22W-PHYSICS-1C Final exam

XIMENG GUO

TOTAL POINTS

70 / 70

QUESTION 1

10 pts

1.1 3 / 3

✓ **+ 3 pts** Correct option selected.

+ 0 pts Incorrect option selected.

1.2 2 / 2

✓ **+ 2 pts** Correct option selected.

+ 0 pts Incorrect option selected.

1.3 3 / 3

✓ **+ 3 pts** Correct option selected.

+ 0 pts Incorrect option selected.

1.4 2 / 2

✓ **+ 2 pts** Correct option selected.

+ 0 pts Incorrect option selected.

QUESTION 2

10 pts

2.1 3 / 3

✓ **- 0 pts** Correct

- 1.5 pts Incorrect formula

- 0.5 pts Numerical error

- 3 pts Blank

- 0.25 pts Minus sign

2.2 4 / 4

✓ **- 0 pts** Correct

- 1.5 pts Incorrect formula

- 1.5 pts Unfinished

- 1 pts Numerical error

- 4 pts Blank

2.3 3 / 3

✓ **- 0 pts** Correct

- 1.5 pts Incorrect formula

- 0.5 pts Numerical error

- 0.5 pts Correct but find something else

- 1 pts Lack of or wrong explanation

- 3 pts Blank

- 1 pts Incomplete

QUESTION 3

10 pts

3.1 3 / 3

✓ **+ 1.5 pts** Correct expression for $$f$$

✓ **+ 1.5 pts** Correct value for $$f$$

3.2 4 / 4

✓ **+ 1 pts** Correct expression for Magnification.

✓ **+ 1 pts** Set Magnification to $$m=\frac{1}{3}$$.

✓ **+ 1 pts** Correct expression for $$f$$.

✓ **+ 1 pts** Correct Value for $$f$$

+ 0 pts Not Attempted

3.3 3 / 3

✓ **+ 1 pts** Correct expression for magnification $$m$$

✓ **+ 1 pts** Correct expression for image distance.

✓ **+ 1 pts** Correct value and direction for image distance.

+ 0 pts Not Attempted

QUESTION 4

10 pts

4.1 3 / 3

✓ **- 0 pts** Correct

- 1.5 pts Incorrect formula

- **0.5 pts** Numerical error for gamma
- **1 pts** Numerical error
- **3 pts** Blank

4.2 3 / 3

✓ **- 0 pts** **Correct**
- **1.5 pts** Incorrect formula
- **1 pts** Numerical error
- **3 pts** Blank

4.3 4 / 4

✓ **- 0 pts** **Correct**
- **1 pts** Incorrect time for strike at t'=0, x'=0
- **1 pts** Incorrect coordinate for strike at t'=0, x'=0
- **1 pts** Incorrect time for strike at t'=0, x'=2
- **1 pts** Incorrect coordinate for strike at t'=0, x'=2
- **0.5 pts** Numerical error
- **1 pts** Numerical error
- **4 pts** Blank

QUESTION 5

5 10 / 10

✓ **+ 1.5 pts** **Correct expression for magnetic field of a solenoid $$B=\mu_o ni$$, where $$n=\frac{N}{l}$$**

✓ **+ 1 pts** **Correct value for $$n=5000 \frac{turns}{m}$$ in $$SI$$ units.**

✓ **+ 1.5 pts** **Correct expression for circular motion for the electron $$F_c=\frac{mv^2}{R}$$ and $$F_B=qvB$$.**

✓ **+ 2 pts** **Set $$F_b=F_c$$ to calculate $$B$$**

✓ **+ 1 pts** **Correct expression for $$B=\frac{mv}{qR}$$ and**

✓ **+ 2 pts** **Use $$B$$ to calculate $$i$$ by plugging it into $$i=\frac{B}{\mu_o n}$$**

✓ **+ 1 pts** **Correct numerical value for $$i$$**

+ 0 pts Not Attempted

QUESTION 6

10 pts

6.1 2 / 2

✓ **- 0 pts** **Correct**
- **1 pts** Incorrect formula
- **0.5 pts** Numerical error

6.2 2 / 2

✓ **- 0 pts** **Correct**
- **1 pts** Incorrect formula
- **0.5 pts** Numerical error
- **0.25 pts** Successive numerical error
- **2 pts** Blank

6.3 3 / 3

✓ **- 0 pts** **Correct**
- **1 pts** Incorrect formula for E
- **1 pts** Incorrect or no formula for B
- **0.5 pts** Numerical error
- **0.25 pts** Numerical error for B
- **3 pts** Blank

