22W-PHYSICS-1C Mid-term 1

XIMENG GUO

TOTAL POINTS

25 / 28

QUESTION 1

8 pts

1.1 Forces on q2 3 / 3

✓ **- 0 pts** Correct

- 3 pts Incorrect

1.2 magnitude of the current 5 / 5

✓ **- 0 pts** Correct

- 3 pts Incorrect magnitude

- 2 pts Incorrect direction

- 5 pts Incorrect magnitude and direction

QUESTION 2

10 pts

2.1 8 / 10

- 0 pts Correct

- 2 pts B12 is incorrect

- 1 pts B23 magnitude is incorrect

✓ **- 1 pts B23 direction is incorrect**

- 2 pts B34 is incorrect

- 1 pts B41 magnitude is incorrect

✓ **- 1 pts B41 direction is incorrect**

- 1 pts Net B is incorrect

- 2 pts Net B is incorrect

QUESTION 3

10 pts

3.1 4.5 / 5

✓ **+ 0.5 pts Written the correct expression for magnetic field due to a straight long wire $$B=\frac{\mu_0 i}{2\pi r}$$**

✓ **+ 0.5 pts Defined correct area element $$A=a\cdot dr$$**

✓ **+ 1 pts Calculated Magnetic flux through a element $$dr$$, i.e. calculated $$d\phi_B$$**

✓ **+ 1 pts Set up the Integral to calculate the Magnetic flux $$\phi_B$$ correctly.**

✓ **+ 2 pts Evaluated the integral correctly to get Magnetic flux $$\phi_B$$**

+ 0 pts Not attempted

- 0.5 Point adjustment

correct approach but made a mistake....

(1) should be $$a$$ and not $$a^2$$ .... as $$dA=a\cdot dr$$

3.2 4.5 / 5

✓ **+ 1 pts Correct expression for induced electromotive force written, $$|\epsilon|=|\frac{d\phi_B}{dt}|$$.**

✓ **+ 1 pts The velocity $$v=\frac{ds}{dt}$$ is written.**

✓ **+ 1 pts The correct expression for $$\epsilon$$ is set up, i.e. $$\epsilon=\frac{d\phi_B}{ds}\frac{ds}{dt}$$ using chain rule.**

✓ **+ 2 pts The expression is evaluated correctly.**

+ 0 pts Not attempted

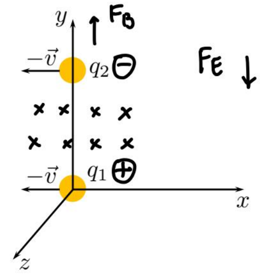
- 0.5 Point adjustment

correct approach but got the derivative wrong....due to incorrect limits....

Problem 1 (8 pts)

Please ***be very careful*** in writing down your answers for these two questions. They are graded by the final answers ONLY, no partial credits for any intermediate steps.

a) (3 pts) In the figure, q_1 is a positive charge while q_2 is a negative charge. They both move at the same speed v, and along the same $-x$ direction. Please determine the direction of the electric force F_E and the magnetic force F_B on the upper charge q_2. Your choice: ___d___



a. both along $+y$

b. both along $-y$ ✓

c. F_E is $+y$, F_B is $-y$

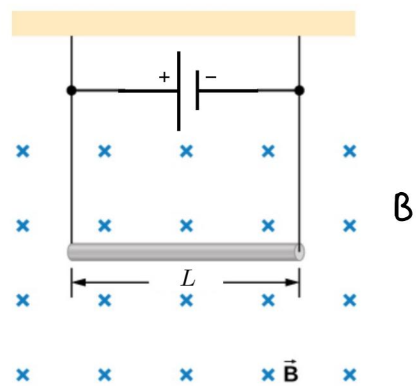
d. F_E is $-y$, F_B is $+y$ ✓ ✓

e. F_E is $+y$, F_B is $+z$

f. F_E is $+y$, F_B is $-z$

g. none of the above

b) (5 pts) A solid conducting wire of length L and mass m is suspended in a horizontal plane by a pair of flexible leads. The wire is then subjected to a constant magnetic field of magnitude B, which is directed as shown. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads? Express your answer in terms of the given variables m, g, L, B.



Magnitude (3 pts): ___$\frac{mg}{LB}$___

Direction (2 pts, e.g., from left to right or from right to left?): ___from left to right___

$F = mg = ILB$

$I = \frac{mg}{LB}$

1.1 Forces on q2 **3 / 3**

✓ **- 0 pts** **Correct**

- 3 pts Incorrect

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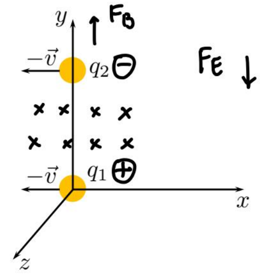
Please ***be very careful*** in writing down your answers for these two questions. They are graded by the final answers ONLY, no partial credits for any intermediate steps.

a) (3 pts) In the figure, q_1 is a positive charge while q_2 is a negative charge. They both move at the same speed v, and along the same $-x$ direction. Please determine the direction of the electric force F_E and the magnetic force F_B on the upper charge q_2. Your choice: d



