat{k}).$

(1 pt) (1 pt) (1 pt) (1 pt).

(c) $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{E_{max}^2}{c\mu_0}\sin^2(kx+\omega t)(-\hat{i}).$

(1 pt) (1 pt) (1 pt) (1 pt).

Problem 5 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

In an ac series circuit, we have $R = 600\ \Omega$, $L = 120$ mH, $C = 0.25\ \mu$F, voltage amplitude $V = 100$ V, and $\omega = 10{,}000$ rad/s.

(a) (2 pts) Find the impedance Z and the voltage amplitude across each circuit element.

(b) (5 pts) Find expressions for the time dependence of the instantaneous current $\boldsymbol{i}$ and the instantaneous voltages across the resistor ($\boldsymbol{v_R}$), inductor ($\boldsymbol{v_L}$), capacitor ($\boldsymbol{v_C}$), and the source ($\boldsymbol{v}$).

(c) (3 pts) Calculate the power factor and the average power delivered to the entire circuit, and to each circuit element.

(a) $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 1000\ \Omega.$ (1 pt)

$X_L = \omega L = 1200\ \Omega, \quad X_C = \frac{1}{\omega C} = 400\ \Omega.$

$I = \frac{V}{Z} = 0.1\,A.$

$V_R = IR = 60\,V, \ V_L = IX_L = 120\,V, \ V_C = IX_C = 40\,V.$ (1 pt).

(b) $i = I\cos(\omega t) = 0.1\cos(10^4 t).$ (1 pt).

$v_R = V_R\cos(\omega t) = 60\cos(10^4 t).$ (1 pt).

$v_L = V_L\cos(\omega t + 90^\circ) = -120\sin(10^4 t).$ (1 pt).

$v_C = V_C\cos(\omega t - 90^\circ) = 40\sin(10^4 t).$ (1 pt).

$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = 53^\circ.$

$v = V\cos(\omega t + \phi) = 100\cos(10^4 t + 53^\circ).$ (1 pt).

(C) Power factor $\cos\phi = 0.6$. (1pt)

Average power:

$P_R = \frac{1}{2} I V_R \cos 0° = 3$ W. (0.5 pt).

$P_L = 0$ W. (0.5 pt).

$P_C = 0$ W. (0.5 pt).

$P = P_R + P_L + P_C = 3$ W. (0.5 pt).

Alternative time Mid-term Exam 2: PHYSICS 1C (Spring 2021)

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Student Name: Liz + Fangi

Student I.D. Number: ______________________________

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$\varepsilon_2 = -M\dfrac{di_1}{dt}$ and $\varepsilon_1 = -M\dfrac{di_2}{dt}$ (mutually induced emfs)

$M = \dfrac{N_2\Phi_{B2}}{i_1} = \dfrac{N_1\Phi_{B1}}{i_2}$ (mutual inductance) $\varepsilon = -L\dfrac{di}{dt}$ (self-induced emf)

$L = \dfrac{N\Phi_B}{i}$ (self-inductance) $U = L\int_0^I i\,di = \dfrac{1}{2}LI^2$ (energy stored in an inductor)

$u = \frac{B^2}{2\mu_0}$ (magnetic energy density in vacuum) $\quad \tau = \frac{L}{R}$ (time constant for an R-L circuit)

$\omega = \sqrt{\frac{1}{LC}}$ (angular frequency of oscillation in an L-C circuit)

$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ (underdamped oscillations in L-R-C series circuit)

$i = I\cos\omega t$ (sinusoidal alternating current) $\quad I_{\text{rav}} = \frac{2}{\pi}I \quad I_{\text{rms}} = \frac{1}{\sqrt{2}}I \quad V_{\text{rms}} = \frac{1}{\sqrt{2}}V$

$V_R = I\,R \quad V_L = IX_L \quad V_C = I\,X_C \quad V = I\,Z$

$Z = \sqrt{R^2 + [\omega L - (1/\omega C)]^2}$ (impedance of an L-R-C series circuit)

$\tan\phi = \frac{\omega L - 1/\omega C}{R}$ (phase angle of an L-R-C series circuit)

$P_{\text{av}} = \frac{1}{2}VI\cos\phi = V_{\text{rms}}I_{\text{rms}}\cos\phi$ (average power into a general ac circuit)

$\omega_0 = \frac{1}{\sqrt{LC}}$ (L-R-C series circuit at resonance) $\quad \frac{V_2}{V_1} = \frac{N_2}{N_1}$ (terminal voltages in a transformer)

$V_1 I_1 = V_2 I_2$ (terminal voltages and currents in a transformer)

$E = cB$ (electromagnetic wave in vacuum) $\quad c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

$\vec{\boldsymbol{S}} = \frac{1}{\mu_0}\vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}$ (Poynting vector in vacuum)

$I = S_{\text{av}} = \frac{E_{\text{max}}B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}{}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_{\text{max}}{}^2 = \frac{1}{2}\epsilon_0\, cE_{\text{max}}{}^2$

(intensity of a sinusoidal electromagnetic wave in vacuum)

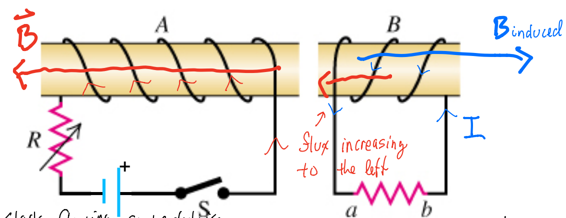
$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$ (flow rate of electromagnetic momentum)

Two constants for your reference: $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$

Problem 1 (8 pts): please ***be very careful*** in writing down your answers for these two questions. They are graded by the final answers ONLY, no partial credits for any intermediate steps.

