Muthupal.

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

40 points

Suppose you find or develop a material within which there can be **no magnetic field** (a superconductor, maybe). You send an electromagnetic (EM) plane wave in vacuum towards the surface of this material, with electric field

$$\vec{E}_I = E_I \cos(kz - \omega t)\hat{x}.$$

All answers in this problem should be in terms of the constants E_1, k, ω, and/or c (speed of light in vacuum).

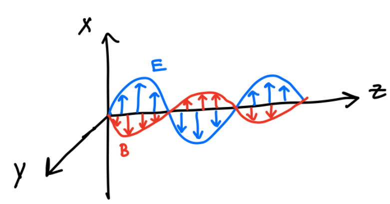
(a): 5 pts

Write down the magnetic field (magnitude & direction) of the incident wave.

From equation 32.4 in the textbook:

$E = cB$, so $B = E/c$

So $B = \frac{E_1 \cos(kz - \omega t)}{c} \hat{y}$ w/ direction of B being parallel to the y axis



(b): 5 pts

Is there any EM wave inside of the material? If so, write down its electric and magnetic fields.

There is no EM wave inside. There is no B field inside the material. The Meissner effect confirms this. EM waves get reflected by conductors

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(c): 8 pts

Write down the electric and magnetic fields (magnitude & direction) of the reflected wave.

After it has been reflected:

$$E_R = E_1 \cos(\omega t + kz)$$ direction: $+x$

$$B_R = \frac{E_1}{c} \cos(\omega t + kz)$$

direction $= -y$

we just flip the signs for reflected waves.

(d): 7 pts

Write down the *total* electric and magnetic fields outside the material. You may simplify the result using trig identities.

$$E_{Tot} = E_1 \left[\cos(\omega t - kz) + \cos(\omega t + kz)\right] \hat{x}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$E_{Tot} = E_1 \left[\cos(\omega t)\cos(kz) + \cancel{\sin(\omega t)\sin(kz)} + (\cos(\omega t)\cos(kz) - \cancel{\sin(\omega t)\sin(kz)})\right]$$

$$E_{Tot} = 2E_1 \cos(\omega t)\cos(kz)\ \hat{x}$$

$$B_{Tot} = \frac{E_1}{c}\left(\cos(\omega t - kz) - \cos(\omega t + kz)\right)$$

$$B_{Tot} = 2\frac{E_1}{c}\sin(\omega t)\sin(kz)\ \hat{y}$$

(e): 7 pts

Write down the Poynting vector outside the material.

$\hat{x} \times \hat{y} = \hat{z}$

x

z ↻ y

$$S = \frac{1}{\mu_0} E \times B \longrightarrow \text{cross } E_{Tot} \times B_{Tot}$$

$$S = \frac{1}{\mu_0}\left[4\frac{E_1^2}{c}\sin(\omega t)\sin(kz)\cos(\omega t)\cos(kz)\right]\hat{z}$$

Problem 1 continued on next page...

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(f): 8 pts

What is the intensity of the wave outside of the material?

We can take the time average of the poynting vector over a full period.

The avg becomes 0 because of the sins & the cos's.

$$\boxed{I = S_{av} = 0}$$

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Problem 2

30 pts

Consider a circuit consisting of a resistor, a capacitor, and an inductor all connected in series, driven by an AC source of fixed magnitude and variable frequency ω. The resistance of the resistor is R, the capacitance of the capacitor is C, and the inductance of the inductor is L. For this problem, you may use without proof any result for impedance derived in lecture, homework, or discussion.

(a): 5 pts

What is the impedance of this circuit? Express your answers in terms of R, L,C, and ω.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L$$
$$X_C = \frac{1}{\omega C}$$

so $\boxed{Z = \sqrt{R^2 (\omega L - \frac{1}{\omega C})^2}}$

(b): 15 pts

(from discussion week 5 too)

At what frequencies ω_+, ω_- will the amplitude of the current through the circuit equal *one-third* the maximum amplitude of current that can flow through the circuit? Express your answers in terms of the given parameters. [HINT 1: What does the impedance need to be for this condition to hold?]. [HINT 2: Make sure your frequencies are positive!]

To maximize I from $I = \frac{V}{Z}$, Z must be Z_{min}. So,

Z_{min} = when $\omega L = \frac{1}{\omega C}$

so, $\omega = \frac{1}{\sqrt{LC}}$ and $Z = R$

$$\underbrace{\frac{1}{3}\frac{V}{R}}_{\frac{1}{3}\cdot I_{max}} = \frac{V}{Z}$$

$$3R = Z$$
$$3R = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$
$$9R^2 = R^2 + (\omega L - 1/\omega C)^2$$

$$8R^2 = (\omega L - 1/\omega C)^2$$

$$\omega L - \frac{1}{\omega C} = R\sqrt{8} \quad \text{OR} \quad \omega L - \frac{1}{\omega C} = -R\sqrt{8}$$

$$\omega^2 LC - 1 = \omega C R\sqrt{8}$$
$$\omega^2 LC - \omega C R\sqrt{8} - 1 = 0$$

use quad. eq. :

$$\omega = \frac{\sqrt{2}CR \pm \sqrt{C(L + 2CR^2)}}{LC}$$

$$\omega^2 LC + \omega C R\sqrt{8} - 1 = 0$$

use quad Formula:

$$\omega = \frac{-\sqrt{2}CR \pm \sqrt{C(L+2CR^2)}}{LC}$$

negative ω answer

Since $\omega > 0$:

$$\boxed{\omega_+ = \frac{\sqrt{2}CR + \sqrt{C(L+2CR^2)}}{LC} \quad \& \quad \omega_- = \frac{\sqrt{2}CR - \sqrt{C(L+2CR^2)}}{LC}}$$

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(c): 10 pts

Describe what changes could be made to R, L, and/or C in order to *double* the maximum possible amount of current flowing through the circuit.

