midterm2

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midterm2

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Name

ID

Section

Physics 1C
Fall 2018
Sivaramakrishnan

Midterm Exam

Problem 1: ____

Problem 2: ____

Problem 3: ____

Problem 4: ____

Total: ____ /100

To get credit for an answer you must show your work!

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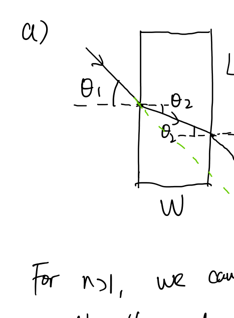
Problem 1: [25 points]

a) [15 pts] A light ray is incident on the longer side of a rectangular block of glass at angle θ from the normal. The block has index of refraction $n > 1$, width W, and length $L > W$. Find the path of the ray through the block and out the other side (draw your answer and compute all relevant angles). Is it possible to find a value of n and θ such that no light makes it all the way out the other side (i.e. it is reflected back)? Why or why not?

b) [5 pts] Suppose instead there is a small light bulb inside the block of glass. Draw a ray diagram to show where the apparent image would be, as viewed by the eye outside the block of glass.

c) [5 pts] Now exchange the location of the eye and bulb so the bulb is outside the block and the observer is inside. Draw a ray diagram to show where the apparent image would be.

a)



$1 \cdot \sin\theta_1 = n\sin\theta_2 \Rightarrow \theta_2 = \arcsin(\frac{1}{n})$

$n\sin\theta_2 = 1 \cdot \sin\theta_3 \Rightarrow \theta_3 = \theta_1$

$\Rightarrow$ the outgoing light is parallel to the incident light.

For $n>1$, we can never find n, θ such that no light makes it out the other side. Because, as what we calculate before, the light will get out will an angle that is parallel to the incoming light.

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b)

c)

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Problem 2: [25 points]

A circuit contains an inductor L, capacitor C, resistor R, and open switch in series. The capacitor initially carries change Q_0. The switch is closed. Applying Kirchhoff's loop rule, we have the relationship $Q/C + IR + L\frac{dI}{dt} = 0$ where Q is the charge on the capacitor.

a) [15 pts] Suppose the charge takes the form $Q(t) = Ae^{t/\tau}\cos(\omega t)$ for some values of τ, ω, A. If $R = 0$, find the value of ω, τ, A such that this is a valid solution to $Q/C + IR + L\frac{dI}{dt} = 0$ with $R = 0$.

b) [5 pts] The solution for ω with $R \neq 0$ is $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$. This solution is only valid for ω real, but at fixed L, ω becomes imaginary for large enough R or C. What do you expect to be the qualitative difference of $I(t)$ between the ω real and imaginary cases? How does this compare to the behavior in an LR circuit (or if you prefer, an RC circuit)?

c) [5 pts] What is I_{rms} in the case of $R = 0$?

a) $\frac{Q}{C} + L\frac{dI}{dt} = 0$, $Q(t) = Ae^{t/\tau}\cos(\omega t)$

$I(t) = \frac{dQ(t)}{dt} = -Ae^{t/\tau}\omega\sin(\omega t) + \frac{1}{\tau}Ae^{t/\tau}\cos(\omega t)$

$\frac{dI(t)}{dt} = -Ae^{t/\tau}\omega^2\cos(\omega t) - \frac{A}{\tau}e^{t/\tau}\omega\sin(\omega t)$
$+ \frac{1}{\tau^2}Ae^{t/\tau}\cos(\omega t) - \frac{1}{\tau}A\omega e^{t/\tau}\sin(\omega t)$
$= (\frac{1}{\tau^2} - \omega^2)Ae^{t/\tau}\cos(\omega t) - 2\frac{\omega}{\tau}Ae^{t/\tau}\sin(\omega t)$

$\Rightarrow \frac{1}{C}Ae^{t/\tau}\cos(\omega t) + L(\frac{1}{\tau^2} - \omega^2)Ae^{t/\tau}\cos(\omega t) - 2\frac{\omega L}{\tau}Ae^{t/\tau}\sin(\omega t) = 0$

$[\frac{1}{C} + L(\frac{1}{\tau^2} - \omega^2)]\cos(\omega t) - \frac{2\omega L}{\tau}\sin(\omega t) = 0.$

$\Rightarrow \begin{cases} \frac{1}{C} + L(\frac{1}{\tau^2} - \omega^2) = 0, \\ \frac{2\omega L}{\tau} = 0 \end{cases} \Rightarrow \begin{cases} \tau = \infty \\ \omega = \frac{1}{\sqrt{LC}} \end{cases}$

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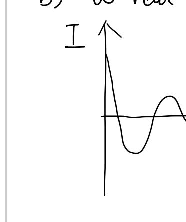
$Q(t) = A\cos(\omega t)$,

at $t=0$, $Q = Q_0$, $\Rightarrow A = Q_0$

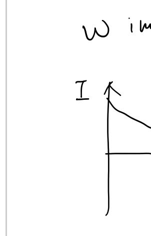
b) ω real: oscillate & decay. (R small)