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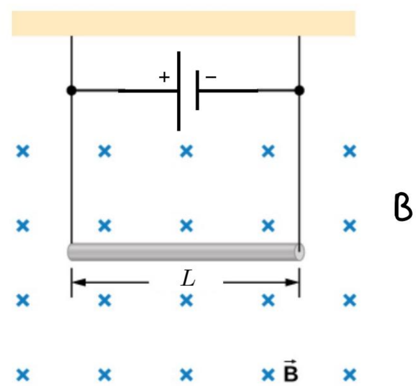

Magnitude (3 pts): $\frac{mg}{LB}$

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1.2 magnitude of the current 5 / 5

- ✓ **- 0 pts** Correct
- **- 3 pts** Incorrect magnitude
- **- 2 pts** Incorrect direction
- **- 5 pts** Incorrect magnitude and direction

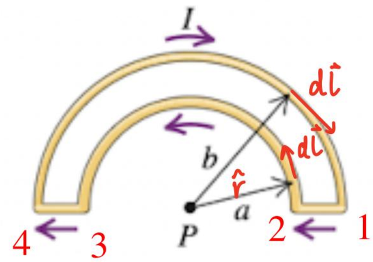
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} \quad \text{(magnetic field due to an infinitesimal current element)}$$

Problem 2 (10 pts)

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{(magnetic field near a long, straight, current-carrying conductor)}$$

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

The wire semicircles shown in the figure have radii ***a*** and ***b***. The current inside the wire is given by ***I***. Let us divide the wire into four pieces: horizontal piece [12], the small semicircle [23], horizontal piece [34], and the large semicircle [41]. The field point *P* is at the center of the semicircle. Please answer the following questions (***specify the magnitude and direction***)



a) (8 pts) find the magnetic field that is produced at the point *P* by each of the four wire segments: [12], [23], [34], and [41].

b) (2 pts) what is the net magnetic field that the entire wires produce at the point *P*?

a). [12]: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\cdot dl\cdot -\hat{i}\times -\hat{i}}{r^2}$

$\because -\hat{i}\times -\hat{i} = 0$

$\therefore d\vec{B} = 0, \quad \vec{B}_{[12]} = 0$

[23]: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\cdot d\vec{l}\times\hat{r}}{r^2}$ $\quad d\vec{l}\times\hat{r} = dl\cdot\sin 90^\circ\, -\hat{k} = dl\cdot -\hat{k}$

$= \frac{\mu_0}{4\pi}\frac{I\cdot dl}{a^2}$

$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\cdot \pi a}{a^2}$

$= \frac{\mu_0 I}{4a}$ $\quad \therefore \vec{B}_{[23]} = \frac{\mu_0 I}{4a}$, direction is $-\hat{k}$ into the page

[34]: similar to [12], $-\hat{i}\times\hat{i} = 0$, so $\vec{B}_{[34]} = 0$

[41]: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\cdot d\vec{l}\times\hat{r}}{r^2}$, $\quad d\vec{l}\times\hat{r} = dl\cdot\sin 90^\circ\,\hat{k} = dl\,\hat{k}$

$= \frac{\mu_0}{4\pi}\frac{I\,dl}{b^2}$

$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\cdot\pi b}{b^2} = \frac{\mu_0 I}{4b}$ $\quad \therefore \vec{B}_{[41]} = \frac{\mu_0 I}{4b}$, direction is $\hat{k}$ out of the page

4

b). net magnetic field: $0 + 0 + \frac{\mu_0 I}{4b}\hat{k} - \frac{\mu_0 I}{4a}\hat{k} = \left(\frac{\mu_0 I}{4a} - \frac{\mu_0 I}{4b}\right) -\hat{k}$

in $-\hat{k}$ direction

2.1 8 / 10

- **- 0 pts** Correct
- **- 2 pts** B12 is incorrect
- **- 1 pts** B23 magnitude is incorrect
- ✓ **- 1 pts B23 direction is incorrect**
- **- 2 pts** B34 is incorrect
- **- 1 pts** B41 magnitude is incorrect
- ✓ **- 1 pts B41 direction is incorrect**
- **- 1 pts** Net B is incorrect
- **- 2 pts** Net B is incorrect

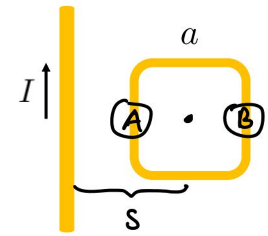
$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$ (magnetic field due to an infinitesimal current element)

$B = \frac{\mu_0 I}{2\pi r}$ (magnetic field near a long, straight, current-carrying conductor)

$\Phi_B = \int B\cos\phi\, dA = \int B_\perp dA = \int \vec{B} \cdot d\vec{A}$ (magnetic flux through a surface)

Problem 3 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.



