Mid-term Exam 2: PHYSICS 1C (Spring 2020)

Time: 2:00PM – 3:50PM, May 14, 2020, Instructor: Prof. Zhongbo Kang

Student Name: ________________________________

Student I.D. Number: ___________________________

Note:

- Please make sure that you have ***read, signed and uploaded*** your "student verification form", without which your exam will not be graded.
- The exam time (in total 1 hour and 50 minutes) is designed in such a way that ideally the actual time for answering the problems is 1 hour and 20 minutes, while the remaining 30 minutes are used to download exam from CCLE, scan and upload your solution to gradescope. Please plan your time properly.
- The exam is open book, and open notes. We provide key physical equations. You can use a calculator.
- Remember to write down each step of your calculations, for partial credits.

Score Sheet (total 60 points): _______________________

Problem 1 (12 points): ___________________________

Problem 2 (8 points): ___________________________

Problem 3 (10 points): ___________________________

Problem 4 (10 points): ___________________________

Problem 5 (10 points): ___________________________

Problem 6 (10 points): ___________________________

Formula Sheet

From mid-term 1:

$\vec{F} = q\vec{v} \times \vec{B}$ (magnetic force on a moving charged particle)

$\Phi_B = \int B\cos\phi \, dA = \int B_\perp \, dA = \int \vec{B} \cdot d\vec{A}$ (magnetic flux through a surface)

$\oint \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for magnetism)

$R = \dfrac{mv}{|q|B}$ (radius of a circular orbit in a magnetic field)

$\vec{F} = I\vec{l} \times \vec{B}$ (magnetic force on a straight wire segment)

$d\vec{F} = I\,d\vec{l} \times \vec{B}$ (magnetic force on an infinitesimal wire section)

$\tau = IBA\sin\phi$ (magnitude of magnetic torque on a current loop)

$\vec{\tau} = \vec{\mu} \times \vec{B}$ (vector magnetic torque on a current loop)

$U = -\vec{\mu} \cdot \vec{B} = -\mu B\cos\phi$ (potential energy for a magnetic dipole)

$\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{q\vec{v} \times \hat{r}}{r^2}$ (magnetic field due to a point charge with constant velocity)

$d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{I\,d\vec{l} \times \hat{r}}{r^2}$ (magnetic field due to an infinitesimal current element)

$B = \dfrac{\mu_0 I}{2\pi r}$ (magnetic field near a long, straight, current-carrying conductor)

$\dfrac{F}{L} = \dfrac{\mu_0 I I'}{2\pi r}$ (two long, parallel, current-carrying conductors) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ (Ampere's law)

New for mid-term 2:

$\varepsilon = -\dfrac{d\Phi_B}{dt}$ (Faraday's law of induction)

$\varepsilon = vBL$ (motional emf; length and velocity perpendicular to uniform $\vec{B}$)

$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ (motional emf; closed conducting loop)

$\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ (stationary integration path)

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \dfrac{d\Phi_E}{dt} \right)_{\text{encl}}$ (Ampere's law for a stationary integration path)

$\varepsilon_2 = -M\dfrac{di_1}{dt}$ and $\varepsilon_1 = -M\dfrac{di_2}{dt}$ (mutually induced emfs)

$M = \dfrac{N_2 \Phi_{B2}}{i_1} = \dfrac{N_1 \Phi_{B1}}{i_2}$ (mutual inductance) $\varepsilon = -L\dfrac{di}{dt}$ (self-induced emf)

$L = \dfrac{N\Phi_B}{i}$ (self-inductance) $U = L\int_0^I i\,di = \dfrac{1}{2}LI^2$ (energy stored in an inductor)

$u = \dfrac{B^2}{2\mu_0}$ (magnetic energy density in vacuum) $\tau = \dfrac{L}{R}$ (time constant for an R-L circuit)

$\omega = \sqrt{\dfrac{1}{LC}}$ (angular frequency of oscillation in an L-C circuit)