6.4 3 / 3

✓ **- 0 pts** **Correct**
- **1.5 pts** Incorrect formula
- **0.25 pts** Successive numerical error
- **0.5 pts** Numerical error
- **3 pts** Blank

QUESTION 7

10 pts

7.1 5 / 5

✓ **- 0 pts** **Correct**
- **2 pts** Incorrect Kirchhoff's laws
- **1.5 pts** Solving differential equation
- **1 pts** Error in expression
- **0.5 pts** Incorrect I_0
- **1 pts** Incorrect I_0
- **3 pts** Incorrect expression with no explanation
- **5 pts** Blank
- **2 pts** Correct general form but with no specification
- **3 pts** Correct but imcomplete

7.2 5 / 5

✓ **- 0 pts** Correct

- 2 pts Error in the expression of P_L

- 1 pts Error in the expression of P_L

- 2 pts Error in the way to find t for maximal P_L

- 1 pts Error in the way to find t for maximal P_L

- 1.5 pts No answer to t for maximal P_L

- 1 pts Incomplete in finding t for maximal P_L

- 1 pts Numerical error

- 0.5 pts Numerical error

- 3 pts Incomplete

- 5 pts Blank

Problem 1 (10 pts)

Please ***be very careful*** in writing down your answers for these two questions. They are graded by the final answers ONLY, no partial credits for any intermediate steps.

(a) (3 pts) An object 0.65 cm tall is placed 10 cm to the left of a thin converging lens, which has a focal length of 5.0 cm. Determine the nature of the image. Your choice: ____c____

a. Enlarged inverted real b. Shrinked erect virtual c. equal inverted real

d. equal inverted virtual e. Enlarged erect real f. Shrinked inverted real

(b) (2 pts) In our universe, the speed of light is a constant $c = 3.0 \times 10^8$ m/s. Now let us imagine an alternative universe where the speed of light is 1,000,000 times less than it is in our universe to see what happen. An airplane has a length of 60 m when measured at rest. When the airplane is moving at 180 m/s in the alternate universe, how long would the plane appear to be to a stationary observer? ____c____

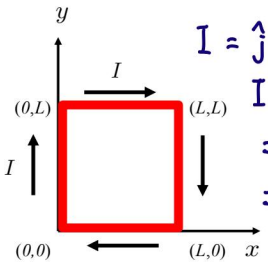
$L = L_0\sqrt{1-\frac{u^2}{c^2}}$

$= 60\sqrt{1-\frac{180^2}{300^2}}$

$= 48\,m$

a. 24 m b. 36 m c. 48 m d. 60 m e. 75 m

(c) (3 pts) The figure shows a square loop of wire that lies in the *xy*-plane. The loop carries a constant current *I* in the clockwise direction. The magnetic field has no *x* component but has both *y* and *z* components: $\vec{B} = \frac{B_0}{L}(z\,\hat{j} + y\,\hat{k})$, where B_0 is a positive constant. Find the direction of the force on the side, that runs from *(0,0)* to *(0,L)*. Your choice: ____a____



$\vec{I} = \hat{j}$

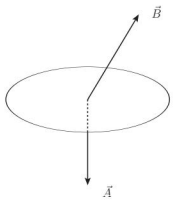
$I\vec{dl} \times \vec{B}$

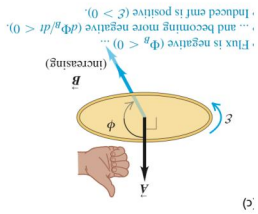
$= \hat{j} \times (\hat{j} + \hat{k})$

$= \hat{i}$

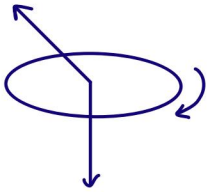
a. $+x$ b. $-x$ c. $+y$ d. $-y$ e. $+z$ f. $-z$ g. 0

(d) (2 pts) The positive directions of the vector area $\vec{A}$ for the conducting loop and the magnetic field $\vec{B}$ are shown in the figure on the right. The magnetic field is increasing, what is the direction of the induced emf. Your choice: ____a____





a. clockwise b. counterclockwise c. undetermined, no enough information



1.1 **3 / 3**

✓ **+ 3 pts** **Correct option selected.**

+ 0 pts Incorrect option selected.

