MT 2 Physics 1C Fall 2014

Name 

Signature

9-digit SID

1	30	/33
2	29	/35
3	32	/32
Total	91	/100

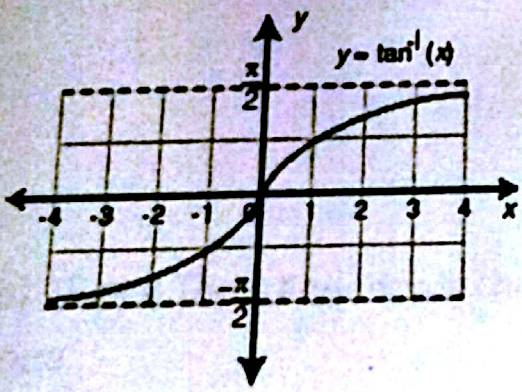
Turn off and put away all electronics, phones, cameras, calculators.

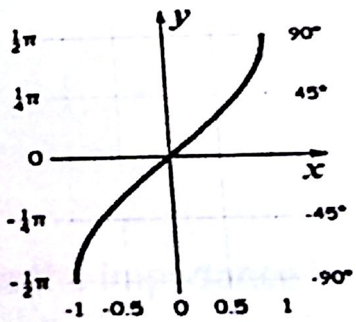
You may have a single-sided one-half page of notes, pencils and eraser on your desk and this exam.

Put all work and calculations on the pages of the exam. It's good advice not to erase anything, just cross out. You may get partial credit on some problems.

The following may or may not be helpful. $\sin 30 = \frac{1}{2}$, $\sin 60 = \frac{\sqrt{3}}{2}$, $\cos 30 = \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{1}{2}$, $\cos 45 = \sin 45 = \frac{\sqrt{2}}{2} \approx 0.7$, $0.707 \approx \frac{1}{1.4}$ $\pi^2 = 9.87 \approx 10$



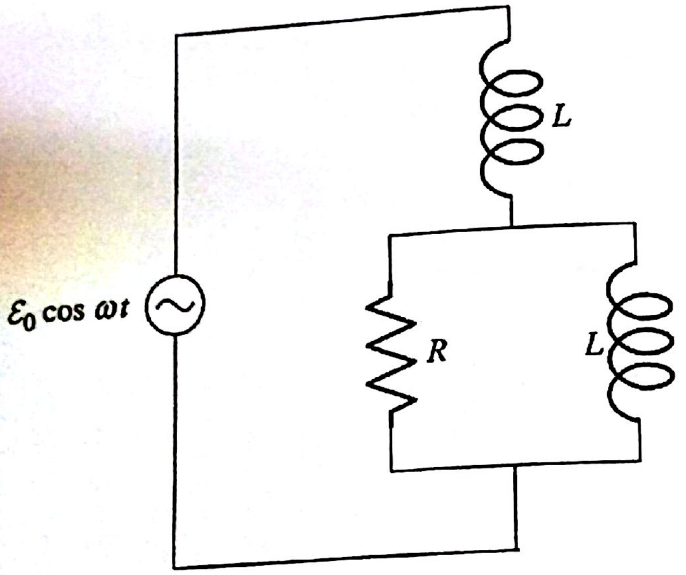


Inverse Sine

$\mu_0 = 4\pi\times10^{-7}$ Tm/A (Tm/A = N/A^2) $\epsilon_0 = 8.85\times10^{-12}$ C^2/Nm^2

1 eV= 1.602×10^{-19} J 1 MeV= 1.602×10^{-13} J 1 nm=10^{-9} m

1. Consider the circuit below with an AC voltage source, $V = V_0 \cos(\omega t)$ and two equal inductors L and a resistor R.



1a). (8 pts) What is the total impedance of the circuit? (Complex form is ok) .

$$\omega L i + \frac{1}{\frac{1}{R}+\frac{1}{\omega L i}} = \frac{R+\omega L i}{\omega L i R} + \omega L i$$

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1b). (5 pts) The frequency is chosen so $\omega = R/L$. Give the impedance in terms of R alone.

$$Ri + \frac{1}{\frac{1}{R}+\frac{1}{Ri}} = Ri + \frac{1}{\frac{1-i}{R}} = Ri + \frac{R}{1-i}$$

$$= Ri + \frac{R(1+i)}{2}$$

$$= \frac{R}{2} + \frac{3}{2}Ri$$

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1c). (8 pts) If the total current though the circuit is written as $I_0 \cos(\omega t + \varphi)$, what are I_0 and φ?

