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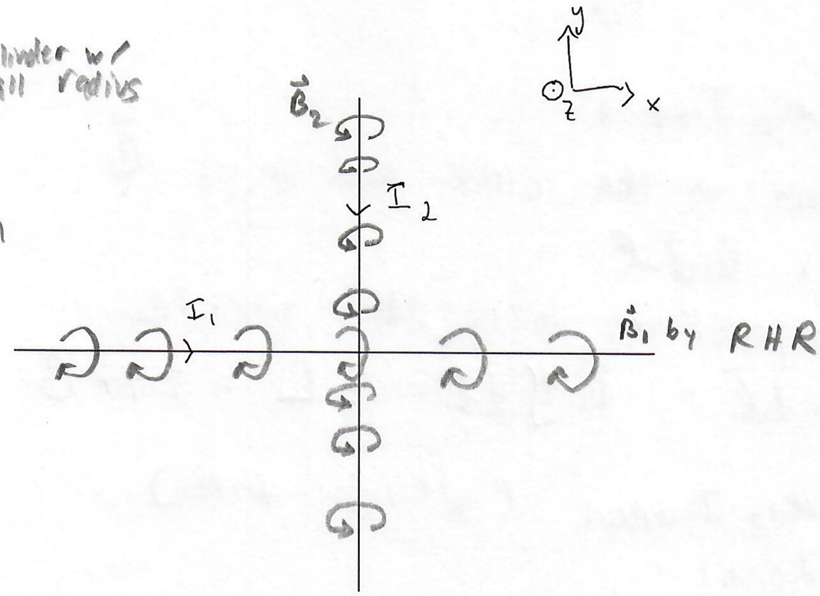
You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

100 points

Consider two infinitely long straight wires lying in the xy-plane. Wire 1 carries current I_1 in the $+\hat{x}$ direction and wire 2 carries current I_2 in the $-\hat{y}$ direction.

straight line has: rotational, translational symmetry) like a cylinder w/ only small radius
is ∞ in a direction



goal: calculate $\vec{B}(x,y,z)$ for each wire and add w/ principle of superposition

(a): 40 points

Calculate the magnetic field $\vec{B}(x,y)$ (magnitude and direction) everywhere in the xy-plane. [In terms of μ_0, I_1, I_2, and/or coordinates.] **Do not** use any results derived in class, show your work starting with either the Biot-Savart law or Ampere's law.

for an indiv wire, field is constant on a parallel line any distance away from the wire. therefore, field strength can only change depending on the distance of this line of equal magnetic strength

(moving parallel to wire → constant mag strength)

Problem 1 continued on next page...

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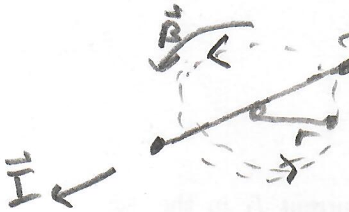

Amperean circle

$r_1 = \sqrt{y^2+z^2}$ $\quad r_2 = \sqrt{x^2+z^2}$

x direc is irrel b/c of translat. sym.

$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$

$d\vec{\ell}$ is tangent to the circle and so is $\vec{B}$

$\therefore\ \vec{B}\cdot d\vec{\ell} = B\,d\ell$

AND r is constant along the circle

so: $\oint \vec{B}\cdot d\vec{\ell} = B\oint d\ell = BL = 2\pi r B$

$2\pi r B = \mu_0 I_{encl}$ (straight wire)

$B = \dfrac{\mu_0 I_{encl}}{2\pi r}$

$B_1 = \dfrac{\mu_0 I_1}{2\pi\sqrt{y^2+z^2}}$ $\quad B_2 = \dfrac{\mu_0 I_2}{2\pi\sqrt{x^2+z^2}}$ if $x\to 0$, $z\to 0$, then coord is on the wire itself so $B\to\infty$ ie undef. which checks out!

problem limits us to x-y plane so $z=0$

$\vec{B}_1 = \dfrac{\mu_0 I_1}{2\pi y}\hat{z}$ $\quad \vec{B}_2 = \dfrac{\mu_0 I_2}{2\pi x}\hat{z}$

from the rhr diagram we see that $\vec{B}_1, \vec{B}_2$ pt in $+\hat{z}$ direc when y and x are respectively positive

now, apply superposition

$$\vec{B}_1+\vec{B}_2 = \frac{\mu_0}{2\pi}\left(\frac{I_1}{y}+\frac{I_2}{x}\right)\hat{z} = \vec{B}(x,y)$$

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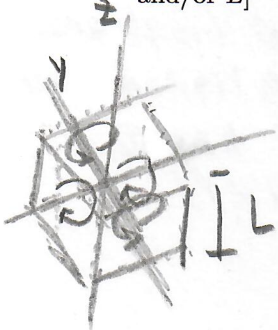
* **(b): 12 points**

Calculate the magnetic flux through a cube of side length L centered at the origin. [In terms of μ_0, I_1, I_2, and/or L]



$$\Phi_B = \int_S \vec{B} \cdot d\vec{A}$$

$$= \int_1 \vec{B} \cdot d\vec{A} + \int_2 \vec{B} \cdot d\vec{A} + \cdots + \int_6 \vec{B} \cdot d\vec{A}$$

the field is a loop - it is either contained entirely (1) in the cube (so no flux) or it passes through just to exit again. (see fig) ~~or~~ (2) or (3) it avoids cube entirely.

net flux of a magnetic field passing through a closed surface must be zero

$$\boxed{\Phi_{B_{net}} = 0}$$

side view:

(2) every entry point has a corresp. exit point

Suppose now a magnetic moment $\vec{\mu} = \mu\hat{z}$ is placed at rest at some location (x, y).