Problem 1 (10 pts)

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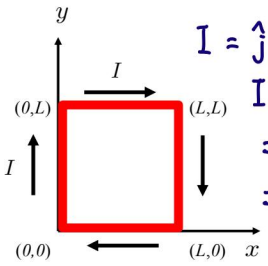

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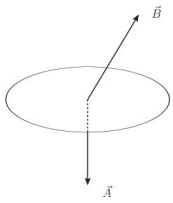

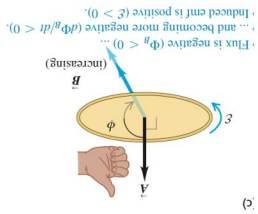

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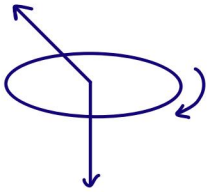

1.2 **2 / 2**

✓ **+ 2 pts** **Correct option selected.**

+ 0 pts Incorrect option selected.

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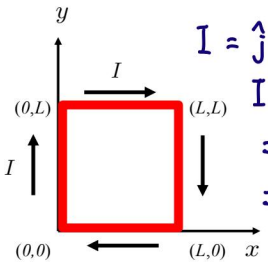

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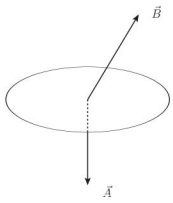

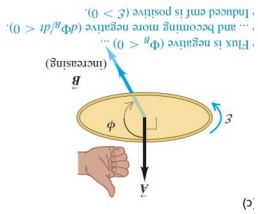

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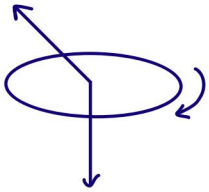

1.3 **3 / 3**

✓ **+ 3 pts** **Correct option selected.**

+ 0 pts Incorrect option selected.

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$= 48\,m$

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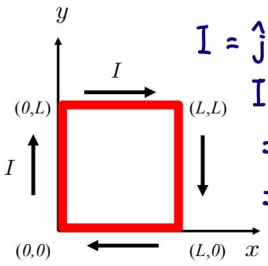

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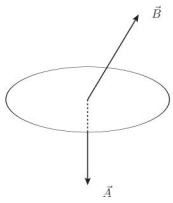

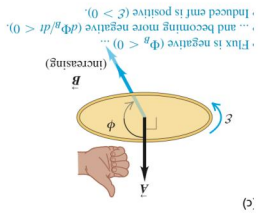

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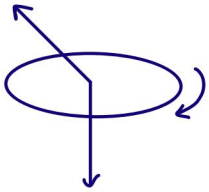


1.4 **2 / 2**

✓ **+ 2 pts** **Correct option selected.**

+ 0 pts Incorrect option selected.

Problem 2 (10 pts)

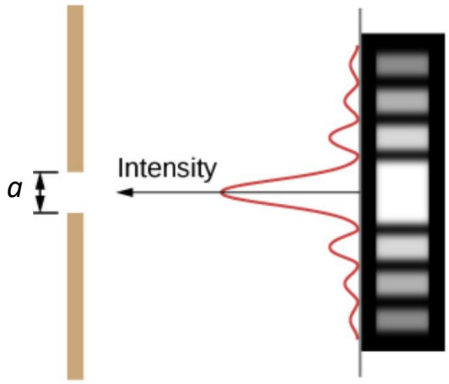
Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

As shown in the figure, Tom performed a single-slit diffraction experiment using a laser beam. The intensity at the center of the diffraction pattern is I_0. Please answer the following questions

(a) (3 pts) What is the intensity at a point in the pattern where there is a 66-radian phase difference between wavelets from the two edges of the slit?

(b) (4 pts) If this point is 7.0° away from the central maximum, how many wavelengths wide is the slit?

(c) (3 pts) Tom kept adjusting the wavelength of the laser beam. When the wavelength of the laser beam is 580 nm, he found that the first diffraction minima are at $\pm 90^\circ$, so the central maximum completely fills the screen. What is the width a of the slit?



(a). $I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = I_0 \left[\frac{\sin(33)}{33}\right]^2 = 9.18 \times 10^{-4} I_0$

(b). $\beta = \frac{2\pi}{\lambda} a \sin\theta$

$a = \frac{\beta\lambda}{2\pi \sin\theta} = \frac{66}{2\pi \cdot \sin(7.0^\circ)} \lambda = 86.2\lambda$

(c). $\sin\theta = \frac{m\lambda}{a}$

$a = \frac{m\lambda}{\sin\theta} = \frac{1 \times 580 \times 10^{-9} m}{\sin(90^\circ)} = 580 nm$

