Mid-term Exam 1: PHYSICS 1C (Spring 2021)

Time: 2:00PM – 3:00PM, April 15, 2020, Instructor: Prof. Zhongbo Kang

Student Name: ___________________________________

Student I.D. Number: ______________________________

Exam Version: **A**

Note:

- Please make sure that you have ***read, signed and uploaded*** your "student verification form", without which your exam will not be graded.
- The exam time (in total 1 hour) is designed in such a way that ideally the actual time for answering the problems is 30 minutes, while the remaining 30 minutes are used to scan and upload your solution to gradescope.
- The exam is open book, and open notes. One page of physical equations is provided. You can use a calculator.
- Remember to write down each step of your calculations, for partial credits.

Score Sheet:

Problem 1 (8 points): ______________________________

Problem 2 (10 points): ______________________________

Problem 3 (7 points): ______________________________

Total (25 points): __________________________________

Formula Sheet

$\vec{F} = q\vec{v} \times \vec{B}$ (magnetic force on a moving charged particle)

$\Phi_B = \int B\cos\phi \, dA = \int B_\perp \, dA = \int \vec{B} \cdot d\vec{A}$ (magnetic flux through a surface)

$\oint \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for magnetism)

$R = \dfrac{mv}{|q|B}$ (radius of a circular orbit in a magnetic field)

$\vec{F} = I\vec{l} \times \vec{B}$ (magnetic force on a straight wire segment)

$d\vec{F} = I\,d\vec{l} \times \vec{B}$ (magnetic force on an infinitesimal wire section)

$\tau = IBA\sin\phi$ (magnitude of magnetic torque on a current loop)

$\vec{\tau} = \vec{\mu} \times \vec{B}$ (vector magnetic torque on a current loop)

$U = -\vec{\mu} \cdot \vec{B} = -\mu B\cos\phi$ (potential energy for a magnetic dipole)

$\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{q\vec{v} \times \hat{r}}{r^2}$ (magnetic field due to a point charge with constant velocity)

$d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{I\,d\vec{l} \times \hat{r}}{r^2}$ (magnetic field due to an infinitesimal current element)

$B = \dfrac{\mu_0 I}{2\pi r}$ (magnetic field near a long, straight, current-carrying conductor)

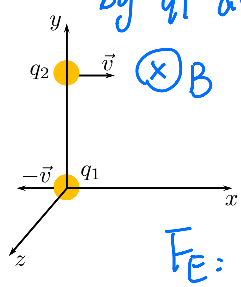
$\dfrac{F}{L} = \dfrac{\mu_0 I I'}{2\pi r}$ (two long, parallel, current-carrying conductors) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$ (Ampere's law)

Problem 1 (8 pts)

Please ***be very careful*** in writing down your answers for these two questions. They are graded by the final answers ONLY, no partial credits for any intermediate steps.

a) (3 pts) In the figure, two positive charges q_1 and q_2 move at the same speed v, but in the opposite direction. q_1 is moving along $-x$, while q_2 is along $+x$ direction. Determine the direction of the electric force F_E and magnetic force F_B on the upper charge q_2.

Your choice: a.



Magnetic field B generated by q_1 at q_2: ⊗B Thus $\vec{F_B}$ ↑

F_E: q_1, q_2 are both positive, thus ↑ $\vec{F_E}$.