$$Re[Z(+)] = \sqrt{\left(\frac{R}{2}\right)^2 + \left(\frac{3}{2}R\right)^2} = \sqrt{\frac{10}{4}R^2} = R\sqrt{\frac{5}{2}}$$

$$I_0 = \frac{V_0}{Z} = \frac{\varepsilon_0}{R}\sqrt{\frac{2}{5}}$$

$$\varphi = \tan^{-1}\left(\frac{3/2\,R}{1/2\,R}\right) = \boxed{\tan^{-1}(3)}$$

X

1d). (8 pts) What is the average power dissipated in the circuit

$$P_{av} = I_{rms} V_{rms} \cos\phi = \frac{1}{2} I_0 V_0 \cos\phi$$

$$= \frac{1}{2}\frac{\varepsilon_0^2}{R}\sqrt{\frac{2}{5}} \cdot \frac{3}{\sqrt{2}\sqrt{5}} = \boxed{\frac{3\varepsilon_0^2}{10R}}$$

$\sqrt{10}$, 3, 1

X

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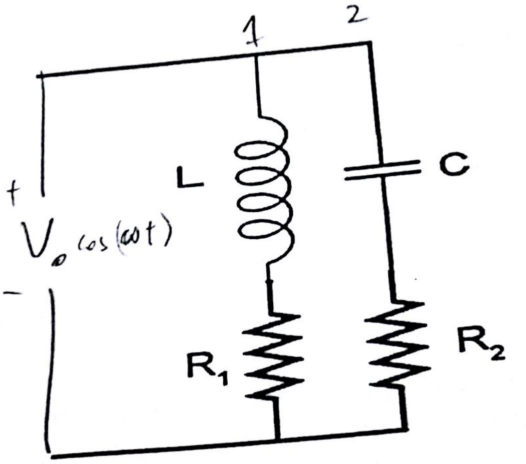
1e). (4 pts) $R_1 = R_2$ in the circuit below. If C = 24 µF and L = 0.6 mH, at what frequency will there be equal current to both resistors?



$X_1 = \omega L_i + R$

$X_2 = R + \frac{1}{\omega C_2}$

$I_1 = I_2$

$Z_1 = \sqrt{(\omega L)^2 + R^2}$

$Z_2 = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

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$\frac{V}{Z_1} = \frac{V}{Z_2}$

$R^2 + \left(\frac{1}{\omega C}\right)^2 = R^2 + (\omega L)^2$

$\omega^2 LC = \pm 1$

$\omega^2 = \pm \frac{1}{LC}$

$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{14}}$ $= \sqrt{\frac{1}{1}}$

