Physics 1C
UCLA
Fall 2018
Sivaramakrishnan

Midterm Exam

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Name

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ID

Problem 1: ____

Problem 2: ____

Problem 3: ____

Problem 4: ____

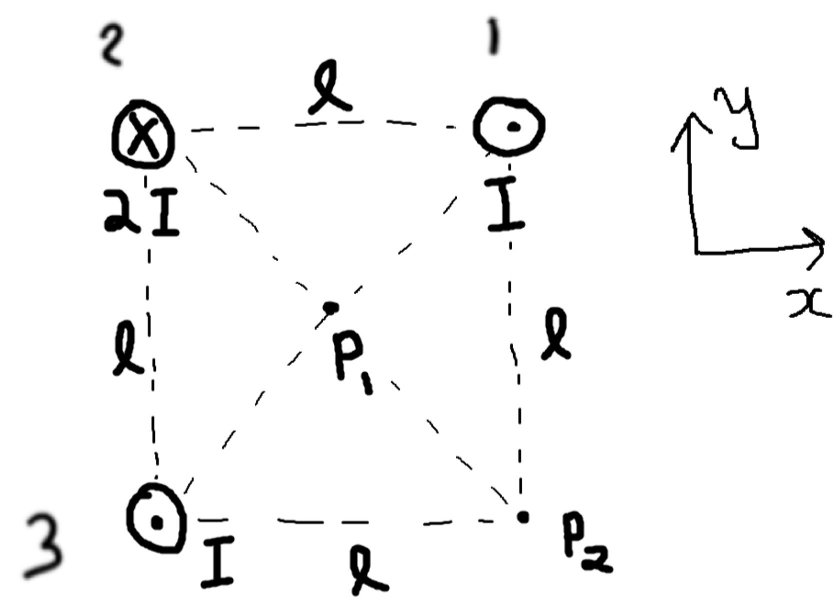
Total: ____ **/100**

Show your work! Answers are given credit according to justification provided.

Problem 1: [25 points]

a) [**5pts**] Use Ampere's law to calculate the magnitude of the magnetic field a perpendicular distance r from an infinitely-long straight wire carrying current I.

b) [**10 pts**] Now consider the following diagram, in which parallel infinitely-long straight wires are placed at three corners of a square of side length l. The wires opposite one another carry current I out of the page, and the third carries current $2I$ into the page. Find the magnetic field at point P_1, the center of the square.



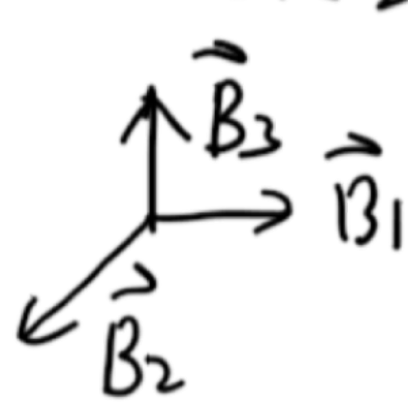
c) [**10 pts**] Find the magnetic field at point P_2, the fourth corner of the square.

a) $2\pi r B = \mu_0 I$, $B = \frac{\mu_0 I}{2\pi r}$.

b) at P_1, $\vec{B}_1 + \vec{B}_3 = 0$ (opposite direction)

$$\vec{B} = \vec{B}_2 = \frac{\mu_0 2I}{2\pi \frac{\sqrt{2}}{2} l}(-\hat{x}-\hat{y})\frac{\sqrt{2}}{2} = -\frac{\mu_0 I}{\pi l}(\hat{x}+\hat{y})$$

c) $\vec{B}$:



