21F-PHYSICS1C-1 Midterm Examination

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TOTAL POINTS

42 / 42

QUESTION 1

1 Inductors in circuits **14 / 14**

- **- 2 pts** Part a. Self-inductance Incorrect, but expression relating the current to potential difference correct.
- **- 2 pts** Part a. Self-inductance Correct, but expression relating the current to potential difference incorrect.
- **- 1 pts** Part b. Current after long time incorrect, $$I_1 = \frac{\epsilon}{R_1}$$ and $$I_2 = \frac{\epsilon}{R_2}$$
- **- 1 pts** Part b. Current at t=0 incorrect, $$I_1 = \frac{\epsilon}{R_1}$$ and $$I_2 = 0$$
- **- 1 pts** Part b. Differential equation for $$I_{2}(t) = \frac{\epsilon}{R_2}(1-e^{-R_{2}t/L})$$ incorrect
- **- 1 pts** Part b. I v.s. t graph incorrect, please refer to midterm solutions on CCLE
- **- 4 pts** Part c. Drawing of the phasor diagram is incorrect, $$V_L$$ is ahead of the current by 90 degree while $$V_R$$ is in phase with current, thus $$V_2$$ phasor will have some phase angle with the current as well
- **- 2 pts** Part c. The drawing of the phasor diagram is incorrect but demonstrated some understanding. $$V_L$$ is ahead of the current by 90 degree while $$V_R$$ is in phase with current, thus $$V_s$$ phasor will have some phase angle with the current as well
- **- 0.5 pts** Part c. Incorrect phase angle for $$I_2$$, $$\phi=\tan^{-1}(\frac{\omega L}{R_2})$$
- **- 0.5 pts** Part c. Incorrect $$I_2$$ amplitude, need to express it in terms of the parameters given in he problem. $$I_2 = \frac{V_s}{\sqrt{R_2^2 + (\omega L)^2}}$$
- **- 0.5 pts** Part c. Incorrect $$I_1$$ amplitude, $$I_1 = \frac{\epsilon}{R_1}$$
- **- 0.5 pts** Part c. Incorrect phase angle for $$I_1$$, $$\phi=\tan^{-1}(\frac{\omega L}{R_2})$$ relative to $$I_2$$
- ✓ **- 0 pts** **Question 1 All correct**
- **- 0.5 pts** Part b. $$I_2$$ at t = 0, incorrect. $$I_2 = 0$$ at t = 0
- **- 4 pts** Part a. Incorrect

QUESTION 2

2 Sources of magnetic fields, magnetic forces **14 / 14**

- ✓ **+ 2 pts** **a) Write the integral expression for Ampere's law: $$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$**
- ✓ **+ 2 pts** **a) Use the integral expression to find an expression for the magnetic field: $$B(r) = \frac{\mu_0 I_{enc}}{2\pi r}$$**
- ✓ **+ 4 pts** **b) Write an expression for the force per unit length $$f_B = IB$$**
- ✓ **+ 2 pts** **b) Use the expression for $$B$$ from (a) to find $$f_B$$ and simplify: $$f_B = \frac{\mu_0 I^2}{4\pi d}$$**
- ✓ **+ 1 pts** **c) Correctly draw the gravitational force (downward)**
- ✓ **+ 1 pts** **c) Correctly draw the magnetic force direction (away from the other wire)**
- ✓ **+ 1 pts** **d) Identify that the tension balances the sum of the gravitational and magnetic forces: $$\vec{T} = \vec{f_G}+\vec{f_B}$$**
- ✓ **+ 1 pts** **d) Use the geometry of the system to relate the angle $$\theta$$ to the forces: $$\tan\theta = \frac{f_B}{f_G}$$**

QUESTION 3

3 Faraday's Law **14 / 14**

✓ + **2 pts** **3(a) Correct. Ans: $$\epsilon= \frac{ -d \phi_B}{dt}$$, $$\epsilon=$$ induced emf, $$\phi_B=$$ mag flux integrated over the surface bounded by the loop used to determine $$\epsilon$$**

+ **1 pts** 3(a) partially correct/ no explanation of terms

+ **0 pts** 3(a) incorrect

✓ + **4 pts** **3(b) correct. Ans: $$-\frac{B\pi r^2}{2}$$**

+ **3 pts** 3b) Correct approach but calculation errors

+ **2 pts** 3(b) Flux in initial or final state correctly calculated but not both

