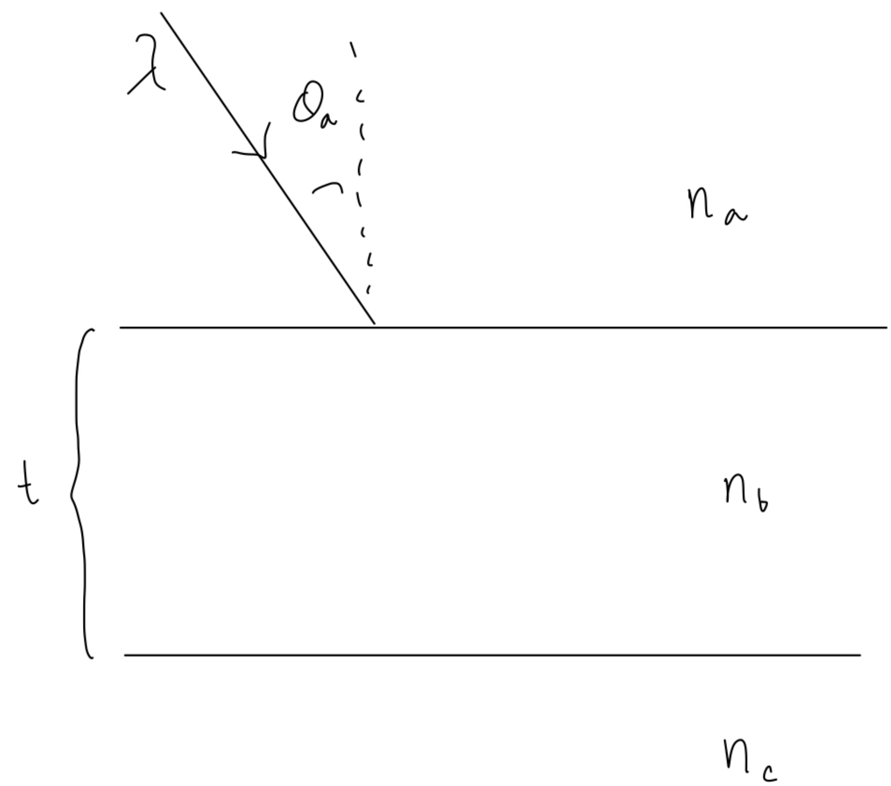
Problem 1

20 points

Consider a thin film of thickness $t = 2.30 \times 10^{-6}$ m and index of refraction $n_b = 1.20$. The film is resting on a material of index of refraction $n_c = 1.13$, and its top face is exposed to air $n_a = 1.00$. What is the shortest wavelength of *visible* light that will interfere destructively when incident on the film at angle $\theta_a = 22.0°$ from the normal? Give your answer in nanometers, to three significant figures. [Note: you may ignore the fact that the wavelength will change upon refraction; this will only very slightly affect the answer.]