$\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$ (underdamped oscillations in L-R-C series circuit)

$i = I\cos\omega t$ (sinusoidal alternating current) $I_{\text{rav}} = \dfrac{2}{\pi} I$ $I_{\text{rms}} = \dfrac{1}{\sqrt{2}} I$ $V_{\text{rms}} = \dfrac{1}{\sqrt{2}} V$

$V_R = I\,R$ $V_L = IX_L$ $V_C = I\,X_C$ $V = I\,Z$

$Z = \sqrt{R^2 + \left[\omega L - (1/\omega C)\right]^2}$ (impedance of an L-R-C series circuit)

$\tan\phi = \dfrac{\omega L - 1/\omega C}{R}$ (phase angle of an L-R-C series circuit)

$P_{\text{av}} = \frac{1}{2} VI\cos\phi = V_{\text{rms}} I_{\text{rms}} \cos\phi$ (average power into a general ac circuit)

$\omega_0 = \dfrac{1}{\sqrt{LC}}$ (L-R-C series circuit at resonance) $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ (terminal voltages in a transformer)

$V_1 I_1 = V_2 I_2$ (terminal voltages and currents in a transformer)

$E = cB$ (electromagnetic wave in vacuum) $c = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}}$

$\vec{\boldsymbol{S}} = \dfrac{1}{\mu_0} \vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}$ (Poynting vector in vacuum)

$I = S_{\text{av}} = \dfrac{E_{\max} B_{\max}}{2\mu_0} = \dfrac{E_{\max}{}^2}{2\mu_0 c} = \dfrac{1}{2}\sqrt{\dfrac{\epsilon_0}{\mu_0}} E_{\max}{}^2 = \dfrac{1}{2}\epsilon_0\, c E_{\max}{}^2$

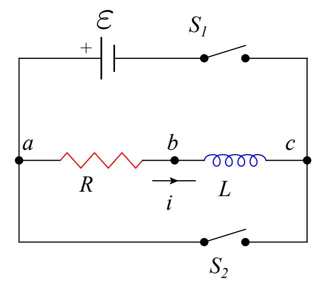
(intensity of a sinusoidal electromagnetic wave in vacuum)

$\dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{S}{c} = \dfrac{EB}{\mu_0 c}$ (flow rate of electromagnetic momentum)

Two constants for your reference: $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$

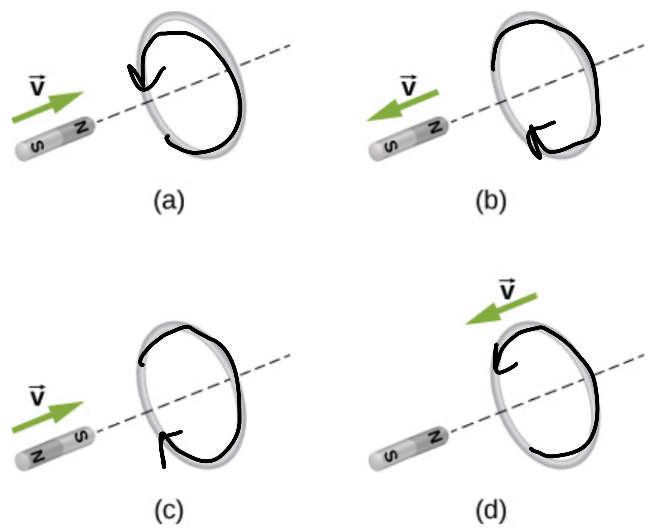
Problem 1 (12 pts): for the questions below, please give *your final answers only*.

(a) (2 pts) In the following figure, the current is flowing in the direction shown, what are the algebraic signs of the potential differences v_{ab} and v_{bc} when switch S_1 is closed and switch S_2 is open: ___a___? What about when S_1 is open and S_2 is closed: ___b___?



a. $v_{ab} > 0, v_{bc} > 0$ b. $v_{ab} > 0, v_{bc} < 0$ c. $v_{ab} < 0, v_{bc} > 0$ d. $v_{ab} < 0, v_{bc} < 0$

(b) (4 pts) For each of the following situations shown in the figure, please indicate the direction of the induced current with an arrow on the conducting loop, observing from the side of the magnet. Please make a clean drawing so that the arrow is unambiguous.