+ **1 pts** 3(b) Correct flux definition but incorrect answer

+ **0 pts** 3(b) incorrect

✓ + **4 pts** **3(c) Correct. Angular Frequency= $$\omega$$, Frequency= $$\frac{\omega}{2\pi}$$amplitude=$$\frac{B\pi r^2\omega}{2}$$**

+ **3 pts** 3(c) Correct method but incomplete/ calculation errors

+ **2 pts** 3(c) Frequency or amplitude correct but not both

+ **1 pts** 3c Partially relevant approach

+ **0 pts** 3(c) Incorrect

✓ + **4 pts** **3(d) correct. Ans: $$\frac{(\pi r^2 \omega B)^2}{8R}$$**

+ **3 pts** 3(d) Correct method but calculation errors

+ **1 pts** 3(d) Partially relevant approach

+ **0 pts** 3(d) incorrect

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Physics 1C Lecture 1, Fall 2021

Midterm Examination

October 29, 2021

Exam rules: Please do not forget to write your name and student ID on the front of the exam! No electronic gadgets of any kind, and the exam is closed book and closed notes. Any numerical answers may be given using one or more significant figures. For example, $4\pi = 10$ is acceptable. If a definite integral appears in an answer, but which you do not know how to solve, then continue on with the rest of the question, assigning an arbitrary constant to take the place of the unsolved integral.

Some equations

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\Phi_B$$

$$\oint (\vec{E} \cdot \hat{n}) dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left[I_{enc} + \varepsilon_0 \frac{d\Phi_E}{dt} \right]$$

$$\oint (\vec{B} \cdot \hat{n}) dA = 0$$

$$\vec{F}_B = q\vec{v} \times \vec{B}; d\vec{F}_B = I d\vec{\ell} \times \vec{B}; \vec{F}_E = q\vec{E}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}; \vec{\mu} = IA\hat{n}$$

$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times (\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3}$$

$$\mathcal{E} = -L\frac{di}{dt}$$

$$U = \frac{1}{2} L I^2$$

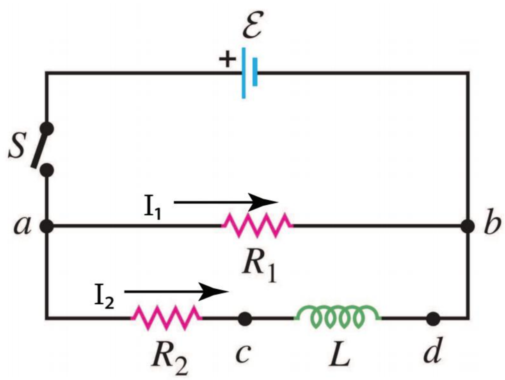
$$X_L = \omega L$$

$$\cos(\omega t + \pi/2) = -\sin(\omega t)$$

$$Z^2 = [R^2 + (X_L - X_C)^2]$$

Problem 1.

Inductors in circuits. (14 points total)



$\leftarrow V_{cd} \rightarrow$

a. Write down the definition of self inductance, followed by an expression relating the current in an inductor to the potential difference across it. (4 points)

Self-inductance occurs when a changing current through a solenoid introduces a change in magnetic flux through the solenoid, resulting in the creation of an emf that opposes the change in current.

Self-inductance $= L = \frac{N \Phi_B}{i_2}$ where N is the number of coils, Φ_B is the magnetic flux through the solenoid, and i_2 is the current traveling in the wire.

$\boxed{\text{Potential difference: } V_{cd} = V_c - V_d = L\frac{di_2}{dt}}$ emf $= \mathcal{E} = -L\frac{di_2}{dt}$

b. An inductor L is connected with two resistors as shown. At $t = 0$, the switch S is closed. *Immediately* after the switch is closed ($t \to 0^+$), what are the currents I_1 and I_2? What are I_1 and I_2 a long time after S is closed? Write down a differential equation for the current $I_2(t)$. Find the solution and sketch it. (4 points)

$t = 0^+$: $I_2 = 0$ as inductor resists change in current and no time has passed.