$\frac{1}{LC} > \frac{R^2}{4L^2} \Rightarrow R < 2\sqrt{\frac{L}{C}}$



ω imaginary: decay (R large)

$R > 2\sqrt{\frac{L}{C}}$,



↳ like a RL circuit, I(t) decay over time with $\tau = \frac{L}{R}$

c) $R=0$, $Q(t) = Q_0\cos(\omega t)$

$I(t) = -\omega Q_0\sin(\omega t)$

$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2\,dt} = \frac{1}{\sqrt{2}}\omega Q_0 = \frac{1}{\sqrt{2}}\frac{Q_0}{\sqrt{LC}}.$

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Problem 3: [25 points]

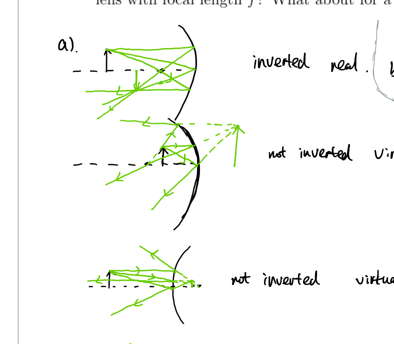
a) [8 pts] Consider an object with some non-zero height (for example a rectangle or arrow) on the optical axis of a mirror. Draw four ray diagrams showing how to find the image for the cases of a concave or convex mirror, with the object inside or outside the focal point. State whether the image is inverted or not, and whether the image is real or virtual.

b) [8 pts] Do the same for a diverging and converging lens.

c) [5 pts] Using ray diagrams and/or the thin lens equation, briefly justify the statement that "solving the converging lens is equivalent to solving the converging mirror" and the same for a diverging lens/mirror.

d) [2 pts] Where is the image located when the object is placed at the focal point of a converging lens with focal length f? What about for a diverging lens?

e) [2 pts] Where is the image located when the object is placed infinitely far from a converging lens with focal length f? What about for a diverging lens?



c) converging lens: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $f > 0$

. . . . mirror: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, $f = \frac{R}{2} > 0$

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diverging lens: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $f < 0$

mirror: . . . . , $f = -\frac{R}{2} < 0$.

the sign of f are same for converging lens and mirror, so their thin lens equation are in same form $\Rightarrow$ equivalent.

So do diverging lens and mirror.

d)

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, $s = f \Rightarrow s' = \infty$

$\frac{1}{s} + \frac{1}{s'} = -\frac{1}{f}$, $s = f \Rightarrow s' = -\frac{f}{2}$

e).

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, $s = \infty$, $s' = f$.

$\frac{1}{s} + \frac{1}{s'} = -\frac{1}{f}$, $s = \infty$, $s' = -f$

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Problem 4: [25 points]

A plane electromagnetic wave is travelling in the z direction. The wavenumber is $k = 2\pi/\lambda = 2m^{-1}$. The magnetic field is found to point in the x direction and has amplitude $B = 1 \times 10^{-8}T$.

Recall: $c = 3 \times 10^8 m/s$, $\mu_0 = (4\pi) \times 10^{-7} T \cdot m/A$, $\epsilon_0 = 8.85 \times 10^{-12} C^2/N \cdot m^2$

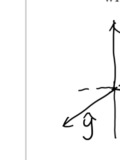
a) [5 pts] What is the angular frequency ω (recall $\omega = 2\pi f$)?

b) [5 pts] What is the direction and amplitude of the electric field?

c) [5 pts] What is the direction and magnitude of the *average* energy flux (energy flow per area per unit time)?

d) [10 pts] Consider a hypothetical plane electromagnetic wave which had a nonzero electric field as usual, but a vanishing magnetic field everywhere. Which of Maxwell's equations would this violate?

Note: Give numerical answers as appropriate, but don't worry about giving precise answers with multiple decimal places.



a) $\omega = ck = 6 \times 10^8 /s$

b) direction: $-\hat{y}$

Amplitude: $E = cB = 3\ T \cdot m/s = 3\ V/m$

c) $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$,

$\langle S \rangle = \frac{1}{2}\frac{1}{\mu_0}E_{max}B_{max} = \frac{3}{8\pi} \times 10^{-1} = 0.012\ J/m^2 s$

d) $\begin{cases} \vec{E} = E_0\hat{y}\cos(kz - \omega t) \\ \vec{B} = 0 \end{cases}$

violate: Faraday's law and Ampere's law.

$\int \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ or $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\int \vec{B} \cdot d\vec{l} = \mu_0 (I_c + \epsilon_0 \frac{d\Phi_E}{dt})_{encl}$ $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$.

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