a. both along $+y$

b. both along $-y$

c. F_E is $+y$, F_B is $-y$

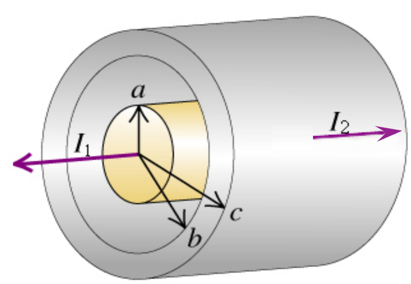
d. F_E is $-y$, F_B is $+y$

e. F_E is $+y$, F_B is $+z$

f. F_E is $+y$, F_B is $-z$

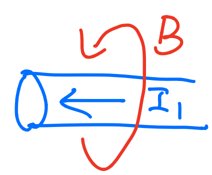
g. none of the above

b) (5 pts) A solid conductor with radius a is supported by insulating disks on the axis of a conducting tube with inner radius b and outer radius c, see the figure. The central conductor and tube carry currents I_1 and I_2 correspondingly in the opposite direction. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude and specify the direction of the magnetic field at points outside the central, solid conductor but inside the tube, i.e., $a < r < b$. Express your answer in terms of the given variables I_1, I_2, r, a, b, c.



Magnitude (3 pts): $\frac{\mu_0 I_1}{2\pi r}$

Direction (2 pts): Counter clockwise. (shown in the plot).



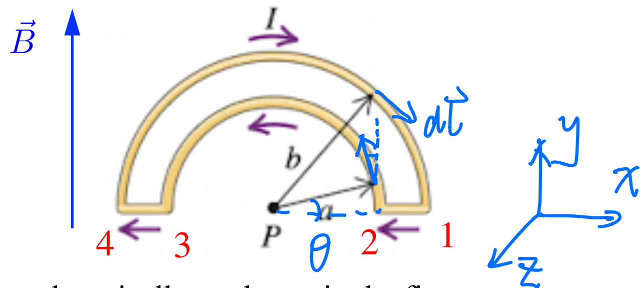
B is only dependent on I_1 and r for $a<r<b$.

(I_2 does not contribute!)

Problem 2 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

The wire semicircles shown in the figure have radii ***a*** and ***b***. The current inside the wire is given by ***I***. Let us divide the wire into four pieces: horizontal piece [12], the small semicircle [23], horizontal piece [34], and the large semicircle [41]. Now a constant uniform magnetic field $\vec{B}$, is uniformly distributed *in the entire space* and pointing upward vertically as shown in the figure. Please answer the following questions (specify the magnitude and direction)



a) (8 pts) find the magnetic force on each of the four wire segments: [12], [23], [34], and [41].

b) (2 pts) what is the net force on the entire loop?

a) [12]: $\vec{F}_{[12]} = \int I\, d\vec{l} \times \vec{B} = I\, l_{[12]}\, B\, (-\hat{k}) = -I(b-a)B\hat{k}$.

[23]: $\vec{F}_{[23]} = \int I\, d\vec{l} \times \vec{B} = \int_0^{\pi} I\, a\, d\theta\, B \sin\theta\, (-\hat{k})$

$= -2IaB\hat{k}$.

[34]: $\vec{F}_{[34]} = \vec{F}_{[12]} = -I(b-a)B\hat{k}$.

[41]: $\vec{F}_{[41]} = \int I\, d\vec{l} \times \vec{B} = \int_0^{\pi} I\, b\, d\theta\, B \sin\theta\, \hat{k}$

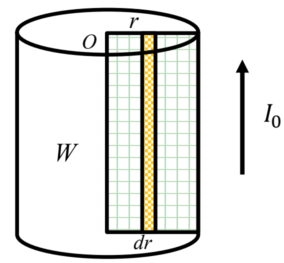
$= 2IbB\hat{k}$.

b) $\vec{F} = \vec{F}_{[12]} + \vec{F}_{[23]} + \vec{F}_{[34]} + \vec{F}_{[41]} = 0\ N$.

Problem 3 (7 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

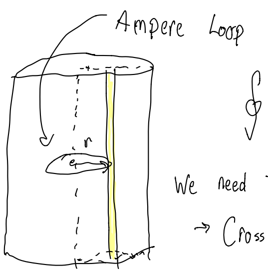
A very long, cylindrical wire of radius R carries a current I_0 uniformly distributed across the cross section of the wire, as shown in the figure.



(a) (3 pts) For the yellow strip that has distance r away from the center O, please compute the magnetic field that goes into the strip.

