Name: answer Key

Student ID number:____________________

Discussion section:____________________

1	10
2	10
3	10
4	30
5	20
6	20
total	100

Physics 1C, Spring 2015

Midterm 1, April 23, 2015 (Thursday, Week 4)

Version B

READ THE FOLLOWING CAREFULLY

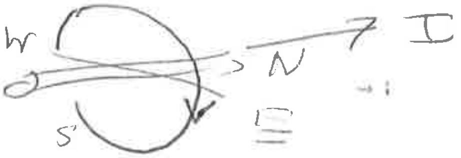
- This exam is closed book. Calculators will not be needed, so no electronic devices should be out as you take the exam. One 3"x5" index card with notes is allowed.
- This exam consists of 6 pages (including this one) with problems numbered 1 through 6; make sure you have been given all pages/problems.
- You have 1 hour to complete the exam.
- Make sure to write your name at the top of each page of this exam. Use the space provided on the exam pages to do your work. You may use the back of the pages also, but please mark clearly which problem you are working on (and also state underneath that problem that you have done work on the back of the page).
- Partial credit will be given. Show as much work/justification as possible (diagrams where appropriate). If you cannot figure out how to complete a particular computation, a written statement of the concepts involved and qualitative comments on what you think the answer should be may be assigned partial credit.
- Have your student ID out and face up next to your exam while you are taking it. Proctors will be checking ID's during the exam.
- Mistakes in grading: If you find a mistake in the grading of your exam, alert the instructor within two days of the exams being returned (this will occur in discussion section following the exam date). DO NOT write on the returned graded exam – you may make a note of the problems you thought were mis-graded on this first page, but any changes/additions to the subsequent pages will negate your chances for a re-grade and may result in the incident being reported to the dean of students. A fraction of the graded exams will be photocopied before they are returned.

Name:________________________________

1) Your professor did his PhD in Berkeley, working in a lab that was 1 km east of the local electric train system (BART), which ran north-south. Because his experiment was sensitive to magnetic fields, he could only take data between 1 AM and 5 AM, when the train system was off. A train pulls 10 kA off the third rail and returns it to ground. Approximate this situation as a long, straight wire carrying 10 kA from south to north.

(-2 for
μ_0 T)

a) **7 pts** What is the magnitude of the resulting ***B*** field in the lab? Give your answer both as a formula and as a value in tesla.



b) **3 pts** In which direction does the ***B*** field point?

$$\oint B\cdot dl = \mu_0 I_{enc} \quad {}^{+2}$$

$$B = \frac{\mu_0 I_{enc}}{2\pi r}{}^{+2} = \frac{4\pi\times10^{-7}\cdot10^{4}}{2\pi\cdot10^{3}}{}^{+2} = \boxed{2\times10^{-6}\ \overset{+1}{T}} \quad a)$$

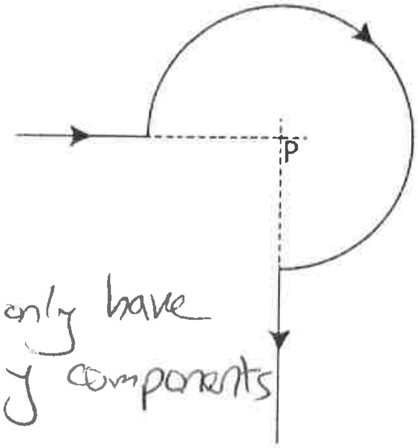
b) It points down, into the ground. +2

2) Consider the wire shown below, carrying current *I* in the direction shown.

a) **4 pts** In which direction does the magnetic field ***B*** point at location P in the diagram?

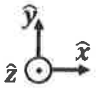
b) **6 pts** Give a convincing (i.e. probably mathematical) justification for your answer to a).

$$dB = \frac{\mu_0}{4\pi} I \frac{d\vec{l}\times\hat{r}}{r^2}$$



a) +4 It points in the $-\hat{z}$ direction (into the paper).

These only have x and y components



b)

$$\left\{\begin{array}{l} d\vec{l} = (dl_x, dl_y, 0) \\ \hat{r} = (\hat{r}_x, \hat{r}_y, 0) \end{array}\right.$$

$$d\vec{l}\times\hat{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dl_x & dl_y & 0 \\ \hat{r}_x & \hat{r}_y & 0 \end{vmatrix}$$

This cross product can only have $\cancel{\hat{z}}$ components.

By the RHR it must point into the page.

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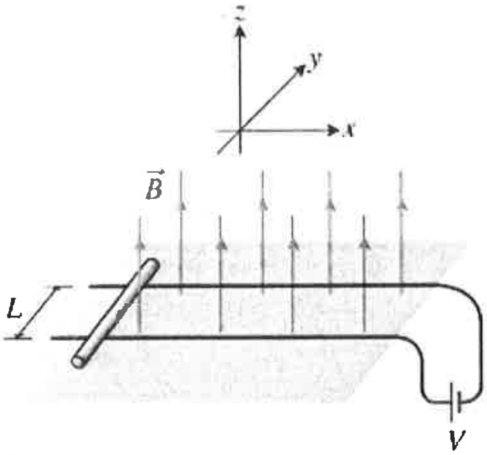
3) A Rail Gun uses electromagnetic forces to accelerate a projectile to very high velocities. The basic mechanism of acceleration is relatively simple and can be illustrated in the following example. A metal rod of mass m and electrical resistance R rests on parallel horizontal rails (that have negligible electric resistance), which are a distance L apart. The rails are also connected to a voltage source V, so a current loop is formed. (See the figure below.)