2.1 **3 / 3**

✓ **- 0 pts** **Correct**

- 1.5 pts Incorrect formula

- 0.5 pts Numerical error

- 3 pts Blank

- 0.25 pts Minus sign

Problem 2 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

As shown in the figure, Tom performed a single-slit diffraction experiment using a laser beam. The intensity at the center of the diffraction pattern is I_0. Please answer the following questions

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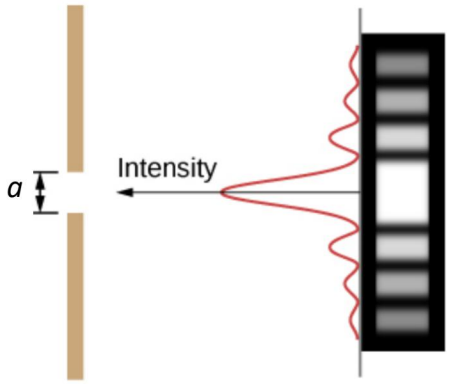

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2.2 4 / 4

✓ **- 0 pts** **Correct**

- 1.5 pts Incorrect formula

- 1.5 pts Unfinished

- 1 pts Numerical error

- 4 pts Blank

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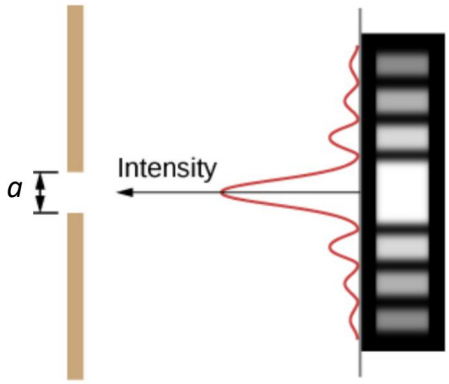

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(c). $\sin\theta = \frac{m\lambda}{a}$

$a = \frac{m\lambda}{\sin\theta} = \frac{1 \times 580 \times 10^{-9} m}{\sin(90^\circ)} = 580 nm$

2.3 3 / 3

✓ **- 0 pts** **Correct**

- 1.5 pts Incorrect formula

- 0.5 pts Numerical error

- 0.5 pts Correct but find something else

- 1 pts Lack of or wrong explanation

- 3 pts Blank

- 1 pts Incomplete

Problem 3 (10 pts)

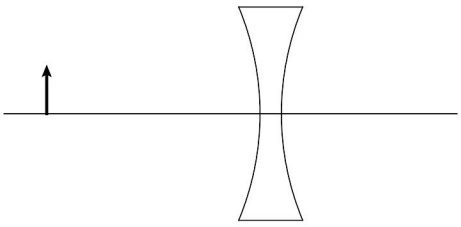
Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

Suppose the absolute values of the radii of curvature of the lens surface in the following figure are both equal to 20.0 cm, and the index of refraction of the glass is $n = 1.50$.

(a) (3 pts) What is the focal length f of the lens? Is f positive or negative?

(b) (4 pts) You want to use this lens to form an erect, virtual image that is 1/3 the height of the object. Where should the object be placed?

(c) (3 pts) Where will the image be? Compute the image distance and point out whether the image is on the left side or the right side of the lens?



(a). $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(-\frac{1}{20.0} - \frac{1}{20.0}\right) = -\frac{1}{20.0}$

$f = \frac{1}{-\frac{1}{20.0}} = -20.0\,\text{cm}$ $\therefore$ f is negative

(b). $\begin{cases} m = \frac{y'}{y} = -\frac{s'}{s} \\ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \end{cases} \Rightarrow \begin{cases} \frac{1}{3} = -\frac{s'}{s} \\ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \end{cases} \Rightarrow \begin{cases} s = -3s' \ ① \\ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \ ② \end{cases}$

Substitute ① into ②: $\frac{1}{-3s'} + \frac{1}{s'} = \frac{1}{-20.0}$

$1 - 3 = \frac{3s'}{20.0}$

$s' = -\frac{40}{3}\,\text{cm}$

$\therefore\ s = -3s' = -3 \times -\frac{40}{3} = 40.0\,\text{cm}$

$\therefore$ object should be placed 40.0 cm left from the center of lens

(c). from (b), $s' = -\frac{40}{3}\,\text{cm} \approx -13.3\,\text{cm}$

$\therefore$ it will be on the left side of the lens

3.1 **3 / 3**

✓ **+ 1.5 pts** Correct expression for **$$f$$**

✓ **+ 1.5 pts** Correct value for **$$f$$**

Problem 3 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

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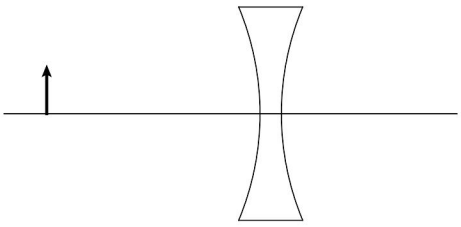

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3.2 4 / 4

✓ **+ 1 pts** Correct expression for Magnification.