To double: I_{max} happens at $I = \frac{V}{R}$

$$2 I_{max} = 2\frac{V}{R} = \frac{V}{\frac{1}{2}R}$$

we could half our resistance

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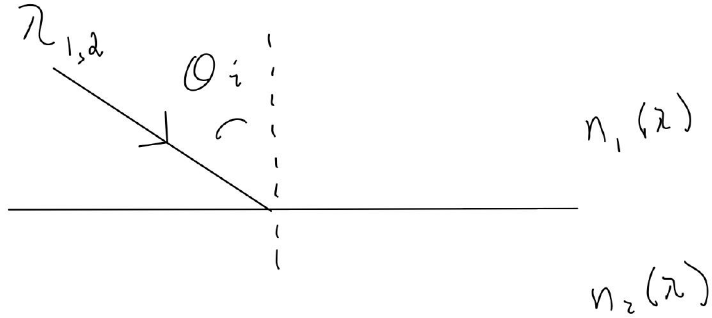
Problem 3

30 pts

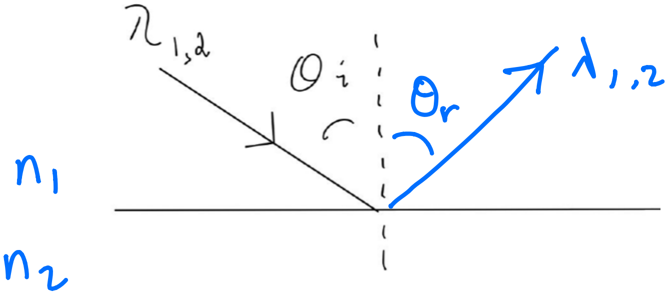
Consider two beams of light, of wavelengths λ_1 and λ_2. The light initially travels through some material with index of refraction $n_1(\lambda) = 1 + e^{-\lambda/\lambda_0}$. Both rays are incident on a second material of refractive index $n_2(\lambda) = 2 + e^{-\lambda/\lambda_0}$ at an angle θ_i. Part of the light is reflected, and part of the light is refracted. [Here λ_0 is some fixed wavelength characteristic of the materials.]



(a): 10 pts

At what angle does each beam of light reflect off of the second material?

The law of Reflection tells us that $\theta_r = \theta_i$ where θ_i is the angle of reflection. So both beams reflect off at an angle of θ_i



Problem 3 continued on next page. . .

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(b): 10 pts θ_b = angle of refraction

At what angle does each beam of light refract into the second material?

$$n_1(\lambda)\sin(\theta_i) = n_2(\lambda)\sin(\theta_b)$$

For λ_1: $(1+e^{-\lambda/\lambda_0})\sin(\theta_i) = (2+e^{-\lambda_1/\lambda_0})\sin(\theta_b)$

$$\sin(\theta_b) = \frac{(1+e^{-\lambda_1/\lambda_0})\sin(\theta_i)}{2+e^{-\lambda_1/\lambda_0}}$$

$$\theta_b = \sin^{-1}\left(\frac{(1+e^{-\lambda_1/\lambda_0})\sin(\theta_i)}{2+e^{-\lambda_1/\lambda_0}}\right) \text{ for } \lambda_1$$

For λ_2: $(1+e^{-\lambda_2/\lambda_0})\sin(\theta_i) = (2+e^{-\lambda_2/\lambda_0})\sin(\theta_b)$

$$\sin(\theta_b) = \frac{(1+e^{-\lambda_2/\lambda_0})\sin(\theta_i)}{2+e^{-\lambda_2/\lambda_0}}$$

$$\theta_b = \sin^{-1}\left(\frac{(1+e^{-\lambda_2/\lambda_0})\sin(\theta_i)}{2+e^{-\lambda_2/\lambda_0}}\right) \text{ for } \lambda_2$$

(c): 10 pts

Do the beams bend towards or away from the normal, or does it depend on the specific values of $\lambda_{1,2}$?

if you graph n_1 vs n_2, you see that n_2 is always larger than n_1 for all wavelengths. The values of λ_1 & λ_2 do not matter because we stay consistent on which λ we use in the law of refraction equation ($n_1(\lambda_1)\sin(\theta_i) = n_2(\lambda_1)\sin\theta_b$ and $n_1(\lambda_2)\sin(\theta_i) = n_2(\lambda_2)\sin(\theta_b)$). Since $n_2 > n_1$, the beams bend toward the normal; a ray going from smaller to bigger index of refraction has a slower wave speed when passing so θ_b with the normal is smaller in the second material than θ_i in the first.