$I_1 = \frac{\mathcal{E}}{R_1}$ as all current travels through the segment ab.

long time: $I_2 = \frac{\mathcal{E}}{R_2}$ as the two resistors are in parallel.

$I_1 = \frac{\mathcal{E}}{R_1}$

Continued →

Using Kirkhoff's Voltage Law:

$$\mathcal{E} - I_2 R_2 - L\frac{dI_2}{dt} = 0$$

$$\mathcal{E} - I_2 R_2 = L\frac{dI_2}{dt}$$

$$\frac{1}{L}\,dt = \frac{dI_2}{\mathcal{E} - I_2 R_2}$$

$$\frac{1}{L}\int_0^t dt = \int_0^i \frac{1}{\mathcal{E} - I_2 R_2}\,dI_2$$

$$u = \mathcal{E} - I_2 R_2$$

$$\frac{du}{dI} = -R_2$$

$$dI = \frac{du}{-R_2}$$

$$\frac{t}{L} = -\frac{1}{R_2}\int_{I_2=0}^{I_2=i} \frac{1}{u}\,du$$

$$-\frac{R_2 t}{L} = \ln(\mathcal{E} - I_2 R_2)\Big|_0^i = \ln(\mathcal{E} - i_2 R_2) - \ln(\mathcal{E})$$

$$-\frac{R_2 t}{L} = \ln\left(1 - \frac{i_2 R_2}{\mathcal{E}}\right)$$

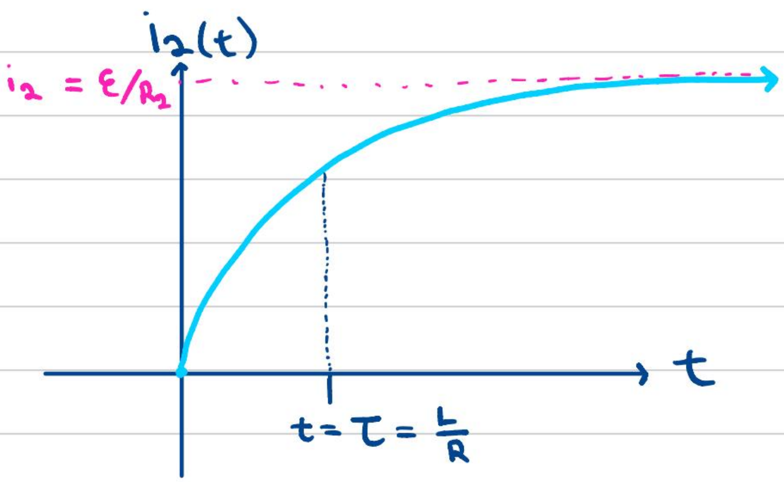
$$e^{-(R_2/L)t} = 1 - \frac{i_2 R_2}{\mathcal{E}}$$

Continued →

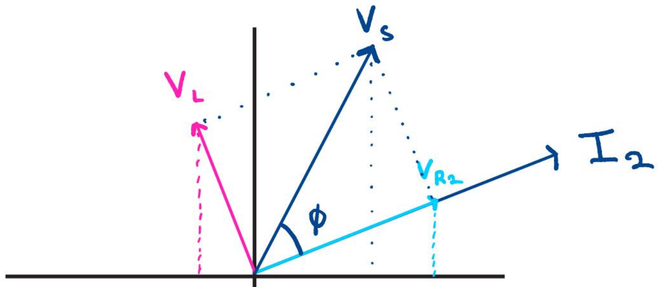
$$\frac{\varepsilon}{R_2} e^{-(R_2/L)t} - \frac{\varepsilon}{R_2} = -i_2$$

$$\boxed{i_2(t) = \frac{\varepsilon}{R_2}\left(1 - e^{-(R_2/L)t}\right)}$$

Sketch:



c. In this part, the battery and switch are replaced by an ac source. Use the axes below to make a phasor diagram for the source potential $v_s(t) = V_s \cos(\omega t + \phi)$, and current $i_2(\cos(\omega t)$. Then use it to solve for the current amplitude I_2, and evaluate the phase angle ϕ. What are the amplitude and phase for $i_1(t)$? (6 points)



v_L is $\frac{\pi}{2}$ ahead of current

$\left(v_L = L\frac{di_2}{dt} = -I_2 \omega L \sin(\omega t)\right.$,

and $\cos(\omega t + \frac{\pi}{2}) = -\sin(\omega t)$,

so $\left. v_L = I_2 \omega L \cos(\omega t + \frac{\pi}{2})\right)$.

v_{R_2} is in phase with current

$(v_{R_2} = i_2 R_2 = I_2 R_2 \cos(\omega t))$.

