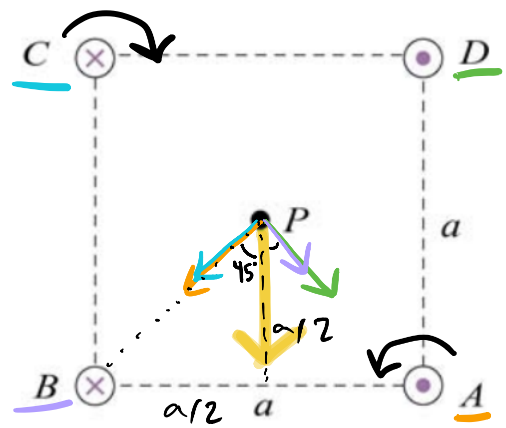
You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

30 points

Consider a configuration of infinitely long parallel current-carrying wires shown in the diagram below:



Wires B and C carry current I into the page, and wires A and D carry current I out of the page. What are the magnitude and direction of the magnetic field at point P? You may use the expression for the magnetic field of infinitely long wires derived in class.

I figured out the arrow directions from the RHR.

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r}$$

$$r = \sqrt{(a/2)^2 + (a/2)^2} = \sqrt{a^2/4 + a^2/4} = \sqrt{a^2/2} = \frac{a}{\sqrt{2}}$$

$$|B| = \frac{\mu_0 I}{2\pi \cdot \frac{a}{\sqrt{2}}} = \frac{\mu_0 I \sqrt{2}}{2\pi a}$$

mag B $= \frac{\mu_0 I \sqrt{2}}{2\pi a}$] add bc same dir. } $\frac{\mu_0 I \sqrt{2}}{\pi a}$ ↙ from BD

mag D $= \frac{\mu_0 I \sqrt{2}}{2\pi a}$

mag A $= \frac{\mu_0 I \sqrt{2}}{2\pi a}$] add } $\frac{\mu_0 I \sqrt{2}}{\pi a}$ ↙ from AC

mag C $= \frac{\mu_0 I \sqrt{2}}{2\pi a}$

$$|B| = \frac{\mu_0 I \sqrt{2}}{\pi a}(\cos 45^\circ) + \frac{\mu_0 I \sqrt{2}}{\pi a}(\cos 45^\circ)$$ angle between arrow & dir.

$$|B| = 2 \cdot \frac{\mu_0 I \sqrt{2}}{\pi a} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{2\mu_0 I}{\pi a} = |B|}$$

The direction is the highlighted yellow arrow, towards the middle of the B & A line.

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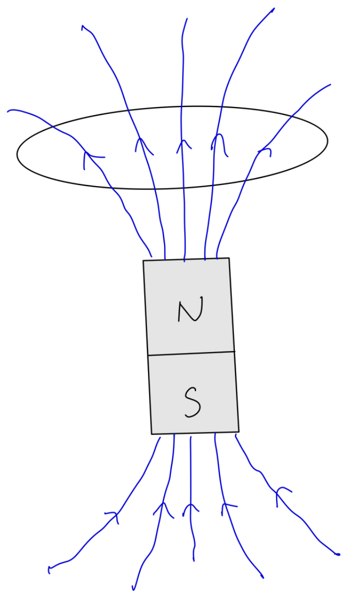
Problem 2

30 points

Consider a loop of wire adjacent to a permanent magnet in the configuration shown.



(a): 10 pts

Hold the magnet stationary and move the loop of wire towards the magnet as shown. Faraday's law,

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

tells us that an emf is induced in the wire due to the change in magnetic flux through the wire. There must be some force that drives the electrons in the wire to form a current. What is this force?

When there is a change in the magnetic flux through a closed loop, an emf is induced in that loop. Since the wire is a conductor, the induced current flows in response to the emf. Holding the mag. stationary & moving the coil creates an induced current caused by induced EMF. The force is the magnetic force because this is motional emf.