(b) (4 pts) Calculate the magnetic flux through the entire rectangle that has a length W.

a.) We can use Ampere's Law to compute $\vec{B}$!



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

We need to calculate I_{enc} → $\sigma = \frac{I}{A}$ → we know I_0 is uniformly distributed over the cross section of the wire

→ Cross Sectional Area of entire wire: $A = \pi R^2$

→ So, $\sigma = \frac{I_0}{\pi R^2}$

→ $I_{enc} = \sigma \cdot \Delta A$

↳ cross sectional area of the Ampere loop: $\Delta A = \pi r^2$

→ $I_{enc} = \sigma \pi r^2$

$= \frac{I_0}{\pi R^2} \pi r^2$

So: $\oint \vec{B} \cdot d\vec{l} = \frac{\mu_0 I_0 \pi r^2}{\pi R^2} = \frac{\mu_0 I_0 r^2}{R^2}$

$B(2\pi r) = \frac{\mu_0 I_0 r^2}{R^2}$

⇒ $\boxed{B = \frac{\mu_0 I_0 r}{2\pi R^2}}$

b.) We need to use $\Phi_B = \vec{B} \cdot \vec{A}$

⇒ $d\Phi_B = \vec{B} \cdot d\vec{A}$ → $\Phi_B = \int_S \vec{B} \cdot d\vec{A}$ → $dA = W \cdot dr$ (Area of a teeny strip of the rectangle with width dr

$$\Phi_B = \int_0^R B \cdot W dr = \int_0^R \frac{\mu_0 I_0 W r}{2\pi R^2} dr = \frac{\mu_0 I_0 W}{2\pi R^2} \int_0^R r dr = \frac{\mu_0 I_0 W}{4\pi R^2} r^2 \Big]_0^R = \frac{\mu_0 I_0 W R^2}{4\pi R^2}$$

$$\boxed{\Phi_B = \frac{\mu_0 I_0 W}{4\pi}}$$

(*Alternative time*) Mid-term Exam 1: PHYSICS 1C (Spring 2021)

Time: 6:00PM – 7:00PM, April 15, 2021, Instructor: Prof. Zhongbo Kang

Student Name: ___________________________________

Student I.D. Number: ______________________________

Exam Version: **A**

Note:

- Please make sure that you have ***read, signed and uploaded*** your "student verification form", without which your exam will not be graded.
- The exam time (in total 1 hour) is designed in such a way that ideally the actual time for answering the problems is 30 minutes, while the remaining 30 minutes are used to scan and upload your solution to gradescope.
- The exam is open book, and open notes. One page of physical equations is provided. You can use a calculator.
- Remember to write down each step of your calculations, for partial credits.

Score Sheet:

Problem 1 (8 points): ______________________________

Problem 2 (10 points): ______________________________

Problem 3 (7 points): ______________________________

Total (25 points): __________________________________

Formula Sheet

$\vec{F} = q\vec{v} \times \vec{B}$ (magnetic force on a moving charged particle)

$\Phi_B = \int B\cos\phi\, dA = \int B_\perp\, dA = \int \vec{B} \bullet d\vec{A}$ (magnetic flux through a surface)

$\oint \vec{B} \bullet d\vec{A} = 0$ (Gauss's law for magnetism)

$R = \dfrac{mv}{|q|B}$ (radius of a circular orbit in a magnetic field)

$\vec{F} = I\vec{l} \times \vec{B}$ (magnetic force on a straight wire segment)

$d\vec{F} = Id\vec{l} \times \vec{B}$ (magnetic force on an infinitesimal wire section)

$\tau = IBA\sin\phi$ (magnitude of magnetic torque on a current loop)

$\vec{\tau} = \vec{\mu} \times \vec{B}$ (vector magnetic torque on a current loop)

$U = -\vec{\mu} \bullet \vec{B} = -\mu B\cos\phi$ (potential energy for a magnetic dipole)