$V_{R_2} = I_2 R_2$

$V_L = I_2 \omega L$

$$V_s = \sqrt{(I_2 R_2)^2 + (I_2 \omega L)^2}$$

$$= I_2 \sqrt{R_2^2 + (\omega L)^2}$$

$$\boxed{I_2 = \frac{V_s}{\sqrt{R_2^2 + (\omega L)^2}}}$$

$$\tan\phi = \frac{V_L}{V_{R_2}} = \frac{I_2 \omega L}{I_2 R_2} = \frac{\omega L}{R_2}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{\omega L}{R_2}\right)}$$

$i_1(t)$ is in phase with the source as the circuit branch only has a resistor whose voltage drop must equal the source voltage.

$V_s = V_{R_1} = i_1 R_1$

so $i_1 = \frac{V_s}{R_1} \Rightarrow i_1$ is in phase with V_s.

$$\therefore v_s = v_{R_1} = i_1 R_1 = V_s \cos\left(\omega t + \overbrace{\tan^{-1}\left(\frac{\omega L}{R_2}\right)}^{\phi}\right)$$

$$\boxed{i_1 = \frac{V_s}{R_1}\cos(\omega t + \phi)} = \frac{V_s}{R_1}\cos\left(\omega t + \tan^{-1}\left(\frac{\omega L}{R_2}\right)\right)$$

Amplitude: $\boxed{I_1 = \frac{V_s}{R_1}}$

1 Inductors in circuits **14 / 14**

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- **- 0.5 pts** Part c. Incorrect phase angle for $$I_2$$, $$\phi=\tan^{-1}(\frac{\omega L}{R_2})$$
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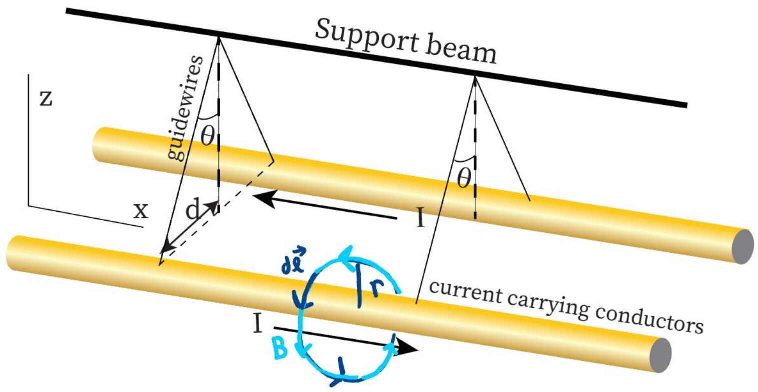
✓ **- 0 pts Question 1 All correct**

- **- 0.5 pts** Part b. $$I_2$$ at t = 0, incorrect. $$I_2 = 0$$ at t = 0
- **- 4 pts** Part a. Incorrect

Problem 2. Sources of magnetic fields, magnetic forces

(14 points total)



a. Write down an integral expression for Ampère's Law, and use it to find the magnetic field of a long, straight wire. Make sure to define the direction of the magnetic field. You may choose to make a diagram as part of your answer. (4 points)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{encl} + \epsilon_0 \frac{d\Phi_E}{dt} \right)$$

B is constant and parallel to $d\vec{l}$ around a circular path:

$$B \cdot \int_0^{2\pi} r \, d\theta = B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$ B points counterclockwise (in the direction indicated by the arrows in the diagram)

In general, B points in a direction determined by the right hand rule (thumb in direction of I, curl fingers in direction of field).