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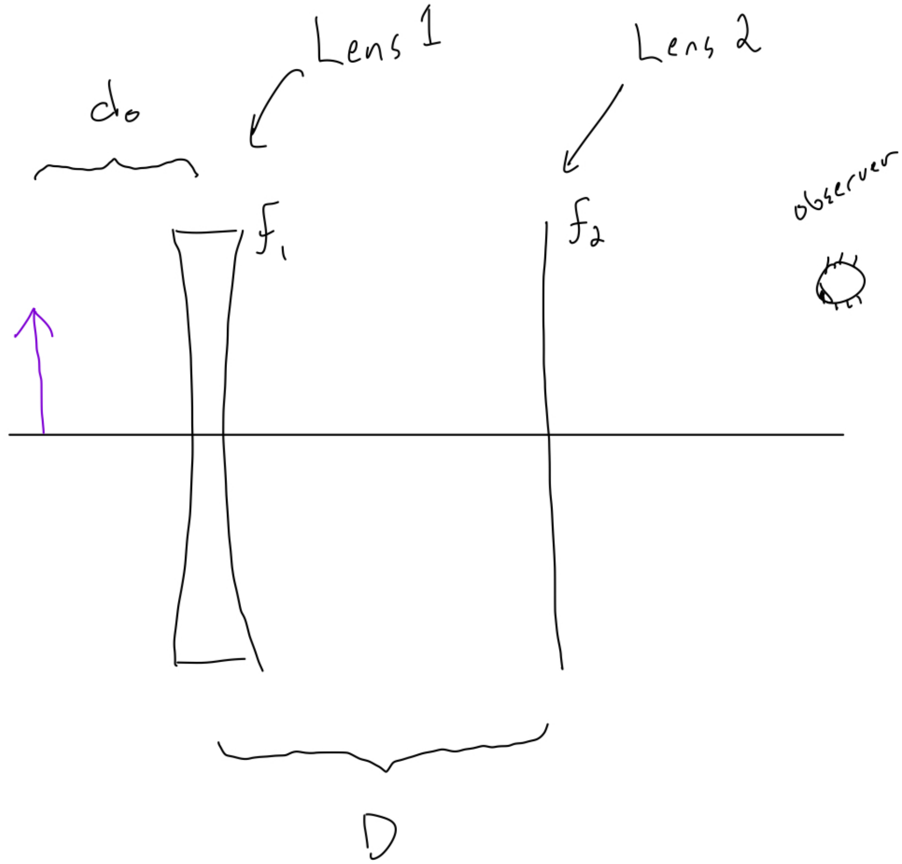
Problem 2

20 points

Consider the thin lens (Lens 1) of focal length $f_1 = -2.00$ cm shown below. An object is placed a distance $d_o = 4.50$ cm to the left of the lens. A second lens (Lens 2, drawn as a single vertical line in the diagram) is then placed $D = 5.00$ cm to the right of Lens 1. What is the focal length Lens 2 must have such that the *final* image of the object seen through the two lenses is inverted and enlarged by a factor of three with respect to the object? Give your answer in centimeters, to three significant figures. Is Lens 2 a converging or diverging lens?



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Problem 3

20 points

Consider an infinitely long cylindrical wire of radius R centered on the z axis. The current density in the wire varies with the distance ρ from the axis as $\vec{J}(\rho) = J_0 \left(1 - \frac{\rho}{R}\right) \hat{z}$, where J_0 is a constant.

(a): 10 points

Find the magnetic field everywhere in space. Recall that current through a surface S is defined as $I = \int_S \vec{J} \cdot d\vec{A}$.

Problem 3 continued on next page. . .

(b): 10 points

Suppose now that the current density falls off as a function of time:

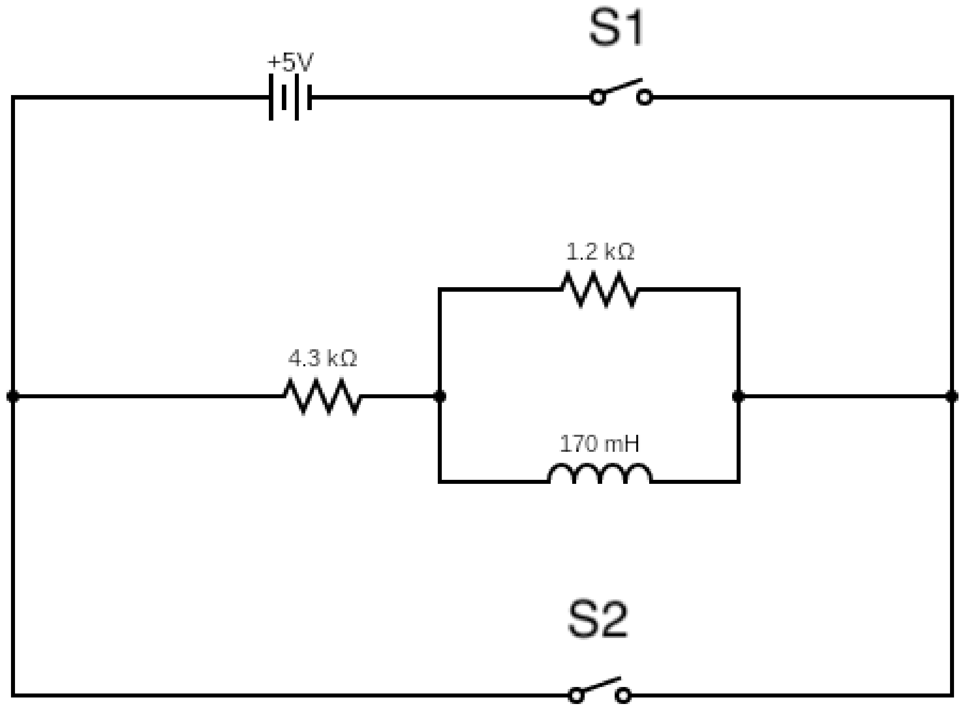
$$\vec{J}(\rho, t) = J_0 \left(1 - \frac{\rho}{R}\right)\left(1 - \frac{t}{\tau}\right)\hat{z}$$

where τ is a constant and $0 \leq t \leq \tau$. Calculate the electric field everywhere outside the wire, in terms of $\mu_0, \epsilon_0, J_0, R, \tau$, and any spatial coordinates and/or time. [Hint: the electric field will point along the z axis. Use Faraday's law and integrate around a rectangle whose base and top are parallel to the z axis.] [Note: you will only be able to calculate the electric field up to an arbitrary constant; this is ok.]

Problem 4

20 points

Consider the circuit drawn below. Let I_1 be the current flowing through the 4.3 kΩ resistor,, I_2 be the current flowing through the 1.2 kΩ resistor, and I_3 be the current flowing through the 170 mH inductor.



(a): 5 points

Suppose S_1 has been closed for a long time, $(-\infty < t < 0)$, and the emf source drives a steady current. Calculate I_1, I_2, I_3.

(b): 5 points

At time $t = 0$, S_1 is opened and S_2 is closed, so that the emf source is no longer connected and current flows through the bottom branch. Calculate I_1, I_2, I_3 just after this happens.

Problem 4 continued on next page. . .

(c): 5 points

Calculate I_1, I_2, I_3 in the limit $t \to \infty$.

(d): 5 points

Calculate the total energy dissipated in all resistors during the time period $0 < t < \infty$.

Problem 5

10 points

(a): 5 points

Consider the following vector field:

$$\vec{V}(r,\theta,\phi) = \alpha\frac{1}{r^2}\hat{r},$$

where α is a constant with units T·m^2, r is the distance from the origin, and $\hat{r}$ is a unit vector pointing away from the origin. According to Maxwell's equations, could this represent a magnetic field? Explain.

(b): 5 points

Consider the following electric and magnetic fields:

$$\vec{E} = \frac{E_0}{\sqrt{2}}\cos\left(\omega(t - z/c)\right)(\hat{y} - \hat{x})$$
$$\vec{B} = \frac{E_0}{c\sqrt{2}}\cos\left(\omega(t - z/c)\right)(-\hat{y} - \hat{x}).$$

Can these fields constitute a traveling electromagnetic plane wave in vacuum? If yes, prove it. If not, explain why not.

Problem 6

10 points

Consider an LRC series circuit driven with an ac source $v(t) = V_0 \cos(\omega t)$. You may use without proof the impedance and phase of an LRC series circuit.

(a): 5 points

Calculate the total energy dissipated in the resistor over one cycle if the system is driven at its resonant frequency.

Problem 6 continued on next page. . .

(b): 5 points

Calculate the total energy dissipated in the resistor over one cycle if the system is driven at *twice* its resonant frequency. Is it greater than or less than the energy found in part (a)?