A long, straight wire shown in the figure carries a current ***I***. A square loop, of side length ***a***, is made of a conducting material and has resistance ***R***. It is positioned so that it is coplanar with the wire, and with one side parallel to it. The center of the loop is at a distance ***s*** from the wire (***s*** > ***a/2***). Please answer the following questions

$\varepsilon = -\frac{d\Phi_B}{dt}$ (Faraday's law of induction)

$\varepsilon = vBL$ (motional emf; length and velocity perpendicular to uniform $\vec{B}$)

$\varepsilon = \oint(\vec{v} \times \vec{B}) \cdot d\vec{l}$ (motional emf; closed conducting loop)

(a) (5 pts) What is the magnetic flux Φ_B going through the loop?

(b) (5 pts) If the loop is now moved at a constant speed ***v*** away from the wire (in the same plane, in the direction perpendicular to the wire), what is the induced emf in the loop, as a function of distance ***s***? *We just need the magnitude of the emf.*

(a).

$$B = \frac{\mu_0 I}{2\pi r}$$

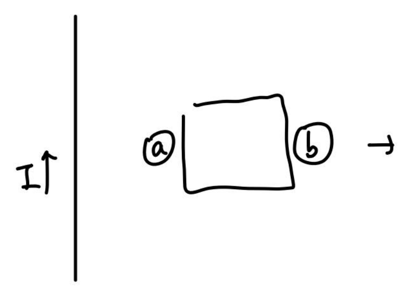
$$\int_{s-\frac{a}{2}}^{s+\frac{a}{2}} \frac{\mu_0 I}{2\pi r} dr = \frac{\mu_0 I}{2\pi}\int_{s-\frac{a}{2}}^{s+\frac{a}{2}} \frac{1}{r} dr = \frac{\mu_0 I}{2\pi} \ln\left(\frac{s+\frac{a}{2}}{s-\frac{a}{2}}\right)$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA = \frac{\mu_0 I}{2\pi} \ln\left(\frac{s+\frac{a}{2}}{s-\frac{a}{2}}\right) a$$

(b).

$$s = v\Delta t \qquad \varepsilon = -\frac{d\Phi_B}{dt}$$

$$\Delta\Phi_B = \frac{\mu_0 I}{2\pi} \ln\left(\frac{s+\frac{a}{2}}{s-\frac{a}{2}}\right) a^2 - \frac{\mu_0 I}{2\pi} \ln\left(\frac{s+\frac{a}{2}+vt}{s-\frac{a}{2}-vt}\right) a^2$$



$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\varepsilon = \int_{A} (\vec{v} \times \vec{B})\, dl + \int_{B} (\vec{v} \times \vec{B})\, dl$$

$$= v\frac{\mu_0 I}{2\pi s} \cdot a - v \cdot \frac{\mu_0 I}{2\pi(s+a)} a$$

$$= \frac{v\mu_0 I a}{2\pi}\left(\frac{1}{s} - \frac{1}{s+a}\right)$$

3.1 **4.5 / 5**

- ✓ + **0.5 pts** **Written the correct expression for magnetic field due to a straight long wire** $B=\frac{\mu_0 i}{2\pi r}$
- ✓ + **0.5 pts** **Defined correct area element** $A=a\cdot dr$
- ✓ + **1 pts** **Calculated Magnetic flux through a element** dr**, i.e. calculated** $d\phi_B$
- ✓ + **1 pts** **Set up the Integral to calculate the Magnetic flux** ϕ_B **correctly.**
- ✓ + **2 pts** **Evaluated the integral correctly to get Magnetic flux** ϕ_B
- + **0 pts** Not attempted
- **- 0.5** **Point adjustment**

  correct approach but made a mistake....

1 should be a and not a^2 .... as $dA=a\cdot dr$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$ (magnetic field due to an infinitesimal current element)

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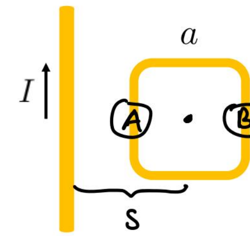

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(a).

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(b).

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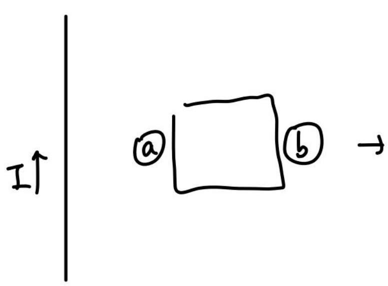

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$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = \int_{A} (\vec{v} \times \vec{B}) dl + \int_{B} (\vec{v} \times \vec{B}) dl$$

$$= v \frac{\mu_0 I}{2\pi s} \cdot a - v \cdot \frac{\mu_0 I}{2\pi(s+a)} a$$

$$= \frac{v \mu_0 I a}{2\pi} \left(\frac{1}{s} - \frac{1}{s+a}\right)$$

3.2 **4.5 / 5**

- ✓ **+ 1 pts** Correct expression for induced electromotive force written, $$|\epsilon|=|\frac{d\phi_B}{dt}|$$.
- ✓ **+ 1 pts** The velocity $$v=\frac{ds}{dt}$$ is written.
- ✓ **+ 1 pts** The correct expression for $$\epsilon$$ is set up, i.e. $$\epsilon=\frac{d\phi_B}{ds}\frac{ds}{dt}$$ using chain rule.
- ✓ **+ 2 pts** The expression is evaluated correctly.
- **+ 0 pts** Not attempted
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