✓ **+ 1 pts** Set Magnification to $$m=\frac{1}{3}$$.

✓ **+ 1 pts** Correct expression for $$f$$.

✓ **+ 1 pts** Correct Value for $$f$$

+ 0 pts Not Attempted

Problem 3 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

Suppose the absolute values of the radii of curvature of the lens surface in the following figure are both equal to 20.0 cm, and the index of refraction of the glass is $n = 1.50$.

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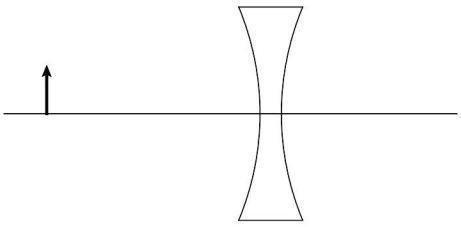

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3.3 3 / 3

✓ **+ 1 pts** Correct expression for magnification $$m$$

✓ **+ 1 pts** Correct expression for image distance.

✓ **+ 1 pts** Correct value and direction for image distance.

+ 0 pts Not Attempted

Problem 4 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

Tom and Mary are astronauts and are now both driving their own spaceship, flying in the space. Tom's spaceship moves with a speed 0.6 c with respect to Mary's spaceship. At the time 0, Tom and Mary's spaceships are both at origin of their corresponding frames. Mary stands at the origin: at $t = 0$, she turns on a laser; and after 0.400 second, she turns it off.

(a) (3 pts) What is the time duration between on and off of the laser as seen by Tom?

(b) (3 pts) A ruler of length 1 m is laid out on the x-axis in Mary's spaceship, what is the length of the ruler as observed by Tom?

(c) (4 pts) In Tom's spaceship, at time 0, a lightning strikes at the origin and another lightning strikes at the position 2.00 m in the x-axis in Tom's frame. Please find the coordinates and times for the two lightning events as seen by Mary in her frame. $x = \gamma(x' + ut')$ and $t = \gamma(t' + ux'/c^2)$

(a). $t_0 = 0.400\,s$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{0.400\,s}{\sqrt{1-0.6^2}} = 0.500\,s$$

(b). $l_0 = 1\,m$

$$l = l_0\sqrt{1-\frac{u^2}{c^2}} = 1 \times \sqrt{1-0.6^2} = 0.8\,m$$

(c). $x = \gamma(x' + ut')$

$$t = \gamma\left(t' + \frac{ux'}{c^2}\right)$$

First lightning : $x = \gamma(0 + u\cdot 0) = 0\,m$; $t = \gamma\left(0 + \frac{0}{c^2}\right) = 0\,s$

Second lightning : $x = \gamma(2 + u\cdot 0) = 2 \times \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = 2 \times \frac{1}{\sqrt{1-0.6^2}} = 2.50\,m$

$$t = \gamma\left(0 + \frac{0.6c \times 2.00}{c^2}\right) = \frac{1}{\sqrt{1-0.6^2}} \cdot \frac{0.6 \times 2.00}{3.0\times 10^8} = 5\times 10^{-9}\,s$$

4.1 3 / 3

✓ **- 0 pts** **Correct**

- 1.5 pts Incorrect formula

- 0.5 pts Numerical error for gamma

- 1 pts Numerical error

- 3 pts Blank

Problem 4 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

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4.2 3 / 3

✓ **- 0 pts** **Correct**

- 1.5 pts Incorrect formula

- 1 pts Numerical error

- 3 pts Blank

Problem 4 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

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4.3 4 / 4

✓ **- 0 pts** **Correct**

- 1 pts Incorrect time for strike at t'=0, x'=0

- 1 pts Incorrect coordinate for strike at t'=0, x'=0

- 1 pts Incorrect time for strike at t'=0, x'=2

- 1 pts Incorrect coordinate for strike at t'=0, x'=2

- 0.5 pts Numerical error

- 1 pts Numerical error

- 4 pts Blank

Problem 5 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

A solenoid with 50 turns per centimeter carries a current I. An electron moves within the solenoid in a circle that has a radius of 4.0 cm and is perpendicular to the axis of the solenoid. We have electron mass $m = 9.1 \times 10^{-31}$ kg, and its charge is given by $e = -1.60 \times 10^{-19}$ C. If the speed of the electron is 8.0×10^{5} m/s, what is the current I?

