211C-PHYSICS1C-2 Midterm 1

RICHARD JIANG

TOTAL POINTS

100 / 100

QUESTION 1

Problem 1 100 pts

1.1 (a) **40 / 40**

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify in specific comments and adjust points below)

1.2 (b) **12 / 12**

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify in specific comments and adjust points below)

1.3 (c) **12 / 12**

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify in specific comments and adjust points below)

1.4 (d) **12 / 12**

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify in specific comments and adjust points below)

1.5 (e) **12 / 12**

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify in specific comments and adjust points below)

1.6 (f) **12 / 12**

✓ **- 0 pts** Correct

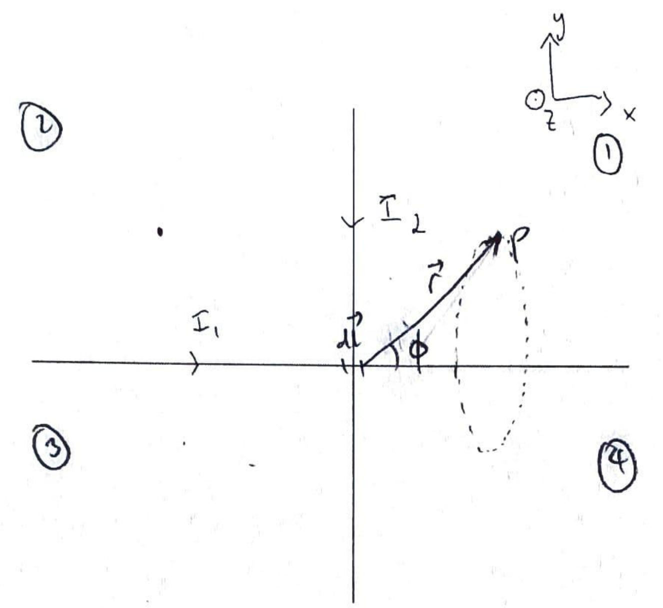
- 0 pts Mistake (will clarify in specific comments and adjust points below)

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

100 points

Consider two infinitely long straight wires lying in the xy-plane. Wire 1 carries current I_1 in the $+\hat{x}$ direction and wire 2 carries current I_2 in the $-\hat{y}$ direction.



(a): 40 points

Calculate the magnetic field $\vec{B}(x, y)$ (magnitude and direction) everywhere in the xy-plane. [In terms of μ_0, I_1, I_2, and/or coordinates.] **Do not** use any results derived in class, show your work starting with either the Biot-Savart law or Ampere's law.

1 Point P: (x,y) Due to I_1: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I_1 d\vec{l}\times\hat{r}}{r^2}$ $\quad d\vec{l} = dx\,\hat{x}$, $r = \sqrt{x^2+y^2}$

$$\vec{B} = \frac{\mu_0}{4\pi} I_1 \int \frac{dx\,\hat{x}\times\hat{r}}{x^2+y^2}$$

$$|\hat{x}\times\hat{r}| = 1\sin\phi$$

$$\hat{x}\times\hat{r} = \sin\phi\cdot\hat{z} = \frac{y}{r}\hat{z} = \frac{y}{\sqrt{x^2+y^2}}\hat{z}$$

$$\Rightarrow \vec{B}\text{ due to } I_1 = \frac{\mu_0 I_1 y}{4\pi}\int_{-a}^{a}\frac{dx}{(x^2+y^2)^{\frac{3}{2}}} = \frac{\mu_0 I_1}{4\pi}\cdot\frac{2a}{y\sqrt{x^2+a^2}}\hat{z}$$

as $a\to\infty$: $\vec{B} = \frac{\mu_0 I_1}{2\pi y}\hat{z}$

Problem 1 continued on next page...

At $\vec{P}$, B due to I_1: $\frac{\mu_0 I_1}{2\pi y}\hat{z}$

B due to I_2: $\frac{\mu_0 I_2}{2\pi x}\hat{z}$ (dir from RHR, mag. from above)

You must show your work to receive credit. An answer written down with no work will receive no credit.

① $\vec{B} = \left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)\hat{z}$

Using results from ①:

② B due to I_1: $\frac{\mu_0 I_1}{2\pi y}\hat{z}$ (dir from RHR)

B due to I_2: $-\frac{\mu_0 I_2}{2\pi x}\hat{z}$ (dir from RHR)

$\vec{B}_{tot} = \left(\frac{\mu_0 I_1}{2\pi y} - \frac{\mu_0 I_2}{2\pi x}\right)\hat{z}$

③ B due to I_1: $-\frac{\mu_0 I_1}{2\pi y}\hat{z}$ (dir from RHR)

" " " I_2: $-\frac{\mu_0 I_2}{2\pi x}\hat{z}$ (" " ")

$\vec{B}_{tot} = -\left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)\hat{z}$

④ B due to I_1: $-\frac{\mu_0 I_1}{2\pi y}\hat{z}$

B due to I_2: $\frac{\mu_0 I_2}{2\pi x}\hat{z}$

$\vec{B}_{tot} = \left(\frac{\mu_0 I_2}{2\pi x} - \frac{\mu_0 I_1}{2\pi y}\right)\hat{z}$

On the axes / wires:

On the +y-axis: $B_{tot} = \frac{\mu_0 I_1}{2\pi y}\hat{z}$ on the −y-axis: $B_{tot} = -\frac{\mu_0 I_1}{2\pi y}\hat{z}$

On the + x-axis: $B_{tot} = \frac{\mu_0 I_2}{2\pi x}\hat{z}$ on the −x-axis: $B_{tot} = -\frac{\mu_0 I_2}{2\pi x}\hat{z}$

1.1 (a) **40 / 40**

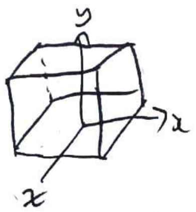
✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify in specific comments and adjust points below)

You must show your work to receive credit. An answer written down with no work will receive no credit.