2a. (2 pts) The electric field of an infinite plane wave can be written

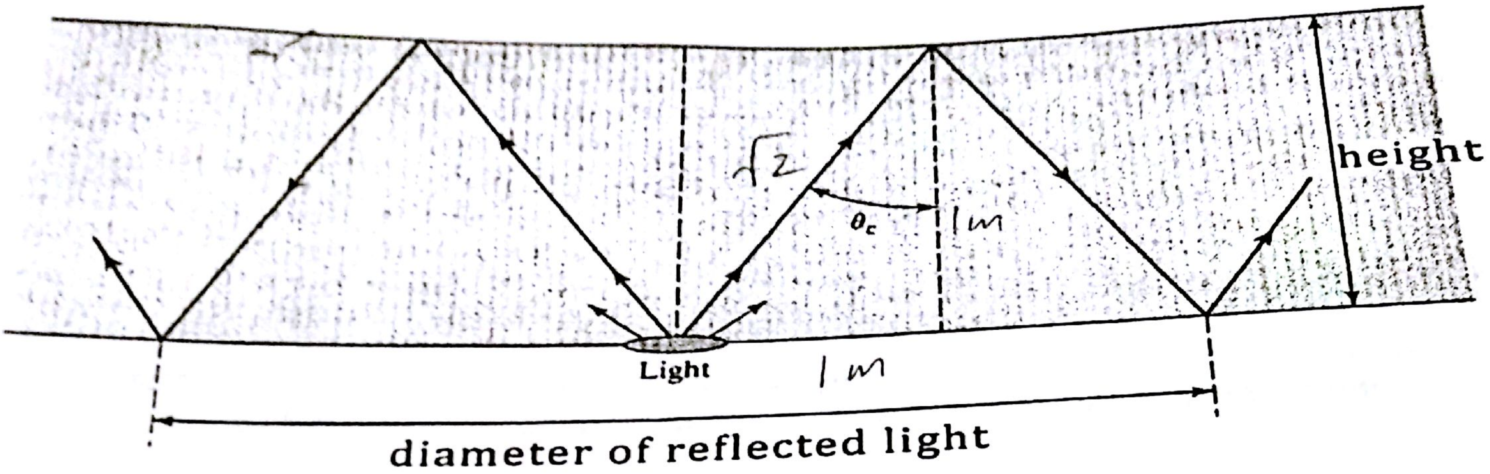
$$E_y(x,t) = E_0 \cos(kx + \omega t)$$

Write an expression for the corresponding magnetic field of the wave.

$E \times B = -x$

$B_0 = \frac{E_0}{c}$

$$B_z(x,t) = -\frac{E_0}{c} \cos(kx + \omega t)$$



2b. (8 pts) A small source of light at the bottom of a pool filled to a 1 m height is viewed from air above. Rays of light reflected at the top surface outline a circle 4 m in diameter on the **bottom** surface. (side view above) What is the refractive index of the fluid?

$$n \sin \theta_c = \sin(90)$$

$$\sin \theta_c = \frac{1}{n}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{n}$$

$$\boxed{n = \sqrt{2}}$$

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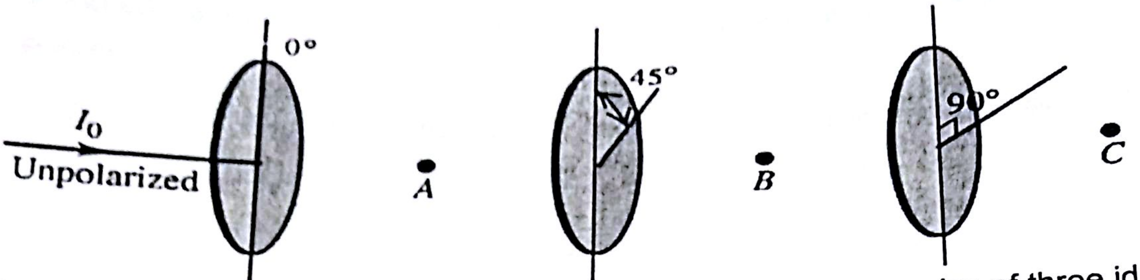

2c. A beam of unpolarized light of intensity I_0 passes through a series of three ideal polarizing filters with their polarizing directions turned to the angles shown above.

What is the light intensity (in terms of I_0) at points A,B.C?

A. (3 pt) $\boxed{\frac{I_0}{2}}$

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B. (3 pt) $\frac{I_0}{2} \cdot \cos^2(45) = \boxed{\frac{I_0}{4}}$

C. (3 pt) $\frac{I_0}{4} \cos^2(45) = \boxed{\frac{I_0}{8}}$

2d. (10 pts) A birefringent crystal has two different index of refractions, 1.51 for the electric field aligned with the x-axis and 1.56 for the electric field along the y-axis. Find the crystal thickness z which will cause a π/2 (or quarter wave) phase difference for the two polarizations of light with a free space wavelength of 500 nm.

$n = \frac{c}{v}$ $\lambda = \frac{\lambda_0}{n}$

- 1.51

wavelengths traveled $= \frac{z}{\lambda} = \frac{zn}{\lambda_0}$

$\frac{z}{\lambda_y} = \frac{z}{\lambda_x} + \frac{\pi}{2}$

$2\pi \cdot \frac{z n_y - z n_x}{\lambda_0} = \frac{\pi}{2}$

$\frac{2\pi z}{\lambda_0}$ ...s traveled

$z = \frac{\frac{\pi}{2}}{2\pi} \frac{\lambda_0}{n_y - n_x} = \frac{\pi}{2} \frac{5 \cdot 10^{-7}}{5 \cdot 10^{-2}} = \frac{\pi}{2} \cdot 1$

$v_x v_x + \frac{\pi}{2}$

A 200 kW radio AM radio station broadcasts 1500 kHz into a hemisphere above the ground.