The rod begins to move if the externally applied vertical magnetic field in which the rod is located reaches the value B. Assume that the rod has a slightly flattened bottom so that it slides instead of rolling. Use g for the magnitude of the acceleration due to gravity.

10 pts Express the coefficient of static friction μ_s in terms of the variables given.



$$\vec{F}_B = q\, v \times B = \frac{q}{L} v\, B \cdot L = \lambda v\, BL = IBL$$

$$\vec{F}_f = \mu m g \qquad\qquad = \frac{V}{R} BL$$

Moves when the forces become equal.

$$F_B = F_f$$

$$\frac{V}{R} BL = \mu m g$$

$$\boxed{\mu = \frac{VBL}{Rmg}}$$

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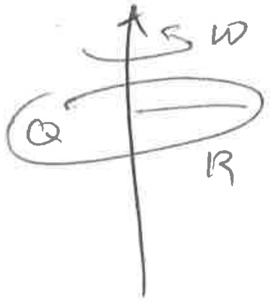
4) A thin disk of dielectric material with radius R has a total charge $+Q$ distributed uniformly over its surface. It rotates at angular frequency ω about an axis perpendicular to the surface of the disk and passing through its center.

a) **15 pts** Find the magnetic field at the center of the disk. Express your answer in terms of the variables R, Q, ω, and appropriate constants.

b) **15 pts** Find the magnetic moment of the disk.



$$dB = \frac{\mu_0}{4\pi} I \frac{dl \times \hat{r}}{r^2}$$

$$\sigma = \frac{Q}{\pi R^2} \qquad K = \sigma v = \sigma \omega r$$

$$\vec{r} = r(\cos\theta, \sin\theta, 0)$$

$$d\vec{l} = r(-\sin\theta, \cos\theta, 0)\,d\theta$$

$$d\vec{l} \times \vec{r} = r^2 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \end{vmatrix} d\theta = -r^2\,d\theta\,\hat{z}$$

$$\vec{B}_{ring} = \int dB = \int \frac{\mu_0}{4\pi} I \cdot \frac{r^2\,d\theta}{r^3} = \frac{\mu_0 I}{2r}$$

Integrate rings to find disk field $B_{disk} = \int dB_{ring}$

$$|B_{disk}| = \int \frac{\mu_0\,dI}{2r} = \int \frac{\mu_0}{2r} K\,dr = \int_0^R \frac{\mu_0 \sigma \omega \cancel{r}\,dr}{2\cancel{r}} = \frac{\mu_0 \sigma \omega}{2} R = \frac{\mu_0 Q}{2\pi R^2} \omega R = \boxed{\frac{\mu_0 Q \omega}{2\pi R}}$$

$$|\vec{\mu}| = \int d\mu = \int d(I \cdot a) = \int \pi r^2 \cdot dI = \int \pi r^2 K\,dr$$

$$= \int_0^R \sigma \omega r \cdot \pi r^2\,dr = \sigma \omega \pi \cdot \int_0^R r^3\,dr$$

$$= \frac{\sigma \omega \pi R^4}{4} = \boxed{\frac{Q \omega R^2}{4}}$$

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5) A proton (proton #1) in a particle accelerator is incident on another proton (proton #2) in a fixed target. Say proton #1 has just enough energy to drive the following reaction:

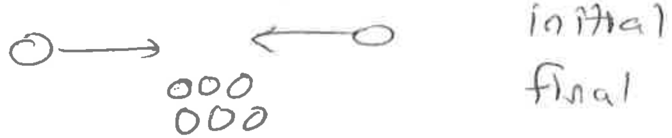
$$p + p \to p + p + p + p + \bar{p} + \bar{p}.$$

In words, 2 protons react to form 4 protons and 2 anti-protons.

a) **10 pts** What is the kinetic energy of proton #1 in the lab frame?

b) **10 pts** How fast are the product protons and anti-protons moving in the lab frame? Both answers should be written in terms of the proton mass m and the speed of light c.

Solve in center-of-mass frame 1st.