$B_1 = B_3 = \frac{\mu_0 I}{2\pi l}$

$B_2 = \frac{\mu_0 2I}{2\pi \sqrt{2} l}$,

$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = 0$

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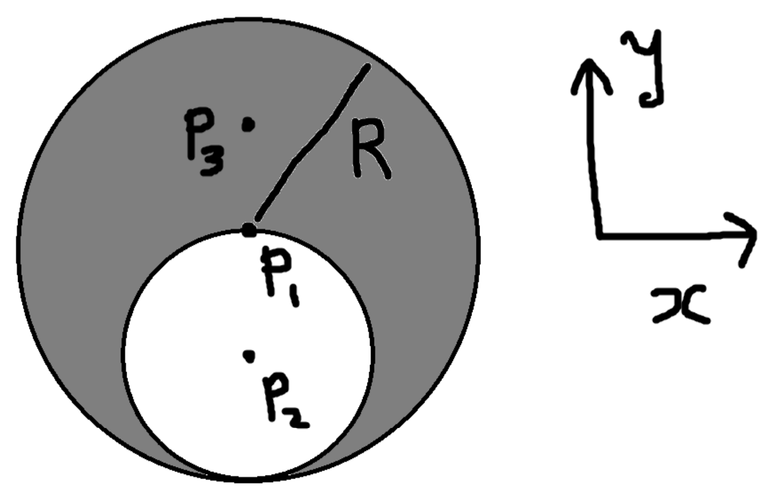
Problem 2: [25 points] **Name**

a) [**5 pts**] Suppose a cylindrical wire of radius R has uniform current density with total current I. Find the magnitude of the magnetic field at a perpendicular distance $r < R$ from the center of the wire.

b) [**10 pts**] Now suppose the cylindrical wire has an off-center cylindrical hole as pictured below, but the current density in the remaining shaded region remains the same as in part a). The hole has diameter R and lies tangent to the circle. What is the magnitude of the magnetic field at point P_1, the center of the circle?

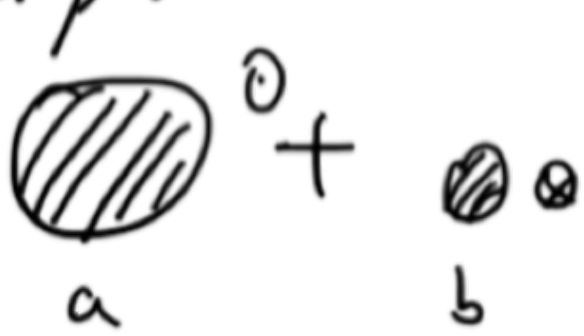


c) [**10 pts**] What is the magnitude of the field at point P_2, the center of the hole?

a) $2\pi r B = \frac{\mu_0 I}{\pi R^2}\pi r^2$, $B = \frac{\mu_0 I}{2\pi R^2} r$

b) Density: $\frac{I}{\pi R^2} = J$.

Superposition.



P_1: $\vec{B} = \vec{B}_a + \vec{B}_b = 0 + \frac{\mu_0 J \pi \left(\frac{R}{2}\right)^2}{2\pi \frac{R}{2}} \hat{x}$

$= 0 + \frac{\mu_0 J}{4} R \hat{x}$

$= \frac{\mu_0 I}{4\pi R} \hat{x}$

P_2: $\vec{B} = \vec{B}_a + \vec{B}_b = \frac{\mu_0 I}{2\pi R^2} \frac{R}{2} \hat{x} + 0 = \frac{\mu_0 I}{4\pi R} \hat{x}$

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Problem 3: [25 points]

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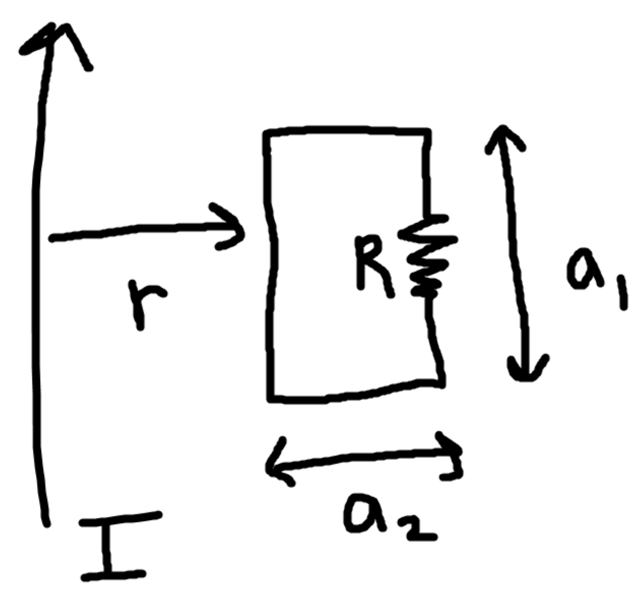
Name