$\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v} \times \hat{r}}{r^2}$ (magnetic field due to a point charge with constant velocity)

$d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l} \times \hat{r}}{r^2}$ (magnetic field due to an infinitesimal current element)

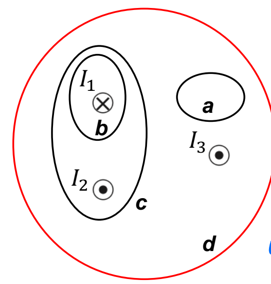
$B = \dfrac{\mu_0 I}{2\pi r}$ (magnetic field near a long, straight, current-carrying conductor)

$\dfrac{F}{L} = \dfrac{\mu_0 II'}{2\pi r}$ (two long, parallel, current-carrying conductors) $\oint \vec{B} \bullet d\vec{l} = \mu_0 I_{encl}$ (Ampere's law)

Problem 1 (8 pts)

Please ***be very careful*** in writing down your answers for these two questions. They are graded by the final answers ONLY, no partial credits for any intermediate steps.

a) (4 pts) The right figure shows, in cross section, several conductors that carry current through the plane of the figure. Four paths labeled a through d, are shown. The line integrals $\oint \vec{B} \cdot d\vec{l}$ over each path (a, b, c, d) are given by A, B, C, D. Note: each integral involves going around the path in the ***counterclockwise*** direction. Express A/B/C/D in terms of I_1, I_2, I_3:



$d\vec{l}$: counter clockwise

A: 0 B: $-\mu_0 I_1$

C: $\mu_0(I_2 - I_1)$ D: $\mu_0(I_2 + I_3 - I_1)$

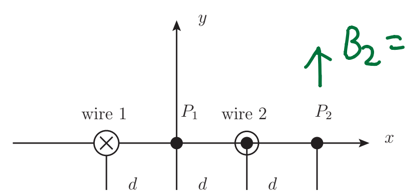
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$.

I_1 generates $\vec{B}$ field clockwise.

$I_2, I_3 \Rightarrow \vec{B}$ field counter clockwise.

b) (4 pts) Shown in the right figure is an end-on view of two long, straight, parallel wires perpendicular to the xy-plane, each carrying a current I but in opposite directions (as indicated by cross and dot in the figure), with d the distance between the relevant vertical lines. What are the directions of $\vec{B}$ field at points P_1 and P_2?

Your choice: a



$\uparrow B_2 = \frac{\mu_0 I}{2\pi d}$

$\downarrow B_2 \downarrow B_1$

$\downarrow B_1 = \frac{\mu_0 I}{2\pi (3d)}$

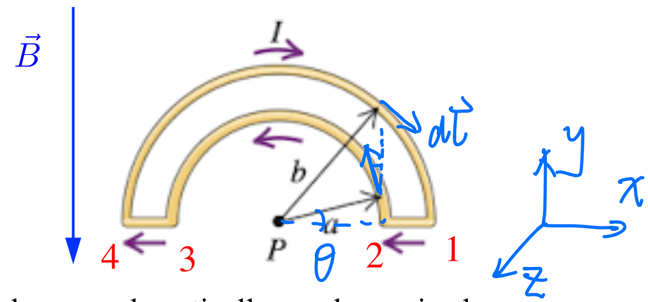
a. P_1: $-y$, P_2: $+y$ b. P_1: $-y$, P_2: undetermined c. P_1: $+y$, P_2: $+y$

d. P_1: $-y$, P_2: 0 e. P_1: 0, P_2: $+y$ f. none of above

Problem 2 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

The wire semicircles shown in the figure have radii ***a*** and ***b***. The current inside the wire is given by ***I***. Let us divide the wire into four pieces: horizontal piece [12], the small semicircle [23], horizontal piece [34], and the large semicircle [41]. Now a constant uniform magnetic field $\vec{B}$, is uniformly distributed *in the entire space* and pointing downward vertically as shown in the figure. Please answer the following questions (specify the magnitude and direction)



a) (8 pts) find the magnetic force on each of the four wire segments: [12], [23], [34], and [41].