(c): – points

Calculate the torque felt by the magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y]

$$\tau = \vec{\mu} \times \vec{B}$$

$$= \mu\hat{z} \times \vec{B}(x, y) = \mu\hat{z} \times B(x,y)\hat{z}$$

$$= \mu\hat{z} \times \left(\frac{\mu_0}{2\pi} \left(\frac{I_1}{y} + \frac{I_2}{x} \right) \hat{z} \right)$$

$\vec{\mu}$ is parallel or antiparallel to $\vec{B}$ so the cross product value is 0

$$\boxed{\tau = 0}$$

this makes sense b/c the moment is already aligned w/ the field ~~and~~ does not need to rotate to align

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(d): 12 points

Calculate the potential energy of the magnetic moment in this magnetic field. [In terms of μ_0, I_1, I_2, μ, x, and/or y].

$U_B = -\vec{\mu} \cdot \vec{B}$

moment and field are already aligned so U_B will be a minimum

$\vec{\mu}$ is $\parallel$ or anti-$\parallel$ to $\vec{B}$ so $|\vec{\mu} \cdot \vec{B}| = \mu B$

$$U_B = -\mu\left(\frac{\mu_0}{2\pi}\left(\frac{I_1}{y} + \frac{I_2}{x}\right)\right)$$

$$= \boxed{-\frac{\mu\mu_0}{2\pi}\left(\frac{I_1}{y} + \frac{I_2}{x}\right)}$$

(e): 12 points

Calculate the force exerted by the magnetic field on this magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y].

$$\vec{F} = -\nabla U_B$$

$$= -\left\langle \frac{\partial U_B}{\partial x}, \frac{\partial U_B}{\partial y} \right\rangle$$

$$\frac{\partial U_B}{\partial x} = -\frac{\mu\mu_0}{2\pi}\left(-\frac{I_2}{x^2}\right) = \frac{\mu\mu_0 I_2}{2\pi x^2} \qquad \frac{\partial U_B}{\partial y} = -\frac{\mu\mu_0}{2\pi}\left(-\frac{I_1}{y^2}\right) = \frac{\mu\mu_0 I_1}{2\pi y^2}$$

$$\vec{F} = \boxed{-\frac{\mu\mu_0}{2\pi}\left\langle \frac{I_2}{x^2}, \frac{I_1}{y^2} \right\rangle}$$

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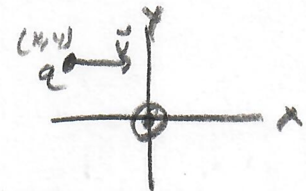
(f): 12 points

Suppose a particle of charge q is placed at a position (x, y) and given velocity $\vec{v} = v\hat{x}$. What electric field could be established at (x, y) so that the net force on the charged particle at that location is zero? [In terms of $\mu_0, I_1, I_2, q, v, x,$ and/or y].

1) find $\vec{F}_B$
2) calculate $\vec{E}$ s.t. $\vec{F}_e$ is equal and opposite



$$\vec{F}_B = q\vec{v}\times\vec{B}$$

$$= q(v\hat{x})\times\left(\frac{\mu_0}{2\pi}\left(\frac{I_1}{y}+\frac{I_2}{x}\right)\hat{z}\right)$$

$$\hat{x}\times\hat{z} = -\hat{y}$$

$$= -qv\left(\frac{\mu_0}{2\pi}\left(\frac{I_1}{y}+\frac{I_2}{x}\right)\right)\hat{y}$$

$$\hat{x}\perp\hat{z} \text{ so } |\vec{v}\times\vec{B}| = vB$$

$$= -\frac{\mu_0 q v}{2\pi}\left(\frac{I_1}{y}+\frac{I_2}{x}\right)\hat{y}$$

$$\vec{F}_E = q\vec{E}$$

must be in $+\hat{y}$ direc

$$\cancel{q}E = \cancel{q}\frac{\mu_0 v}{2\pi}\left(\frac{I_1}{y}+\frac{I_2}{x}\right)$$

$$E = \frac{\mu_0 v}{2\pi}\left(\frac{I_1}{y}+\frac{I_2}{x}\right)$$

$$\boxed{\vec{E} = \frac{\mu_0 v}{2\pi}\left(\frac{I_1}{y}+\frac{I_2}{x}\right)\hat{y}}$$