$B = \mu_0 n I$

$F = evB = \frac{mv^2}{R}$

$\therefore \quad B = \frac{mv}{eR}$

$\mu_0 n I = \frac{mv}{eR}$

$I = \frac{mv}{eR\mu_0 n}$

$= \frac{9.1 \times 10^{-31} \times 8.0 \times 10^{5}}{-1.6 \times 10^{-19} \times 4.0 \times 10^{-2} \times 4\pi \times 10^{-7} \times 50 \times 100}$

$\approx -0.018\,A$

magnitude of current I: $|-0.018\,A| = 0.018\,A$

5 **10 / 10**

✓ **+ 1.5 pts** Correct expression for magnetic field of a solenoid $$B=\mu_o ni$$, where $$n=\frac{N}{l}$$

✓ **+ 1 pts** Correct value for $$n=5000 \frac{turns}{m}$$ in $$SI$$ units.

✓ **+ 1.5 pts** Correct expression for circular motion for the electron $$F_c=\frac{mv^2}{R}$$ and $$F_B=qvB$$.

✓ **+ 2 pts** Set $$F_b=F_c$$ to calculate $$B$$

✓ **+ 1 pts** Correct expression for $$B=\frac{mv}{qR}$$ and

✓ **+ 2 pts** Use $$B$$ to calculate $$i$$ by plugging it into $$i=\frac{B}{\mu_o n}$$

✓ **+ 1 pts** Correct numerical value for $$i$$

+ 0 pts Not Attempted

Problem 6 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

A new professor in the university recently built his own laser. This laser can produce light of wavelength 500 nm and a power of 0.500 mW spread over a cylindrical beam with 1.00 mm in radius. $\lambda = 500\times10^{-9}\,m$ $P = 0.500\times10^{-3}\,W$ $r = 1.00\times10^{-3}\,m$

$$p_{rad} = \frac{S_{av}}{c} = \frac{I}{c} \quad \text{(radiation pressure, wave totally absorbed)} \qquad (32.32)$$

If the wave is totally reflected, the momentum change is twice as great, and

$$p_{rad} = \frac{2S_{av}}{c} = \frac{2I}{c} \quad \text{(radiation pressure, wave totally reflected)} \qquad (32.33)$$

(a) (2 pts) What is the intensity of this laser beam?

(b) (2 pts) What average pressure does this laser exert on a totally reflecting surface?

(c) (3 pts) What are the maximum values of the electric and magnetic fields?

(d) (3 pts) What is the average energy density in the laser beam?

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_{max}^2 = \frac{1}{2}\epsilon_0 c E_{max}^2 \qquad (32.29)$$

I - intensity!

(a). $I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{0.500\times10^{-3}}{\pi\times(1.00\times10^{-3})^2} = 159.1549\ W/m^2 \approx 159\ W/m^2$

(b). $P_{rad} = \frac{2I}{c} = \frac{2\times159.1549}{3.0\times10^8} = 1.061033\times10^{-6} \approx 1.06\times10^{-6}\ Pa$

(c). $\because I = \frac{1}{2}\epsilon_0 c E_{max}^2$

$\therefore E_{max} = \sqrt{\frac{2I}{\epsilon_0 c}}$

$= \sqrt{\frac{2\times159.1549}{8.85\times10^{-12}\times3.0\times10^8}}$

$= 346.25236$

$\approx 346\ V/m$

$B_{max} = \frac{E_{max}}{c} = \frac{346.25236}{3.0\times10^8} = 1.1541745\times10^{-6} \approx 1.15\times10^{-6}\ T$

(d). $u_{av} = \frac{1}{2}\epsilon_0 E_{max}^2$

$= \frac{1}{2}\times8.85\times10^{-12}\times346.25236^2$

$= 5.3051633\times10^{-7}$

$\approx 5.31\times10^{-7}\ J/m^3$

6.1 2 / 2

✓ **- 0 pts** **Correct**

- 1 pts Incorrect formula

- 0.5 pts Numerical error

Problem 6 (10 pts)

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6.2 2 / 2

✓ **- 0 pts** **Correct**

- 1 pts Incorrect formula

- 0.5 pts Numerical error

- 0.25 pts Successive numerical error

- 2 pts Blank

Problem 6 (10 pts)

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6.3 3 / 3

✓ **- 0 pts** **Correct**

- 1 pts Incorrect formula for E

- 1 pts Incorrect or no formula for B

- 0.5 pts Numerical error

- 0.25 pts Numerical error for B

- 3 pts Blank

Problem 6 (10 pts)

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If the wave is totally reflected, the momentum change is twice as great, and

$$p_{rad} = \frac{2S_{av}}{c} = \frac{2I}{c} \quad \text{(radiation pressure, wave totally reflected)} \qquad (32.33)$$

(a) (2 pts) What is the intensity of this laser beam?