(b): 12 points

Calculate the magnetic flux through a cube of side length L centered at the origin. [In terms of μ_0, I_1, I_2, and/or L]



Magnetic flux through a cube is zero.

Magnetic Gauss's Law: $\oint \vec{B} \cdot d\vec{A} = 0$

Suppose now a magnetic moment $\vec{\mu} = \mu\hat{z}$ is placed at rest at some location (x, y).

(c): – points

Calculate the torque felt by the magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y]

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$= \mu\hat{z} \times \left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x} \right)\hat{z}$$

$$= 0.$$

$\hat{z} \times \hat{z} = 0$ as they are parallel.

Problem 1 continued on next page. . .

1.2 (b) **12 / 12**

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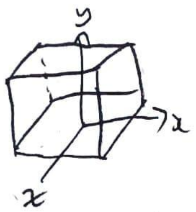
(b): 12 points

Calculate the magnetic flux through a cube of side length L centered at the origin. [In terms of μ_0, I_1, I_2, and/or L]



Magnetic flux through a cube is zero.

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Suppose now a magnetic moment $\vec{\mu} = \mu\hat{z}$ is placed at rest at some location (x, y).

(c): – points

Calculate the torque felt by the magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y]

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$= \mu\hat{z} \times \left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x} \right)\hat{z}$$

$$= 0.$$

$\hat{z} \times \hat{z} = 0$ as they are parallel.

Problem 1 continued on next page...

1.3 (c) 12 / 12

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You must show your work to receive credit. An answer written down with no work will receive no credit.

(d): 12 points

Calculate the potential energy of the magnetic moment in this magnetic field. [In terms of $\mu_0, I_1, I_2, \mu, x,$ and/or y].

$$U_B = -\vec{\mu}\cdot\vec{B}$$

$$= -\left(\mu\hat{z}\cdot\left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)\hat{z}\right) \qquad (\cos 0 = 1)$$

$$= -\frac{\mu\mu_0}{2\pi}\left(\frac{I_1}{y} + \frac{I_2}{x}\right)$$

(e): 12 points

Calculate the force exerted by the magnetic field on this magnetic moment. [In terms of $\mu_0, I_1, I_2, \mu, x,$ and/or y].

$$\vec{F} = -\nabla U_B = \nabla(\vec{\mu}\cdot\vec{B})$$

$$\frac{\partial}{\partial x}(\hat{\mu}\cdot\vec{B}) = -\frac{\mu\mu_0 I_2}{2\pi}\frac{1}{x^2}$$

$$\frac{\partial}{\partial y}(\vec{\mu}\cdot\vec{B}) = -\frac{\mu_0\mu I_1}{2\pi}\cdot\frac{1}{y^2}$$

$$\vec{F} = \left\langle -\frac{\mu\mu_0}{2\pi}\frac{I_2}{x^2}, -\frac{\mu\mu_0}{2\pi}\frac{I_1}{y^2}\right\rangle$$

Problem 1 continued on next page...

1.4 (d) **12 / 12**

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- **- 0 pts** Mistake (will clarify in specific comments and adjust points below)

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(d): 12 points

Calculate the potential energy of the magnetic moment in this magnetic field. [In terms of μ_0, I_1, I_2, μ, x, and/or y].

$$U_B = -\vec{\mu}\cdot\vec{B}$$

$$= -\left(\mu\hat{z}\cdot\left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)\hat{z}\right) \qquad (\cos 0 = 1)$$

$$= -\frac{\mu\mu_0}{2\pi}\left(\frac{I_1}{y} + \frac{I_2}{x}\right)$$

(e): 12 points

Calculate the force exerted by the magnetic field on this magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y].

$$\vec{F} = -\nabla U_B = \nabla(\vec{\mu}\cdot\vec{B})$$

$$\frac{\partial}{\partial x}(\vec{\mu}\cdot\vec{B}) = -\frac{\mu\mu_0 I_2}{2\pi}\frac{1}{x^2}$$

$$\frac{\partial}{\partial y}(\vec{\mu}\cdot\vec{B}) = -\frac{\mu_0\mu I_1}{2\pi}\frac{1}{y^2}$$

$$\vec{F} = \left\langle -\frac{\mu\mu_0}{2\pi}\frac{I_2}{x^2}, -\frac{\mu\mu_0}{2\pi}\frac{I_1}{y^2}\right\rangle$$

Problem 1 continued on next page...

1.5 (e) **12 / 12**

- ✓ **- 0 pts** Correct
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(f): 12 points

Suppose a particle of charge q is placed at a position (x, y) and given velocity $\vec{v} = v\hat{x}$. What electric field could be established at (x, y) so that the net force on the charged particle at that location is zero? [In terms of μ_0, I_1, I_2, q, v, x, and/or y].

$$F = q\vec{v} \times \vec{B}$$

$$= q\left(v\hat{x} \times \left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)\hat{z}\right)$$

$$= qv\left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)(-\hat{y})$$

$(\hat{x} \times \hat{z} = -\hat{y})$

$F_E = -F_B$ gives $F_{net} = 0$.

$$F_E = qE$$

$$E = v\left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)\hat{y}$$

1.6 (f) **12 / 12**

- ✓ **- 0 pts** Correct
- **- 0 pts** Mistake (will clarify in specific comments and adjust points below)