midterm1

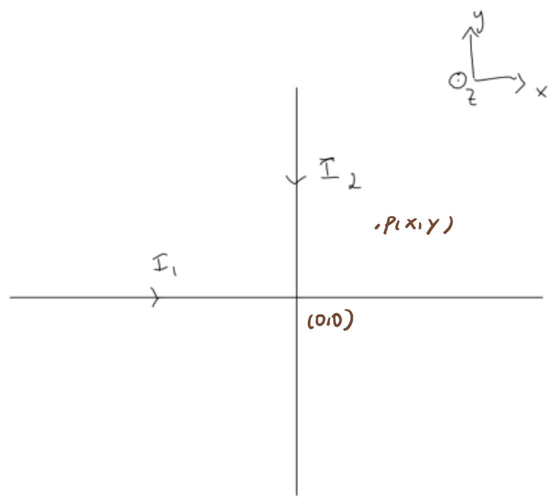
Saturday, August 14, 2021 1:49 AM

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

100 points

Consider two infinitely long straight wires lying in the xy-plane. Wire 1 carries current I_1 in the $+\hat{x}$ direction and wire 2 carries current I_2 in the $-\hat{y}$ direction.



(a): 40 points

Calculate the magnetic field $\vec{B}(x, y)$ (magnitude and direction) everywhere in the xy-plane. [In terms of μ_0, I_1, I_2, and/or coordinates.] **Do not** use any results derived in class, show your work starting with either the Biot-Savart law or Ampere's law.

By Biot-Savart law,

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times \vec{r}}{r^2}$$

For wire 2, we pick a point P that is x from wire 2

From the figure, $r = \sqrt{x^2+y^2}$

Then, $B_2 = \frac{\mu_0}{4\pi}\int \frac{I_2 d\vec{l} \times \vec{r}}{r^2}$

$$= \frac{\mu_0 I_2}{4\pi}\int_{-\infty}^{\infty} \frac{x\,dy}{(x^2+y^2)^{3/2}} \approx \frac{\mu_0 I_2}{2\pi x}$$

By symmetry, $B_2 = \frac{\mu_0 I_2}{2\pi r_2}$, r is distance to I_2

Similarly, $B_1 = \frac{\mu_0 I_1}{2\pi r_1}$, r is distance to I_1

For $P(x,y)$, if we choose the intesection point viewed from top as (0,0), then $r_1 = y$, $r_2 = x$

By right hand rule.

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi y} \cdot \hat{z}$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi x} \cdot \hat{z}$$

$$\therefore \vec{B}(x,y) = \hat{z}\left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right), \quad x \neq 0,\ y \neq 0$$

$$\vec{B}(x,y) = \frac{\hat{z}\mu_0 I_1}{2\pi y}, \quad x = 0,\ y \neq 0$$

$$\vec{B}(x,y) = \frac{\hat{z}\mu_0 I_2}{2\pi x}, \quad x \neq 0,\ y = 0$$

$$\vec{B}(x,y) = 0, \quad x = 0,\ y = 0$$

Problem 1 continued on next page...

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Problem 1 continued on next page...

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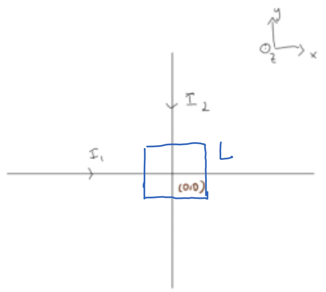
(b): 12 points

Calculate the magnetic flux through a cube of side length L centered at the origin. [In terms of $\mu_0, I_1, I_2,$



If we choose the point (0,0) shown
in figure as the origin

By definition. $\phi_B = \int B\cos\phi \, dA = \int \vec{B} \cdot d\vec{A}$

By Gauss Law of Magnetism, $\oint \vec{B} \cdot d\vec{A} = 0$ for closed surface

$\therefore \quad \Phi_B$ for cube $= 0$

Suppose now a magnetic moment $\vec{\mu} = \mu\hat{z}$ is placed at rest at some location (x, y).

(c): – points

Calculate the torque felt by the magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y]

$$\vec{B}(x,y) = \hat{z}\left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)$$

$$\vec{\mu} = \mu\hat{z}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$= \mu\hat{z} \times \hat{z}\left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x}\right)$$

$$= 0$$

Problem 1 continued on next page...