(a) (2 pts) In the figure, the switch S is open originally. Now let us close it, please determine the direction of the current in the resistor ab.

Your choice: a



Current starts flowing counter clockwise. RHR for solenoids tells us that the magnetic field in solenoid A is pointing to the left. The solenoid gradually allows more current to flow through it, causing B to increase to the left.

As the mag. field in solenoid A increases to the left, solenoid B experiences an increase in magnetic flux to the left. To counteract this, a magnetic field is induced in solenoid B pointing in the opposite direction, so $\vec{B}_{induced}$ points to the right. Using RHR for solenoids, we see that the current must flow from a to b.

a. From point a to b

b. From point b to a

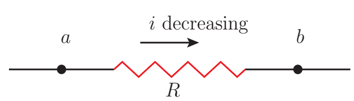
c. Cannot be determined, not enough information

(b) (4 pts) In the following four situations, please determine the sign of potential difference V_{ab} between point a and b? For each of them, choose your answers from:

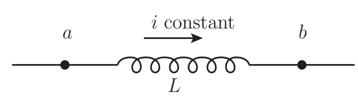
a. $V_{ab} > 0$ b. $V_{ab} < 0$ c. $V_{ab} = 0$ d. V_{ab} cannot be determined

$V = IR$

No dependence on $\frac{dI}{dt}$



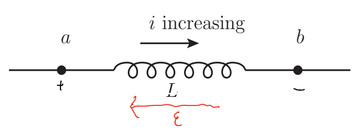
Your choice: a



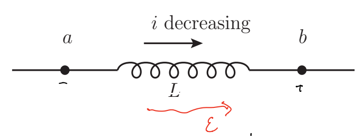
Your choice: c

$V = L\frac{dI}{dt} = L \cdot 0 = 0$

$V_{ab} = L\frac{dI}{dt} > 0$



Your choice: a



Your choice: b

$V_{ab} = L\frac{dI}{dt} < 0$

(c) (2 pts) You plan to take your hair dryer to Europe, where the electric outlets put out 240 V instead of 120 V as seen in the US. The dryer puts out 1600 W at 120 V. Denote the resistance of your dryer as R_1 when operated at 120 V, and the resistance as R_2 for your dryer appear to have when operated at 240 V. Values of (R_1, R_2) = ___d___ Ω

a. (9.0, 9.0) b. (36.0, 9.0) c. (36.0, 36.0) d. (9.0, 36.0)

(See example 31.9 in the textbook!)

We will need a step-down transformer → Need to convert the 240 V a.c. to the 120 V a.c. we require to use the hair dryer

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{120\text{ V}}{240\text{ V}} = \frac{1}{2}$$

We know: $P = V^2/R$

In the U.S., $1600\text{ W} = \frac{(120\text{ V})^2}{R}$

$$R = \frac{(120\text{ V})^2}{1600\text{ W}}$$

$$R_{120} = 9.0\,\Omega$$

In Europe, the apparent resistance is

$$R_{240} = \frac{R_{120}}{(N_2/N_1)^2}$$

$$= \frac{R_{120}}{(\frac{1}{2})^2}$$

$$= 4R_{120}$$

$\therefore R_{240} = 4(9.0\,\Omega)$

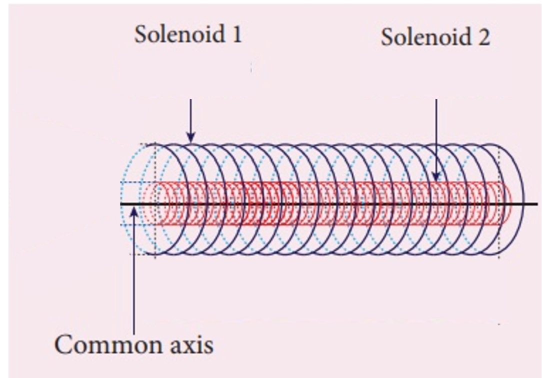
$R_{240} = 36.0\,\Omega$

⇒ So, $\boxed{(R_{120}, R_{240}) = (9.0\,\Omega, 36.0\,\Omega)}$

Problem 2 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

As shown in the figure, the solenoid 1 is 50 cm long and has a cross-sectional diameter of 3.0 cm. This solenoid has a winding of 2000 turns per meter, and the electric current through the windings is changing at a rate $\frac{dI}{dt} = 1.5\ A/s$. A small coil (the red one called solenoid 2 in the figure) consisting of *N* = *20* turns has a cross-sectional diameter of 1.0 cm, and it is placed in the middle of the solenoid such that the plane of the coil is perpendicular to the central axis of the solenoid. Please determine the magnitude of the emf induced in this coil (i.e., solenoid 2).