(c) (2 pts) An electromagnetic plane wave propagates in the vacuum. Its electric field $\vec{E}(x,t) = E_{\max}\cos(kx + \omega t)\,\hat{j}$, please determine the direction of the Poynting vector. Your choice: ___c___

a. $+y$ b. $-y$ c. $-x$ d. $+x$ e. $-z$ f. $+z$

g. not enough information, cannot be determined

(d) (2 pts) In a *L*-*R*-*C* series circuit with AC current, the voltage amplitudes V_R, V_L, V_C for resistor *R*, the inductor *L*, and the capacitor *C* depend on the angular frequency ω. As ω increases, V_R will be___c___; V_L will be___a___

a. increasing b. decreasing c. unchanged d. undetermined

(e) (2 pts) A transformer on a utility pole steps the rms voltage down from 12 kV to 240 V. What is the ratio of the number of secondary turns to the number of primary turns? $\frac{N_2}{N_1} =$ __0.02__

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{240}{12000}$$

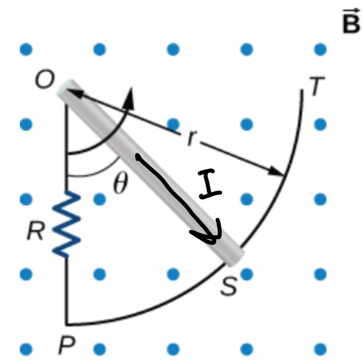
If the input current to the transformer is 2.0 A, what is the output current? $I_{\text{out}} =$ __100 A__

$$\frac{I_2}{I_1} = \frac{V_1}{V_2}$$

$$I_2 = I_1\left(\frac{V_1}{V_2}\right)$$

Problem 2 (8 pts)

As shown in the figure, a metal rod *OS* is rotating in a horizontal plane around the point *O*. The rod slides along a wire that forms a circular arc *PST* of radius *r*. The system is in a constant magnetic field $\vec{B}$ that is directed out of the page, and we rotate the rod at a constant angular velocity ω. Please answer the following questions



a) (3 pts) Start specifically from Faraday's law $\varepsilon = -\frac{d\Phi_B}{dt}$ to compute the current *I* in the closed loop *OSPO*? Assume that the resistor *R* furnishes all of the resistance in the closed loop. Provide the magnitude only. (*hint: you can compute the magnetic flux Φ_B for the loop area covered by OSPO.*)

b) (3 pts) Start specifically from motional emf formula $\varepsilon = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$ to compute the current *I* in the closed loop *OSPO*? Indicate the direction of the current in the figure.

c) (2 pts) Calculate the power dissipated in the resistor.

a) $\Phi_B = \vec{B} \cdot \vec{A}$ $\qquad \theta = \omega t$

$A = \left(\frac{\theta}{2\pi}\right) \pi r^2$

$= \frac{1}{2} \theta r^2$

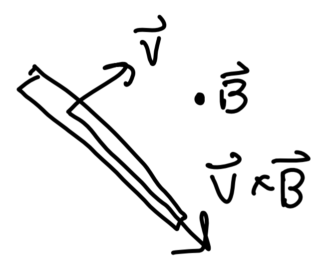
$\frac{d\Phi_B}{dt} = \frac{d}{dt}\left(B \cdot \frac{1}{2} \omega t r^2\right)$

$= \frac{1}{2} B \omega r^2$

$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{1}{2} B \omega r^2$

$I = \frac{|\varepsilon|}{R} = \boxed{\frac{B\omega r^2}{2R}}$

b) $v = \omega r$



$\varepsilon = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$= \int_0^r \omega r B \, dr$