2 2e. (3 pt) At 5 km from the station antenna, what is the intensity of the EM wave?

$$\frac{200 \cdot 10^3}{2\pi \cdot (5000)^2} = \frac{0.2 \cdot 10^6}{2\pi \cdot 25 \cdot 10^6} = \boxed{\frac{1}{500\pi}}$$

2 2f. (3 pt) At 5 km from the antenna, what is the maximum electric field in the wave?

$$I = S_{AV} = \frac{EB}{2\mu_0} = \frac{E^2}{2\mu_0 c}$$

$$E = \sqrt{2\mu_0 c I} = \sqrt{2 \cdot 4\pi \cdot 10^{-7} \cdot 3 \cdot 10^8 \cdot \frac{1}{500\pi}}$$

$$= \sqrt{\frac{24}{50}} = \sqrt{\frac{12}{25}} = \boxed{\frac{2}{5}\sqrt{3}}$$

3a. (5 pts) I had trouble seeing things far away and my prescription was -5 diopters. What was the far point of my uncorrected eye?
(If you don't know what a diopter is, use a focal length of 30 cm for partial credit)

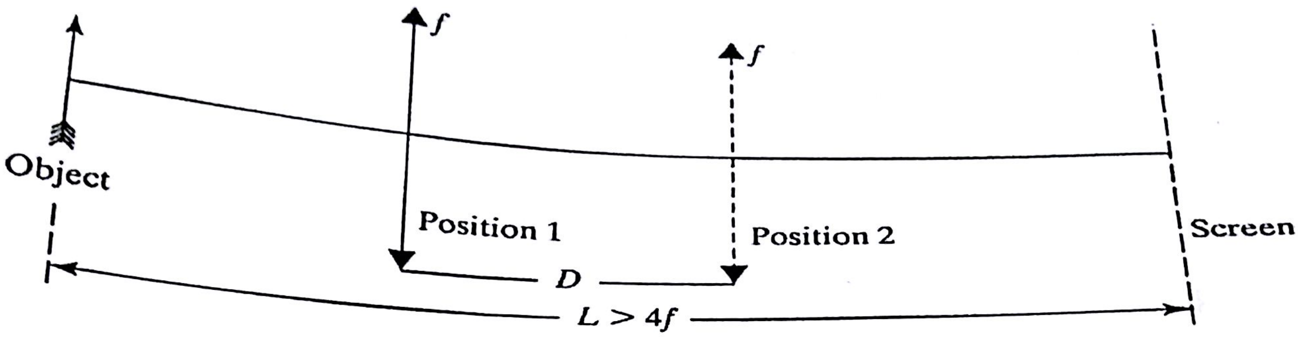
$$P = \frac{1}{f} \qquad f = -\frac{1}{5} \text{ m}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$s = \infty$$

$$\frac{1}{s'} = \frac{1}{f}$$

$$\boxed{s' = -0.2 \text{ m} = 20 \text{ cm from eye}}$$



3b. 8(pt) Bessel's method for finding the focal length of a lens is described in the figure above. The object and screen are at a fixed distance L greater than 4 times the unknown focal length. The single lens is moved between the object and the screen. The image is in focus on the screen for two different positions of the lens separated by distance D. What is the focal length of the lens in terms of L and D?

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s} + \frac{1}{L-s} = \frac{1}{f}$$

$$L - 2s = D$$

$$s = \frac{L-D}{2}$$

$$\frac{1}{\frac{L-D}{2}} + \frac{1}{L - \frac{L-D}{2}} = \frac{1}{f}$$

$$\frac{2}{L-D} + \frac{2}{L+D} = \frac{1}{f}$$

$$\frac{4L}{L^2 - D^2} = \frac{1}{f}$$

$$f = \frac{L^2 - D^2}{4L}$$

✓

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3c. (5 pt) A 10 cm tall object is 60 cm to the left of a convex converging lens of focal length 40 cm. How far away from the lens would you put a screen to see the image in focus?

$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{40} - \frac{1}{60} = \frac{1}{20}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{120}$

120 cm ✓

3d. (4 pt) How tall would the image be on the screen?

$m = -\frac{120}{60} = -2$

−20 cm ✓

3e. (5 pt) Now we remove the screen and put a second converging lens with a focal length of 90 cm about 300 cm to the right of the first lens. What is the location of the image after passing through the two lenses?

$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{90} - \frac{1}{180} = \frac{1}{180}$

180 cm ✓

3f. (5 pt) What is the height of the final image?

$m_2 = -\frac{180}{180} = -1$

20 cm ✓