Midterm

Friday, January 29, 2021 5:37 PM

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

26 points

Consider an infinitely long cylindrical wire of radius R. The current density in the wire varies with the distance ρ from the axis as $\vec{J}(\rho) = \alpha \rho e^{-\rho/R}\hat{z}$, where α is a constant.

(a): 20 points

Find the magnetic field everywhere in space. Recall that current through a surface S is defined as $I = \int_S \vec{J} \cdot d\vec{A}$. You may look up (use Wolfram Alpha) any integrals you encounter. You do not have to prove either Ampere's law or the Biot-Savart law.

$r < R$:

$$B = \frac{\mu_0 I_{encl}}{2\pi r}$$

$$I = \int_S \vec{J} \cdot d\vec{A} = \int_0^{2\pi}\int_0^{r} \vec{J}(\rho)\rho \, d\rho \, d\theta$$

$$B = \frac{\mu_0 \alpha}{2\pi r}\int_0^{2\pi}\int_0^{r} \rho^2 e^{-\frac{\rho}{R}} \, d\rho \, d\theta = \frac{\mu_0 \alpha}{\cancel{2}\cancel{\pi} r}\left(\cancel{4}^{2}\pi R^3 - \cancel{2}\pi R e^{-r/R}\left(r^2 + 2rR + 2R^2\right)\right)$$

$$= \frac{\mu_0 \alpha}{r}\left(2R^3 - Re^{-r/R}\left(r^2 + 2rR + 2R^2\right)\right)$$

$r \geq R$: $$B = \frac{\mu_0 \alpha}{2\pi r}\int_0^{2\pi}\int_0^{R} \rho^2 e^{-\rho/R} \, d\rho \, d\theta$$

$$= \frac{\mu_0 \alpha}{\cancel{2}\cancel{\pi} r} \cdot \frac{\cancel{2}(2e-5)\cancel{\pi}R^3}{e}$$

$$= \frac{\mu_0 \alpha (2e-5)R^3}{er}$$

Circulates around wire

Problem 1 continued on next page...

You must show your work to receive credit. An answer written down with no work will receive no credit.

(b): 6 points

Suppose now that the current density oscillates back and forth in time:

$$\vec{J}(\rho) = \alpha\rho e^{-\rho/R}\cos(\omega t)\hat{z},$$

where ω is the angular frequency of the oscillation. If the magnetic field you found in part (a) is denoted $\vec{B}(\vec{r})$, would the total magnetic field produced by this oscillating current density be equal to $\vec{B}(\vec{r}, t) = \vec{B}(\vec{r})\cos(\omega t)$? Why or why not?

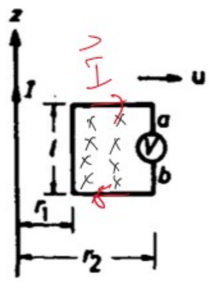
No, there is a small self-inductance in a straight wire that will resist the change in current by creating an induced emf that opposes the current. This induced emf causes the induced magnetic field to not follow $\frac{d\vec{J}}{dt}$ exactly, instead it partially resists and lags behind the change.

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 2

25 points

Consider an infinite wire carrying current I in the $+\hat{z}$ direction. A square of Ohmic wire of side length l and resistance R is placed at $t = 0$ to the right of the wire, its closest side a distance r_1 away and its farthest side a distance $r_2 = r_1 + l$ away. The loop is then pulled with constant speed u away from the wire. **Find the direction and magnitude of the current in the square loop as a function of time.** You may take as given the magnetic field of an infinite current-carrying wire, as well as all of Maxwell's equations.



$$\left. \ln|r_1+\rho| \right|_{\rho=0}^{l} = \ln|r_1+l| - \ln|r_1|$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \quad \mathcal{E} = IR$$

magnetic field due to wire:

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$-\frac{d\Phi}{dt} = \frac{d}{dt}\left(l \int_0^l \frac{-\mu_0 I}{2\pi(r_1+\rho)}\, d\rho \right)$$

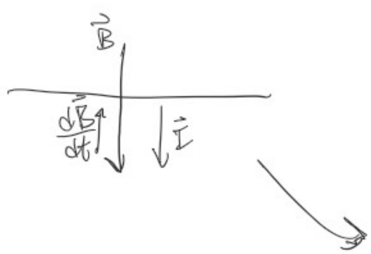
$$= \frac{d}{dt}\left(\frac{-l\mu_0 I}{2\pi} \ln\left| \frac{r_1(t)+l}{r_1(t)} \right| \right)$$

$$= \frac{-l\mu_0 I}{2\pi} \cdot -\frac{l \frac{d}{dt} r_1(t)}{(l + r_1(t))\, r_1(t)}$$

$r_1(t)$: changing position of left side of loop

$$I = \frac{l^2 \mu_0 I u}{2\pi R\,(l + r_1 + tu)(r_1 + tu)}$$

r_1: initial position of left side of loop



CW when facing loop from reader's perspective

You must show your work to receive credit. An answer written down with no work will receive no credit.