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(b): 10 pts

Determine the direction of the current that flows by considering the force you found in part (a)

I can use the RHR. Since the loop is moving towards the magnet the motion causes an increasing upward flux through the loop. So, the induced magnetic field is down to oppose the flux change. To produce this induced field, the induced current must be clockwise. (Thumb points down, fingers curl CW)

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(c): 10 pts

Now consider an equivalent situation in which you hold the loop of wire still and move the magnet towards the loop. Faraday's law still tells us that a current flows due to the changing magnetic flux. What force acts on the electrons to drive the current in this case? Justify your answer.

The electric force. Since the magnet is moving & the loop is stationary, the loop is not in a magnetic field so it can't be magnetic forces moving the electrons. So, it must be an induced electric field in the conductor caused by the changing electric flux

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Problem 3

40 pts

A long cylindrical conductor of radius R carries a uniform current density $\vec{J} = J\hat{z}$ that runs parallel to the axis of the cylinder (the z-axis). A time-varying electric field is established everywhere in space and is given by $\vec{E} = E_0 \cos(\omega t)\hat{z}$. Using Ampere's law, compute the magnetic field in the following regions: [YOU MUST SHOW ALL WORK; YOU MAY NOT USE RESULTS FROM LECTURE OR PSETS]

(a): 20 pts $J = \frac{I}{\pi R^2}$

$r > R$

when $r > R$, we treat it like an infinitely long wire. This is because the mag. field outside any cylindrically symmetric current distribution is the same as if the entire current were concentrated along the axis of distribution.

So, $B = \frac{\mu_0 I}{2\pi r}$ $(I_c + I_D)$, but we have to consider $\vec{E}$.

So, we need to add the component $\varepsilon_0 \frac{d\Phi_E}{dt}$ to I (for I_d)

So $B = \frac{\mu_0}{2\pi r}\left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt}\right) = B = \frac{\mu_0}{2\pi r}\left(i_c + \varepsilon_0 \frac{d}{dt}(E \cdot A)\right)$

$B = \frac{\mu_0}{2\pi r}\left(i_c + \varepsilon_0 A \cdot \frac{d}{dt}(E_0 \cos(\omega t))\right)$

$= \frac{\mu_0}{2\pi r}\left(i_c + \varepsilon_0 A \cdot (-E_0 \omega \sin(\omega t))\right)$

$B = \frac{\mu_0}{2\pi r}\left(i_c - \varepsilon_0 A E_0 \omega \sin(\omega t)\right)$

$A = \pi r^2$, $i_c = J\pi R^2$

so, $B = \frac{\mu_0}{2r}\left(JR^2 - \varepsilon_0 E_0 r^2 \omega \sin(\omega t)\right)$

RHR says B is in CCW direction in terms of $\hat{\rho}$ (cylindrical coords)

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(b): 20 pts

$r < R$

$$\oint \vec{B} \cdot \underbrace{d\ell}_{} = \mu_0 I_{encl}$$

we know path is $2\pi r$ because the mag. is the same at every point on the circular integration path.

$$B \cdot 2\pi r = \mu_0 (I_c + I_d)$$

$$I_{encl} = J(\pi r^2)$$

$$J = \frac{I}{\pi R^2}$$

$$I_{encl} = \frac{I r^2}{R^2}$$

$$B \cdot 2\pi r = \mu_0 \left(\frac{I_c r^2}{R^2} + \overbrace{\varepsilon_0 A \cdot (-E_0 \omega \sin(\omega t))}^{\text{same as } I_d \text{ in part a.}} \right)$$

$$B = \frac{\mu_0}{2\pi r} \left(\frac{I_c r^2}{R^2} - \varepsilon_0 A E_0 \omega \sin(\omega t) \right)$$

$$I_c = J\pi R^2, \quad A = \pi r^2$$

$$B = \frac{\mu_0}{2} \left(Jr - \varepsilon_0 E_0 r \, \omega \sin(\omega t) \right)$$

RHR says B is in CCW direction in terms of $\hat{\rho}$ (cylindrical coords)