(b) (2 pts) What average pressure does this laser exert on a totally reflecting surface?

(c) (3 pts) What are the maximum values of the electric and magnetic fields?

(d) (3 pts) What is the average energy density in the laser beam?

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_{max}^2 = \frac{1}{2}\epsilon_0 c E_{max}^2 \qquad (32.29)$$

I - intensity!

(a). $I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{0.500\times10^{-3}}{\pi\times(1.00\times10^{-3})^2} = 159.1549\ W/m^2 \approx 159\ W/m^2$

(b). $P_{rad} = \frac{2I}{c} = \frac{2\times159.1549}{3.0\times10^8} = 1.061033\times10^{-6} \approx 1.06\times10^{-6}\ Pa$

(c). $\because I = \frac{1}{2}\epsilon_0 c E_{max}^2$

$\therefore E_{max} = \sqrt{\frac{2I}{\epsilon_0 c}}$

$= \sqrt{\frac{2\times159.1549}{8.85\times10^{-12}\times3.0\times10^8}}$

$= 346.25236$

$\approx 346\ V/m$

$B_{max} = \frac{E_{max}}{c} = \frac{346.25236}{3.0\times10^8} = 1.1541745\times10^{-6} \approx 1.15\times10^{-6}\ T$

(d). $u_{av} = \frac{1}{2}\epsilon_0 E_{max}^2$

$= \frac{1}{2}\times8.85\times10^{-12}\times346.25236^2$

$= 5.3051633\times10^{-7}$

$\approx 5.31\times10^{-7}\ J/m^3$

6.4 3 / 3

✓ **- 0 pts** **Correct**

- 1.5 pts Incorrect formula

- 0.25 pts Successive numerical error

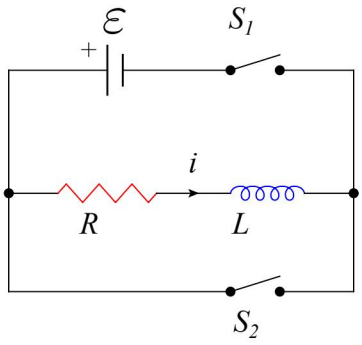
- 0.5 pts Numerical error

- 3 pts Blank

Problem 7 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

For a *R-L* circuit as shown in the figure, the voltage $\varepsilon = 12.0$ V, the resistance $R = 100\ \Omega$, and the inductance $L = 0.01$ H. Suppose both switches are open to begin with, and then at some initial time $t = 0$, we close switch S_1 (leave S_2 still open). The current i is shown in the figure.



(a) (5 pts) Please obtain an expression of the current i as a function of time t. If one denotes the value of the current as I_0 when $t \to \infty$, please determine I_0.

(b) (5 pts) Please calculate the instantaneous power P_L in the inductor as a function of t. At what value of t, P_L is a maximum?

(a). $\mathcal{E} - iR - L\frac{di}{dt} = 0$

$\mathcal{E} - iR = L\frac{di}{dt}$

$\frac{di}{i - \frac{\mathcal{E}}{R}} = -\frac{R}{L}dt$

$\int_0^i \frac{1}{i' - \frac{\mathcal{E}}{R}}\, di' = \int_0^t -\frac{R}{L}\, dt'$

$\ln\left(\frac{i - \frac{\mathcal{E}}{R}}{-\frac{\mathcal{E}}{R}}\right) = -\frac{R}{L}t$

$\therefore\ i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right) = \frac{12}{100}\left(1 - e^{-\frac{100}{0.01}t}\right) = 0.12\left(1 - e^{-10000t}\right)$

$I_0 = \lim_{t\to\infty} i = \frac{\mathcal{E}}{R}\left(1 - e^{-\infty}\right) = \frac{\mathcal{E}}{R}(1 - 0) = \frac{\mathcal{E}}{R} = \frac{12.0}{100} = 0.12\ \text{A}$