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(d): 12 points

Calculate the potential energy of the magnetic moment in this magnetic field. [In terms of μ_0, I_1, I_2, μ, x, and/or y].

If $x \neq 0, y \neq 0$ $\quad U_B = -\vec{\mu} \cdot \vec{B}$

$$= -(\mu \hat{z}) \cdot \hat{z} \left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x} \right)$$

$$= -\mu \left(\frac{\mu_0 I_1}{2\pi y} + \frac{\mu_0 I_2}{2\pi x} \right)$$

$$= -\frac{\mu \mu_0}{2\pi} \left(\frac{I_1}{y} + \frac{I_2}{x} \right) \quad (x \neq 0, y \neq 0)$$

If $x = 0, y \neq 0$,

$U_B = -\vec{\mu} \cdot \vec{B}$

$= -(\mu \hat{z}) \cdot \hat{z} \left(\frac{\mu_0 I_1}{2\pi y} \right)$

$= -\frac{\mu \mu_0 I_1}{2\pi y}$

If $x \neq 0, y = 0$

$U_B = -\frac{\mu \mu_0 I_2}{2\pi x}$

If $x = 0, y = 0$, $U_B = 0$

(e): 12 points

Calculate the force exerted by the magnetic field on this magnetic moment. [In terms of μ_0, I_1, I_2, μ, x, and/or y].

If $x \neq 0, y \neq 0$

$U_B = -\frac{\mu \mu_0}{2\pi} \left(\frac{I_1}{y} + \frac{I_2}{x} \right)$

$\vec{F} = -\nabla U_B = \frac{\partial}{\partial x} \left(\frac{\mu \mu_0}{2\pi} \left(\frac{I_1}{y} + \frac{I_2}{x} \right) \right) \cdot \hat{x}$

$+ \frac{\partial}{\partial y} \left(\frac{\mu \mu_0}{2\pi} \left(\frac{I_1}{y} + \frac{I_2}{x} \right) \right) \cdot \hat{y}$

$= -\frac{\mu \mu_0 I_2}{2\pi} \cdot \frac{1}{x^2} \cdot \hat{x} - \frac{\mu \mu_0}{2\pi} I_1 \frac{1}{y^2} \cdot \hat{y}$

If $x \neq 0, y = 0$

$U_B = -\frac{\mu \mu_0}{2\pi} \frac{I_2}{x}$

$\vec{F} = -\nabla U_B = -\frac{\mu \mu_0 I_2}{2\pi} \frac{1}{x^2} \cdot \hat{x}$

If $x = 0, y \neq 0$, $\quad \vec{F} = -\nabla U_B = -\frac{\mu \mu_0}{2\pi} I_1 \frac{1}{y^2} \cdot \hat{y}$

If $x = 0, y = 0$, $\quad \vec{F} = 0$

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(f): 12 points

Suppose a particle of charge q is placed at a position (x, y) and given velocity $\vec{v} = v\hat{x}$. What electric field could be established at (x, y) so that the net force on the charged particle at that location is zero? [In terms of μ_0, I_1, I_2, q, v, x, and/or y].

If $x \neq 0, y \neq 0$

$$\vec{F} = q\vec{v} \times \vec{B}$$
$$= q(v\hat{x}) \times \hat{z}\frac{\mu_0}{2\pi}\left(\frac{I_1}{y} + \frac{I_2}{x}\right)$$
$$= \frac{qv\mu_0}{2\pi}\left(\frac{I_1}{y} + \frac{I_2}{x}\right)(-\hat{y})$$
$$\vec{F}_{net} = 0 = \vec{F}_{mag} + \vec{F}_{elec}$$
$$\therefore \vec{F}_{ele} = \vec{E} \cdot q = \frac{qv\mu_0}{2\pi}\left(\frac{I_1}{y} + \frac{I_2}{x}\right) \cdot (\hat{y})$$
$$\vec{E} = \frac{v\mu_0}{2\pi}\left(\frac{I_1}{y} + \frac{I_2}{x}\right) \cdot (\hat{y})$$

If $x \neq 0, y = 0$ $\quad \vec{E} = \frac{v\mu_0}{2\pi}\left(\frac{I_2}{x}\right)\hat{y}$

If $x = 0, y \neq 0$ $\quad \vec{E} = \frac{v\mu_0}{2\pi}\left(\frac{I_1}{y}\right)\hat{y}$

If $x = 0, y = 0$, $\vec{E} = 0$