b) (2 pts) what is the net force on the entire loop?

a) [12]: $\vec{F}_{[12]} = \int I\, d\vec{l} \times \vec{B} = I\, l_{[12]} B\,(\hat{k}) = I(b-a)B\hat{k}.$

[23]: $\vec{F}_{[23]} = \int I\, d\vec{l} \times \vec{B} = \int_0^{\pi} I\, a\, d\theta\, B \sin\theta\, (\hat{k})$

$= 2IaB\hat{k}.$

[34]: $\vec{F}_{[34]} = \vec{F}_{[12]} = I(b-a)B\hat{k}.$

[41]: $\vec{F}_{[41]} = \int I\, d\vec{l} \times \vec{B} = \int_0^{\pi} I\, b\, d\theta\, B \sin\theta\, (-\hat{k})$

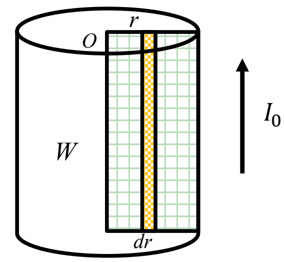
$= -2IbB\hat{k}.$

b) $\vec{F} = \vec{F}_{[12]} + \vec{F}_{[23]} + \vec{F}_{[34]} + \vec{F}_{[41]} = 0\ N.$

Problem 3 (7 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

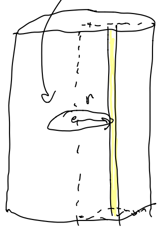
A very long, cylindrical wire of radius R carries a current I_0 uniformly distributed across the cross section of the wire, as shown in the figure.



(a) (3 pts) For the yellow strip that has distance r away from the center O, please compute the magnetic field that goes into the strip.

(b) (4 pts) Calculate the magnetic flux through the entire rectangle that has a length W. (Hint: recall the definition of magnetic flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$, here $d\vec{A}$ is the area element.

a.) We can use Ampere's Law to compute $\vec{B}$!



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

We need to calculate I_{enc} → $\sigma = \frac{I}{A}$ → we know I_0 is uniformly distributed over the cross section of the wire

→ Cross Sectional Area of entire wire: $A = \pi R^2$

→ So, $\sigma = \frac{I_0}{\pi R^2}$

→ $I_{enc} = \sigma \cdot \Delta A$

↳ cross sectional area of the Ampere loop: $\Delta A = \pi r^2$

→ $I_{enc} = \sigma \pi r^2$

$= \frac{I_0}{\pi R^2} \pi r^2$

So: $\oint \vec{B} \cdot d\vec{l} = \frac{\mu_0 I_0 \pi r^2}{\pi R^2} = \frac{\mu_0 I_0 r^2}{R^2}$

$B(2\pi r) = \frac{\mu_0 I_0 r^2}{R^2}$

$\Rightarrow \boxed{B = \frac{\mu_0 I_0 r}{2\pi R^2}}$

b.) We need to use $\Phi_B = \vec{B} \cdot \vec{A}$

$\Rightarrow d\Phi_B = \vec{B} \cdot d\vec{A} \rightarrow \Phi_B = \int_S \vec{B} \cdot d\vec{A} \rightarrow dA = W \cdot dr$ (Area of a teeny strip of the rectangle with width dr

$\Phi_B = \int_0^R B \cdot W dr = \int_0^R \frac{\mu_0 I_0 W r}{2\pi R^2} dr = \frac{\mu_0 I_0 W}{2\pi R^2} \int_0^R r dr = \frac{\mu_0 I_0 W}{4\pi R^2} r^2 \Big]_0^R = \frac{\mu_0 I_0 W R^2}{4\pi R^2}$

$\boxed{\Phi_B = \frac{\mu_0 I_0 W}{4\pi}}$