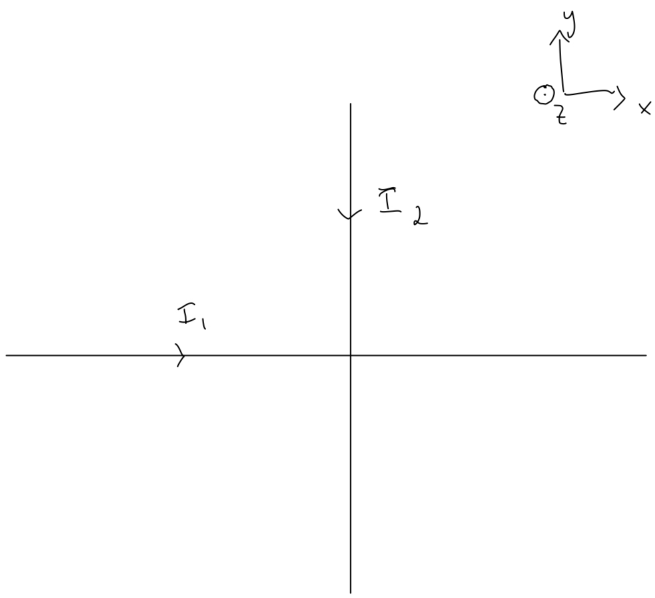
You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

– points

Consider two infinitely long straight wires lying in the xy-plane. Wire 1 carries current I_1 in the $+\hat{x}$ direction and wire 2 carries current I_2 in the $-\hat{y}$ direction.



(a): – points

Calculate the magnetic field $\vec{B}(x, y)$ (magnitude and direction) everywhere in the xy-plane. [In terms of μ_0, I_1, I_2, and/or coordinates.] **Do not** use any results derived in class, show your work starting with either the Biot-Savart law or Ampere's law.

On this plane, the field from I_1 and I_2 are

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi y}\hat{z}$$
$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi x}\hat{z}.$$

The total field is thus

$$\vec{B} = \frac{\mu_0}{2\pi}\left(\frac{I_2}{x} + \frac{I_1}{y}\right)\hat{z}$$

For full points, students should show how they arrived at $\vec{B}_1$ and $\vec{B}_2$.

(a): – points

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(b): – points

Calculate the magnetic flux through a cube of side length L centered at the origin. [In terms of μ_0, I_1, I_2, and/or L]

The magnetic flux through *any* closed surface is zero.

(b): – points

Suppose now a magnetic moment $\vec{\mu} = \mu\hat{z}$ is placed at rest at some location (x, y).

(c): – points

Calculate the torque felt by the magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y]

The magnetic moment is parallel to the magnetic field, so the moment feels no torque.

(c): – points

(d): – points

Calculate the potential energy of the magnetic moment in this magnetic field. [In terms of μ_0, I_1, I_2, μ, x, and/or y] .

$$U_B = -\vec{\mu} \cdot \vec{B} = -\frac{\mu_0 \mu}{2\pi}\left(\frac{I_2}{x} + \frac{I_1}{y}\right).$$

(d): – points

(e): – points

Calculate the force exerted by the magnetic field on this magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y].

$$\vec{F} = -\nabla U_B = -\frac{\mu_0 \mu}{2\pi}\left(\frac{I_2}{x^2}\hat{x} + \frac{I_1}{y^2}\hat{y}\right).$$

(e): – points

(f): – points

Suppose a particle of charge q is placed at a position (x, y) and given velocity $\vec{v} = v\hat{x}$. What electric field could be established at (x, y) so that the net force on the charged particle at that location is zero? [In terms of μ_0, I_1, I_2, q, v, x, and/or y].

The magnetic force at that location is

$$\vec{F}_B = q\vec{v} \times \vec{B} = -\frac{\mu_0 q v}{2\pi}\left(\frac{I_2}{x} + \frac{I_1}{y}\right)\hat{y}.$$

The electric field that could oppose this force is thus

$$\vec{E} = \frac{\mu_0 v}{2\pi}\left(\frac{I_2}{x} + \frac{I_1}{y}\right)\hat{y}$$

(f): – points