Mutual Inductance!

$$M = \frac{N_2 \Phi_{B2}}{I_1}$$

Φ_{B2} := flux through Solenoid 2 due to the $\vec{B}$ field created by the current in the windings of Solenoid 1.

We know: $B_{\text{Solenoid}} = \mu_0 n I$

And $\Phi_B = \vec{B} \cdot \vec{A}$

So, $B_1 = \mu_0 n_1 I_1$

$\Phi_{B2} = B_1 A_2 = \mu_0 n_1 I_1 A_2$

Then, $M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_2(\mu_0 n_1 \cancel{I_1} A_2)}{\cancel{I_1}}$

$M = \mu_0 n_1 N_2 A_2$

We also know:

$$|\mathcal{E}_2| = \left| M \frac{dI_1}{dt} \right|$$

So, $|\mathcal{E}_2| = \left| \mu_0 n_1 N_2 A_2 \frac{dI_1}{dt} \right|$

Now we just need to plug and chug!

$\mu_0 = 4\pi \times 10^{-7}\ \frac{T \cdot m}{A}$

$n_1 = 2000\ m^{-1}$

$N_2 = 20$

$A_2 = \pi(.005)^2\ m^2$

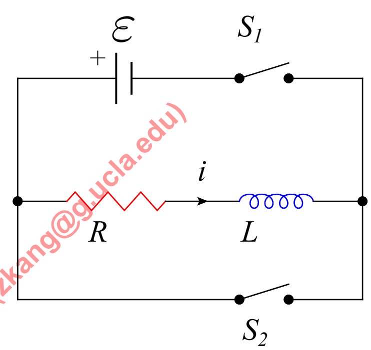
$\frac{dI}{dt} = 1.5\ A/s$

$\Rightarrow |\mathcal{E}_2| = (4\pi \times 10^{-7}\ \frac{T \cancel{m}}{\cancel{A}})(2000\ \cancel{m^{-1}})(20)(\pi(.005^2)\ m^2)(1.5\ \frac{\cancel{A}}{s}) = 5.92 \times 10^{-6}\ \frac{T \cdot m^2}{s} = 5.92 \times 10^{-6}\ \frac{V \cdot \cancel{s}}{\cancel{m^2}} \cdot \frac{\cancel{m^2}}{\cancel{s}} = \boxed{5.92 \times 10^{-6}\ V}$

Problem 3 (12 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

For a *R*-*L* circuit as shown in the figure, the voltage $\varepsilon = 12.0$ V, the resistance $R = 100\ \Omega$, and the inductance $L = 0.01$ H. Suppose both switches are open to begin with, and then at some initial time $t = 0$, we close switch S_1 (leave S_2 still open). The current i is shown in the figure.



(a) (4 pts) Please obtain an expression of the current i as a function of time t. If one denotes the value of the current as I_0 when $t \to \infty$, please determine I_0. (*Note: Do not just copy the final expression from the notes or book, but including intermediate steps please.*)

(b) (4 pts) Please calculate the instantaneous power P_L in the inductor as a function of t. At what value of t, P_L is a maximum?

(c) (4 pts) When the current becomes I_0, one then resets our stopwatch to redefine the initial time, we open switch S_1 but close switch S_2 at $t = 0$. Obtain an expression of the current i as a function of time t.

Problem 4 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

The electric field associated with an electromagnetic wave traveling in vacuum is given by

$\vec{E}(x,t) = \left(20\,\frac{\mathrm{V}}{\mathrm{m}}\right)\sin(kx + \omega t)\,\hat{\jmath}$, where $k = 10^7\,\mathrm{m}^{-1}$.

(a) (2 pts) What is the wavelength λ and period T of the wave?

(b) (4 pts) Please derive the expression for the wave corresponding to the magnetic field $\vec{B}$.

(c) (4 pts) Please derive the expression for the Poynting vector $\vec{S}$.

Problem 5 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

In an ac series circuit, we have $R = 600\ \Omega$, $L = 120$ mH, $C = 0.25\ \mu$F, voltage amplitude $V = 100$ V, and $\omega = 10{,}000$ rad/s.

(a) (2 pts) Find the impedance Z and the voltage amplitude across each circuit element.

(b) (5 pts) Find expressions for the time dependence of the instantaneous current $\boldsymbol{i}$ and the instantaneous voltages across the resistor ($\boldsymbol{v_R}$), inductor ($\boldsymbol{v_L}$), capacitor ($\boldsymbol{v_C}$), and the source ($\boldsymbol{v}$).

(c) (3 pts) Calculate the power factor and the average power delivered to the entire circuit, and to each circuit element.