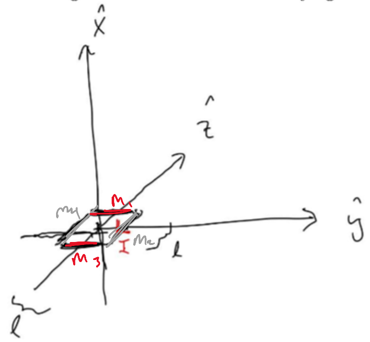
Problem 3

25 pts

Suppose that the magnetic field in some region of space is given by $\vec{B} = \alpha z \hat{x}$, where α is some constant. Find the net force on a square loop of side length l carrying current I lying in the yz plane and centered at the origin. The direction of the current is clockwise as seen from the region $x > 0$. You may use without proof the equation for the force of a magnetic field on a current-carrying wire.

$\hat{x}$

Find the net force on a square loop of side length l carrying current I lying in the yz plane and centered at the origin. The direction of the current is clockwise as seen from the region $x > 0$. You may use without proof the equation for the force of a magnetic field on a current-carrying wire.



for m_1:

$\vec{B} = \alpha\left(\frac{l}{2}\right)\hat{x}$

$\vec{F} = Il(-\hat{y})\times\alpha\left(\frac{l}{2}\right)\hat{x}$

$= \frac{Il^2\alpha}{2}\hat{z}$

$\hat{z} = -\hat{y}\times\hat{x}$

for m_3:

$\vec{B} = \alpha\left(\frac{-l}{2}\right)\hat{x}$

$\vec{F} = Il(\hat{y})\times a\left(\frac{-l}{2}\right)\hat{x}$

$= \frac{Il^2 a}{2}\cdot\hat{z}$

$\hat{z} = \hat{y}\times -\hat{x}$

sum $\boxed{Il^2\alpha\hat{z}}$ F_{net}

m_4 and m_2 cancel out by symmetry, as $\vec{B}$ is only dependent on z

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 4

24 pts (6 pts each)

(a)

Suppose the magnetic field in some region of space is given by $\vec{B}(x,y,z) = B_0[\sin(k_x x)\hat{y} + \sin(k_y y)\hat{z} + \sin(k_z z)\hat{x}]$, where $k_{x,y,z}$ and B_0 are some constants. In this region of space, I place a can of soda (represented by a cylinder of height h and radius r) with its base centered at the origin. What is the magnetic flux through the surface of the soda can?

Gauss's Law: Magnetic flux through any closed surface is $\boxed{\text{zero}}$, because a closed surface cannot surround a magnetic monopole, they don't exist

Problem 4 continued on next page. . .

Rombes Physics 1C: Midterm 1 *PROBLEM 4*

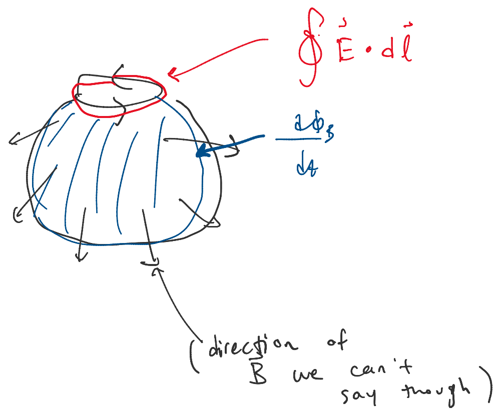
You must show your work to receive credit. An answer written down with no work will receive no credit.

(b)

Suppose you've got a fishbowl. To the rim of this fishbowl, you have fastened a loop of conductive wire of resistance R, not attached to any circuitry. You attach an ammeter to this loop in order to measure the current running through it. You begin to wiggle a permanent magnet around in the vicinity of the fishbowl; at some instant, your ammeter records a current of magnitude I. What can you say *quantitatively* about the electric and/or magnetic fields that are experienced by the glass of the fishbowl, just from this measurement?

The glass of the fishbowl feels a magnetic (magnetic flux) electric and/or magnetic fields that are experienced by the glass of the fishbowl, just from this measurement? (magnetic flux)

The glass of the fishbowl feels a magnetic field passing through it that changes at a rate of IR over time.



You must show your work to receive credit. An answer written down with no work will receive no credit.

(c)

Suppose that some happy day in the future, scientists manage to find and isolate a magnetic monopole. If this magnetic monopole were placed at rest one meter away from an electric monopole, also at rest, would the two monopoles exert any force on each other? Why or why not?

No, just like the electric monopole, which doesn't create a magnetic field unless it's moving, the magnetic monopole would not create an electric field unless it's moving. The two stationary particles would not interact with each other because they wouldn't create the other one's respective field

You must show your work to receive credit. An answer written down with no work will receive no credit.

(d)

Before you turn on a lightswitch, the current running through the switch-bulb circuit is zero. After you turn on the switch, the current running through the circuit is $I > 0$. Suppose your electrician comes and rewires your lightswitch by including an inductor of inductance L between the switch and the bulb. When you turn the lightswitch on, how could you tell that something was wrong? Explain.

The lightbulb would turn on slowly.
There would initially be an induced emf

The lightbulb would turn on slowly.
There would initially be an induced emf in inductor: $\mathcal{E} = -L\frac{di}{dt}$, which opposes the increasing current, and so there would be a delay before the lightbulb reaches the maximum current passing through it, and the maximum power it uses