(b). $P_L = V_L i = L\frac{di}{dt}\cdot i = Li\frac{di}{dt}$

From (a), $i = 0.12\left(1 - e^{-10000t}\right)$

$\frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-(R/L)t}$

$\therefore\ \frac{di}{dt} = 1200\, e^{-10000t}$

$\therefore\ P_L = Li\frac{di}{dt} = 0.01 \times 0.12\left(1 - e^{-10000t}\right) \times 1200\, e^{-10000t}$

$= 1.44\, e^{-10000t}\cdot\left(1 - e^{-10000t}\right)$

(continued on next page)

7.1 5 / 5

✓ **- 0 pts** **Correct**

- 2 pts Incorrect Kirchhoff's laws

- 1.5 pts Solving differential equation

- 1 pts Error in expression

- 0.5 pts Incorrect I_0

- 1 pts Incorrect I_0

- 3 pts Incorrect expression with no explanation

- 5 pts Blank

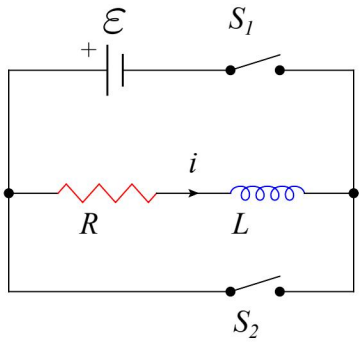
- 2 pts Correct general form but with no specification

- 3 pts Correct but imcomplete

Problem 7 (10 pts)

Please make sure to write down ***intermediate steps*** of your calculations, for partial credits.

For a *R-L* circuit as shown in the figure, the voltage $\varepsilon = 12.0$ V, the resistance $R = 100\ \Omega$, and the inductance $L = 0.01$ H. Suppose both switches are open to begin with, and then at some initial time $t = 0$, we close switch S_1 (leave S_2 still open). The current i is shown in the figure.



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$\mathcal{E} - iR = L\frac{di}{dt}$

$\frac{di}{i - \frac{\mathcal{E}}{R}} = -\frac{R}{L}dt$

$\int_0^i \frac{1}{i' - \frac{\mathcal{E}}{R}}\, di' = \int_0^t -\frac{R}{L}\, dt'$

$\ln\left(\frac{i - \frac{\mathcal{E}}{R}}{-\frac{\mathcal{E}}{R}}\right) = -\frac{R}{L}t$

$\therefore\ i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right) = \frac{12}{100}\left(1 - e^{-\frac{100}{0.01}t}\right) = 0.12\left(1 - e^{-10000t}\right)$

$I_0 = \lim_{t\to\infty} i = \frac{\mathcal{E}}{R}\left(1 - e^{-\infty}\right) = \frac{\mathcal{E}}{R}(1 - 0) = \frac{\mathcal{E}}{R} = \frac{12.0}{100} = 0.12\ \text{A}$

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From (a), $i = 0.12\left(1 - e^{-10000t}\right)$

$\frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-(R/L)t}$

$\therefore\ \frac{di}{dt} = 1200\, e^{-10000t}$

$\therefore\ P_L = Li\frac{di}{dt} = 0.01 \times 0.12\left(1 - e^{-10000t}\right) \times 1200\, e^{-10000t}$

$= 1.44\, e^{-10000t}\cdot\left(1 - e^{-10000t}\right)$

(continued on next page)

to calculate maximum P_L :

let $\frac{d}{dt}(P_L) = 0$

$$\frac{d}{dt}\left(1.44\, e^{-10000t} - 1.44\, e^{-20000t}\right) = 0$$

$$-14400\, e^{-10000t} + 28800\, e^{-20000t} = 0$$

$$-14400 + 28800\, e^{-10000t} = 0$$

$$e^{-10000t} = \frac{14400}{28800}$$

$$-10000t = \ln(0.5)$$

$$t = \frac{\ln(0.5)}{-10000} \approx 6.93 \times 10^{-5}\ s$$

$\therefore$ at $t = 6.93 \times 10^{-5}\ s$, P_L is a maximum

7.2 5 / 5

✓ **- 0 pts** **Correct**

- 2 pts Error in the expression of P_L

- 1 pts Error in the expression of P_L

- 2 pts Error in the way to find t for maximal P_L

- 1 pts Error in the way to find t for maximal P_L

- 1.5 pts No answer to t for maximal P_L

- 1 pts Incomplete in finding t for maximal P_L

- 1 pts Numerical error

- 0.5 pts Numerical error

- 3 pts Incomplete

- 5 pts Blank