$= \omega B \left[\frac{1}{2} r^2\right]_0^r$

$= \frac{1}{2} \omega B r^2$

$I = \frac{\varepsilon}{R} = \boxed{\frac{\omega B r^2}{2R}}$

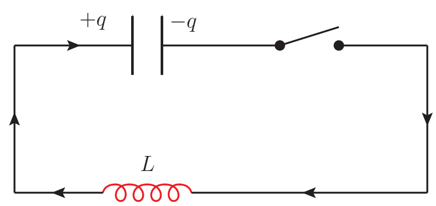
(c) $P = \frac{\mathcal{E}^2}{R} = \frac{1}{R}\left(\frac{1}{2}B\omega r^2\right)^2$

$$= \frac{1}{R}\left(\frac{1}{4}B^2\omega^2 r^4\right)$$

$$= \boxed{\frac{B^2\omega^2 r^4}{4R}}$$

Problem 3 (10 pts)

In a *L-C* circuit as shown in the figure, one closes the switch at some initial time $t = 0$. We find that the maximum charge on the capacitor is 2.0×10^{-6} C and the maximum current through the inductor is 8.0 mA. Please answer the following questions



a) (2 pts) *Briefly* describe *qualitatively* how the current behaves in such a *L-C* circuit. You are **not** supposed to compute anything.

b) (3 pts) What is the period of the circuit?

c) (2 pts) How much time elapses between an instant when the capacitor is fully charged and the next instant when it is completely uncharged?

d) (3 pts) If the capacitance is $C = 1.0\mu F$, what is the self-inductance L?

a) The current oscillates by changing direction back and forth, and the charges on the capacitor also switch sides back and forth.

b) $I_{max} = \omega Q$

$$\omega = \frac{I_{max}}{Q} \qquad T = \frac{2\pi}{\omega} = \frac{2\pi Q}{I_{max}} = \boxed{0.0016\ s}$$

c) $\frac{T}{4} = \boxed{3.9 \times 10^{-4}\ s}$

d) $\omega = \frac{1}{\sqrt{LC}} = \frac{I_{max}}{Q}$

$$\sqrt{LC} = \frac{Q}{I_{max}} \qquad LC = \frac{Q^2}{I_{max}^2}$$

$$L = \frac{Q^2}{C I_{max}^2} = \boxed{0.0625\ H}$$

Problem 4 (10 pts)

An electromagnetic wave traveling in $-z$ direction in vacuum has a wavelength $\lambda = 6 \times 10^{-7}$ m. The magnetic field associated with such an electromagnetic wave is along $+x$ direction and has an amplitude $B_{\max} = 6 \times 10^{-8}$ T. Note for the vectors below, your expression should reflect the direction.

(a) (3 pts) Please derive the expression for the wave corresponding to the magnetic field $\vec{B}$.

(b) (3 pts) Please derive the expression for the wave corresponding to the electric field $\vec{E}$.

(c) (2 pts) Please derive the expression for the instantaneous Poynting vector $\vec{S}$.

(d) (2 pts) Find the instantaneous values of the total energy density u.

(a) $k = \frac{2\pi}{\lambda}$ $\quad \omega = ck = \frac{2\pi c}{\lambda}$

$$\vec{B}(z,t) = B_{max}\cos(kz + \omega t)\,\hat{i}$$

$$= (6\times10^{-8}\,T)\cos\left[(1.04\times10^{7}\,m^{-1})\,z + (3.14\times10^{15}\,rad/s)\,t\right]\hat{i}$$

(b) $E_{max} = c\,B_{max}$

$$\vec{E}(z,t) = cB_{max}\cos(kz + \omega t)\,\hat{j}$$

$$= \left(18\,\frac{V}{m}\right)\cos\left[(1.04\times10^{7}\,m^{-1})\,z + (3.14\times10^{15}\,rad/s)\,t\right]\hat{j}$$