A infinite straight wire carries current I. A rectangular loop is placed a distance r from the wire. In this problem, ignore any self-inductance effects (if you don't know what these are, don't worry, we haven't learnt this yet).



a) [10 pts] Suppose that $a_1 = a_2 = a$. What is the magnetic flux through the loop?

b) [10 pts] Suppose now that the current in the straight wire is time dependent, $I = I(t) = I_0 e^{-bt}$, where $b > 0$. If the loop has resistance R, what current will flow through the loop and in which direction?

c) [5 pts] In addition to the time-dependence of $I(t)$ above, suppose also that the loop's length changes in time according to $a_1(t) = af(t)$. What is the sign of $f'(t)$ (i.e. should the loop should grow or shrink) so that there is no induced current? Justify with a brief explanation or by finding $f'(t)$.

a). $B = \frac{\mu_0 I}{2\pi r}$, $\quad \Phi = a \cdot \int_r^{r+a} \frac{\mu_0 I}{2\pi r'}\, dr' = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{r+a}{r}\right)$, ⊗

b) direction: ↻ CW.

$$|I| = \left|\frac{\varepsilon}{R}\right| = \frac{1}{R}\left|\frac{d}{dt}\Phi\right| = \frac{1}{R}\frac{\mu_0 a}{2\pi}\ln\left(\frac{r+a}{r}\right)\left|\frac{dI}{dt}\right|$$

$$= \frac{\mu_0 a}{2\pi R}\ln\left(\frac{r+a}{r}\right) I_0 b e^{-bt}$$

c) no current $\Rightarrow \frac{d\Phi}{dt} = 0$, B is ↓, A must ↑, so $f'(t) > 0$.