r = distance from current conducting wire

b. Consider the long wire segments in the figure above (only a section of the two straight-line segments is shown). Each wire carries current I, but in opposite directions, and they are supported by identical guidewires of length ℓ. The conducting wires are in static equilibrium (stable position), and separated by distance $2d$. What is the magnetic force of interaction *per unit length* f_B between the two segments? You should assume that the length of the segments satisfies the condition $L \gg d$. Your answer should be written in terms of I, and d. (6 points)

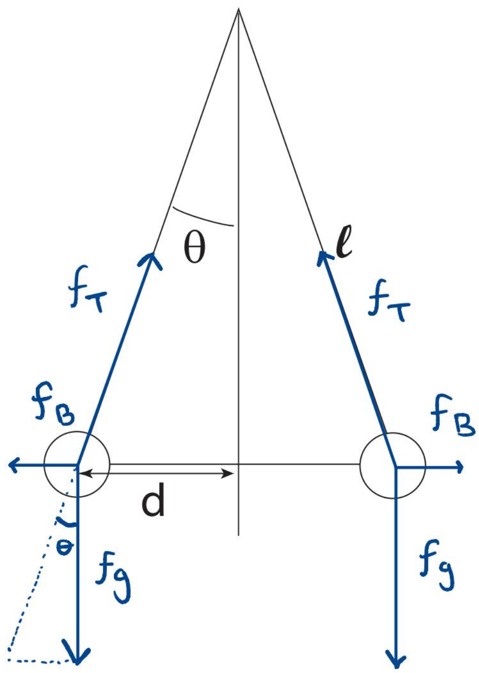
$F = I\vec{L} \times \vec{B}$ ($\vec{L}$ & $\vec{B}$ perpendicular)

$F = ILB$ in direction away from other wire (repulsive)

$f_B = \frac{F}{L} = IB \rightarrow B = \frac{\mu_0 I}{2\pi (2d)}$

$$\boxed{f_B = \frac{\mu_0 I^2}{4\pi d}}$$

c. In the drawing below, sketch the direction of the forces, f_B, f_g acting on the wire segments. f_g is the gravitational force normalized per unit length. (2 points)



d. Taking the forces (per length) acting on each of the wire segments to be f_B and the gravitational force $f_G = \lambda g$, write down the ratio f_B/f_G in terms of the angle θ. λ is the mass per length of each straight wire. (2 points)

$$f_B = \frac{\mu_0 I^2}{4\pi d} \qquad f_G = \lambda g$$

In equilibrium, $\vec{f}_B + \vec{f}_G = \vec{f}_T$ (tension in guidewire)

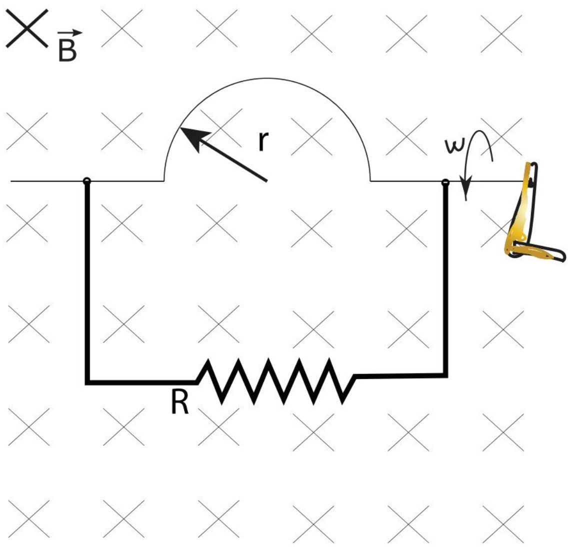
$$\therefore \boxed{\frac{f_B}{f_G} = \tan\theta} \rightarrow \frac{\mu_0 I^2}{4\pi d} = \tan\theta$$

2 Sources of magnetic fields, magnetic forces 14 / 14

- ✓ + **2 pts** a) Write the integral expression for Ampere's law: $$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$
- ✓ + **2 pts** a) Use the integral expression to find an expression for the magnetic field: $$B(r) = \frac{\mu_0 I_{enc}}{2\pi r}$$
- ✓ + **4 pts** b) Write an expression for the force per unit length $$f_B = IB$$
- ✓ + **2 pts** b) Use the expression for $$B$$ from (a) to find $$f_B$$ and simplify: $$f_B = \frac{\mu_0 I^2}{4\pi d}$$
- ✓ + **1 pts** c) Correctly draw the gravitational force (downward)
- ✓ + **1 pts** c) Correctly draw the magnetic force direction (away from the other wire)
- ✓ + **1 pts** d) Identify that the tension balances the sum of the gravitational and magnetic forces: $$\vec{T} = \vec{f_G}+\vec{f_B}$$
- ✓ + **1 pts** d) Use the geometry of the system to relate the angle $$\theta$$ to the forces: $$\tan\theta = \frac{f_B}{f_G}$$