(c) $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$

$$= \frac{EB}{\mu_0}(-\hat{k})$$

$$= \frac{(18\cos(kz+\omega t))(6\times10^{-8}\cos(kz+\omega t))}{\mu_0}(-\hat{k})$$

$$= -\left(0.859\,\frac{W}{m^2}\right)\cos\left[(1.04\times10^7\,m^{-1})z+(3.14\times10^{15}\,rad/s)t\right]\hat{k}$$

(d) $S = cu$

$$u = \frac{S}{c} = \boxed{\underbrace{(2.86\times10^{-9})}_{J/m^3}\cos\left[(1.04\times10^7\,m^{-1})z+(3.14\times10^{15}\,rad/s)t\right]}$$

Problem 5 (10 pts)

A capacitor with capacitance C whose plates have area A and separation distance d is connected to a resistor R and a battery of voltage V. The current starts to flow at time $t = 0$. Please answer the following questions

a) (3 pts) What is the voltage $V_C(t)$ between the plates at time t? (*hint: recall what you have learned on RC circuit. You might refer to eTextbook section 26.4.*)

b) (4 pts) Compute the displacement current I_d between the capacitor plates at time t.

c) (3 pts) From the properties of the capacitor, find the conducting current I_c in the RC circuit. Compare your answer to the displacement current, what is their relation?

a) $V_c(t) = \boxed{V\left(1 - e^{-\frac{t}{RC}}\right)}$

b) $V_c = Ed$

$E = \frac{V_c}{d}$

$\Phi_E = EA = \frac{V_c}{d} A$

$\frac{d\Phi_E}{dt} = \frac{A}{d}\left(\frac{dV_c}{dt}\right) = \left(\frac{A}{d}\right)\frac{d}{dt}\left(V - Ve^{-\frac{t}{RC}}\right)$

$= \frac{A}{d}\left(\frac{1}{RC} Ve^{-\frac{t}{RC}}\right)$

$C = \epsilon_0 \frac{A}{d}$

$= \frac{AV}{dRC} e^{-\frac{t}{RC}} = \frac{V}{R} e^{-\frac{t}{RC}}$

$i_D = \epsilon_0 \frac{d\Phi_E}{dt} = \boxed{\frac{V}{R} e^{-\frac{t}{RC}}}$

c) $q = CV\left(1 - e^{-\frac{t}{RC}}\right)$

$$I_c = \frac{dq}{dt} = \frac{CV}{RC} e^{-\frac{t}{RC}} = \boxed{\frac{V}{R} e^{-\frac{t}{RC}}}$$

$$\boxed{I_c = I_d}$$

Problem 6 (10 pts)

A 200 Ω resistor, a 1.00 H inductor, and a 4.00 μF capacitor are connected in series across a voltage source that has voltage amplitude 40.0 V.

(a) (3 pts) At what angular frequency will the impedance be smallest? What is the impedance Z at this angular frequency?

(b) (2 pts) Assume the angular frequency of 200 rad/s for the rest of questions. What is the power factor for this circuit?

(c) (2 pts) What is the average power delivered to the capacitor, and to the inductor?

(d) (3 pts) At time t = 10.0 ms, please compute the instantaneous current i and instantaneous voltage v.

(a) $\omega_0 = \frac{1}{\sqrt{LC}} = \boxed{500 \text{ rad/s}}$

$Z_{min} = R = \boxed{200\Omega}$

(b) power factor $= \cos\phi = \frac{R}{Z}$

$= \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \boxed{0.187}$

(c) $P_{L,av} = P_{C,av} = \boxed{0 \text{ W}}$

(d) $I = \frac{V}{Z}$

$i(t) = I\cos(\omega t) = \frac{V}{Z}\cos(\omega t)$

$i(10\text{ ms}) = \boxed{-0.0156 \text{ A}}$

$$v(t) = V\cos(\omega t + \phi) \qquad \phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$v(10\,\text{ms}) = \boxed{32.6\ \text{V}}$$