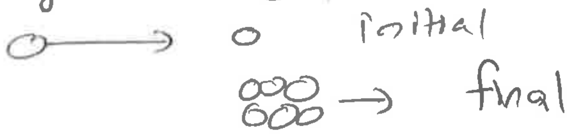
Final invariant mass = $6mc^2$ = initial inv. mass

$$(\underbrace{E_1+E_2}_{\text{equal}})^2 - (\underbrace{p_1+p_2}_{0\text{ in CM frame}})^2c^2 = (6mc^2)^2$$

$$(2E)^2 = (6mc^2)^2 \qquad E = 3mc^2 = \gamma mc^2$$

$$K = E - mc^2 = 2mc^2$$

In fixed target frame



the invariant mass is still $6mc^2$

$$(E_1 + \underset{\text{target not moving}}{mc^2})^2 - (p_1 + \underset{0}{p_2})^2c^2 = (6mc^2)^2$$

$E_1^2 - p_1^2c^2 = (mc^2)^2$ for proton #1.

$$E_1^2 + 2E_1mc^2 + m^2c^4 - p_1^2c^2 = 36m^2c^4$$

$$2E_1mc^2 + m^2c^4 + m^2c^4 = 36m^2c^4$$

$$E_1 = \frac{34}{2}mc^2 = 17mc^2$$

$$K = E_1 - mc^2 = \boxed{16mc^2}$$

$$\boxed{\frac{v}{c} = \frac{2\sqrt{2}}{3}}$$

|Velocity| of fixed target frame = |velocity| of one of the protons in the CM frame.

$1-\beta^2 = \frac{1}{\gamma^2}$ $\quad \beta^2 = 1 - \frac{1}{\gamma^2}$

$\gamma = 3 = \frac{1}{\sqrt{1-\beta^2}}$

$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

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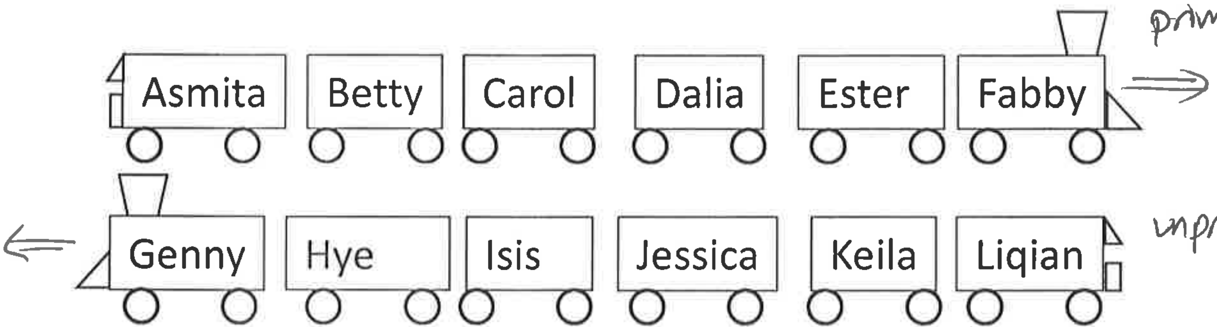
6) Consider two trains passing each other in opposite directions, as shown below:



⟶ version A has 0.8, which changes all of the numbers

Each train observes the other to be moving at velocity $\boxed{v = 0.6c}$ (the upper moves to the right, and the lower to the left) relative to itself. A woman with a watch is in the center of the train car labeled with her name, and all the women on the same train agree that their watches are synchronized. Each train car is L=12 m long in its rest frame. Neglect the distance between cars. Give each answer as a formula and as a number. $\gamma = \frac{1}{\sqrt{1-.36}} = \frac{1}{\sqrt{.64}} = \frac{1}{0.8} = \frac{5}{4}$

a) **3 pts** How far is Carol from Ester, according to Fabby?

same train $2L = 24$ m

b) **3 pts** How far is Carol from Ester, according to Genny?

different trains: Lorentz contraction $\frac{2L}{\gamma} = \frac{24}{5/4} = \frac{96}{5} = 19.2$ m

c) **4 pts** According to Betty, how long does it take Isis's watch to tick off τ = 60 s?

different trains: time dialation $\gamma\tau = 75$ s

When Betty and Hye pass each other, their watches both read $t_0' = t_0$ = 01:00:00 (in the format hh:mm:ss).

d) **5 pts** When Betty's watch reads 01:00:00, what does Hye observe Jessica's watch to read?

$$\left.\begin{array}{l}\text{Hye and Jessica are synchronized. } t_H = t_J \\ \text{Hye and Betty are synchronized. } t_H = t_B'\end{array}\right\} t_J = t_B' = 01:00:00$$

e) **5 pts** When Betty's watch reads 01:00:00, what does Betty observe Jessica's watch to read? (Warning – this problem is subtle.)

The short route: $t_J = \gamma\left(t' + \frac{ux'}{c^2}\right)$

Realize $x' = \frac{x_J}{\gamma}$, $t' = 0$. Then $t_J = \gamma \cdot \frac{u x_J}{\gamma c^2} = \frac{u x_J}{c^2} = \frac{2uL}{c^2}$

$= \frac{2 \cdot 0.6 \cdot 12\ \text{m}}{3\times10^{8}\ \text{m/s}} = 4.8\times10^{-8}\ \text{s}$

Thus the watch time $= t_0 + t_J$

$= 01:00:00 + 4.8\times10^{-8}$ s.

A slightly longer, but equally successful route is use $t_J = \gamma(t' + ux'/c^2)$ and $x_J' = \gamma(x_J - u t_J)$, solve for t_J, and use $t' = 0$, $x_J = 2L$.