Problem 3. Faraday's Law

Shown is a semi-circular wire loop, free to rotate in a uniform magnetic field. The radius of the semicircular part is r. The direction of the rotations is such that $\vec{\omega}$ is directed horizontal to the right, the magnetic field is directed into the page. Note: the part of the circuit containing the resistor R (heavier lines) is not free to rotate. (14 points total)



a. Write down Faraday's Law, which relates a magnetically induced emf to the rate of change of magnetic flux through the current loop. (fill in the blank). Define terms introduced below. (2 points)

Magnetic flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Rate of change of flux $= \frac{d\Phi_B}{dt} = \frac{d}{dt} \int \vec{B} \cdot d\vec{A}$

$\vec{B}$ = magnetic field

$\vec{A}$ = area of loop $\cdot \hat{n}$, where $\hat{n}$ = normal vector to loop

Faraday's Law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$, or $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

b. What is the change in magnetic flux through the semi-circular part on rotating it through a quarter turn, from an orientation with the magnetic field directed orthogonal to the plane of the loop (shown in the figure), to where $\mathbf{B}$ is in the plane of the loop? (4 points)

$\Phi_{B\,\text{initial}} = \int \vec{B} \cdot d\vec{A}$

$\vec{B} \parallel \vec{A}$ and uniform, so:

$\Phi_{B_i} = B \cdot A = B \cdot \frac{\pi r^2}{2}$

$\Phi_{\text{final}} = 0$ as $\vec{B} \perp \vec{A}$.

$$\boxed{\Delta \Phi_B = \Phi_{Bf} - \Phi_{B_i} = -\frac{B\pi r^2}{2}}$$

$\phi = \omega t$

c. The loop is rotating at angular frequency ω. What is the frequency and amplitude of the induced emf? (4 points)

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$-\mathcal{E} = \frac{d\Phi_B}{dt} = -\frac{1}{2}\pi B r^2 \omega \sin(\omega t)$$

$dA = r\,dr\,d\theta$

$$\Phi_B = \int_0^r \int_0^{\pi} B r \cos(\omega t)\, dr\, d\theta$$

$$= \int_0^r \pi B r \cos(\omega t)\, dr$$

$$= \frac{1}{2}\pi B r^2 \cos(\omega t)$$

$$\boxed{\mathcal{E} = \frac{\pi}{2} B r^2 \omega \sin(\omega t)}$$

$$\boxed{f = \frac{2\pi}{\omega}}$$

$$\boxed{\text{amplitude} = \frac{\pi}{2} B r^2 \omega}$$

d. Assuming the mechanical friction between rotating and fixed conductors in the circuit is negligible, how much time-averaged power must be supplied from an external source to rotate the loop at ω? (4 points)

$$P = vi = \frac{v^2}{R} = \frac{\pi^2 B^2 r^4 \omega^2 \sin^2(\omega t)}{4R}$$

$$\langle P \rangle_t = \left\langle \frac{v^2}{R} \right\rangle_t$$

Time average of $\sin^2(x) = \frac{1}{2}$, so we can replace $\sin^2(\omega t)$ with $\frac{1}{2}$.

$$\boxed{\therefore \langle P \rangle_t = \frac{\pi^2 B^2 r^4 \omega^2}{8R}}$$

3 Faraday's Law **14 / 14**

- ✓ **+ 2 pts** **3(a) Correct. Ans: $$\epsilon= \frac{ -d \phi_B}{dt}$$, $$\epsilon=$$ induced emf, $$\phi_B=$$ mag flux integrated over the surface bounded by the loop used to determine $$\epsilon$$**
  - **+ 1 pts** 3(a) partially correct/ no explanation of terms
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