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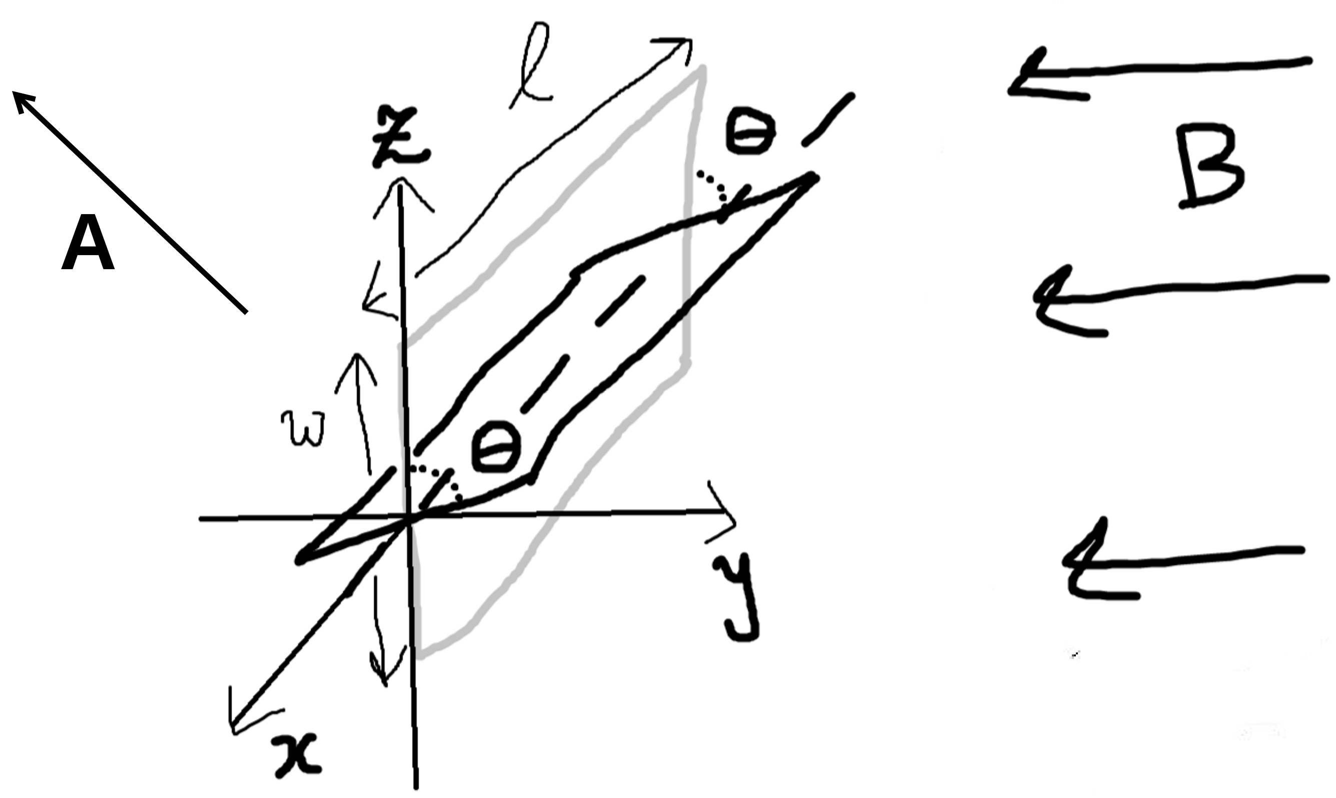
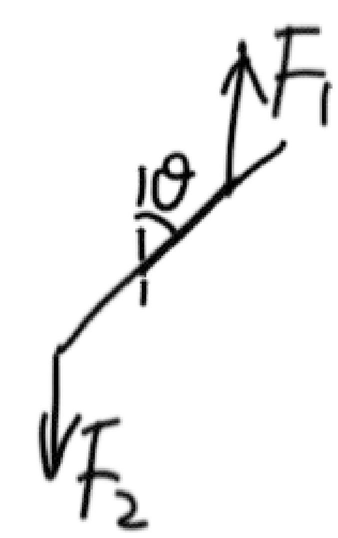
Problem 4: **[25 points]** ______________________ **Name**

The rectangular loop of wire with length l and width w pictured below is rotating about its center in a constant magnetic field $\vec{B} = \text{-}B\hat{y}$. The angular speed of rotation is fixed by hand to be $\omega \frac{rad}{s}$ and the axis of rotation is aligned with the x-axis as pictured. At $t = 0$, the loop is oriented at $\theta = 0$, in the $x - z$ plane. We will only consider half a revolution of the wire in this problem: $\theta = 0$ to $\theta = \pi$.





a) [10 pts] As a function of time t, what is the induced emf in the circuit?

b) [5 pts] Now suppose the wire has resistance R. What is the net force acting on the wire as a result of the external magnetic field as a function of t?

c) [10 pts] What is the net torque about the axis of rotation? To specify the direction, recall that $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ points from the axis of rotation to the point at which $\vec{F}$ acts.

a) $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(BA\cos\theta) = \omega BA\sin\theta$, $A = wl$

b) $I = \frac{\omega BA\sin\theta}{R}$, net force: $F_1 = BLI \uparrow$, $F_2 = BLI \downarrow$, net: 0.

c). $\vec{\tau} = \hat{x}\, 2 \times \frac{w}{2} \sin\theta\, BLI = \hat{x}\, w \sin\theta\, Bl \frac{\omega BwL\sin\theta}{R}$

$= \hat{x} \frac{B^2 l^2 w^2 \sin^2\theta}{R} \omega$

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