211C-PHYSICS1C-2 Final Exam

RICHARD JIANG

TOTAL POINTS

83 / 100

QUESTION 1

1 Problem 1 **15 / 20**

- **- 0 pts** Correct
- ✓ **- 0 pts Mistake (will clarify with comments and adjust points below)**
- **- 5 Point adjustment**

  -5 used 2t when this is not normal incidence

QUESTION 2

2 Problem 2 **20 / 20**

- ✓ **- 0 pts Correct**
- **- 0 pts** Mistake (will clarify with comments and adjust points below)

QUESTION 3

Problem 3 20 pts

3.1 (a) **10 / 10**

- ✓ **- 0 pts Correct**
- **- 0 pts** Mistake (will clarify with comments and adjust points below)

3.2 (b) **6 / 10**

- **- 0 pts** Correct
- ✓ **- 0 pts Mistake (will clarify with comments and adjust points below)**
- **- 4 Point adjustment**

  -4 integrated incorrectly

QUESTION 4

Problem 4 20 pts

4.1 (a) **5 / 5**

- ✓ **- 0 pts Correct**
- **- 0 pts** Mistake (will clarify with comments and adjust points below)

4.2 (b) **3 / 5**

- **- 0 pts** Correct
- ✓ **- 0 pts Mistake (will clarify with comments and adjust points below)**
- **- 2 Point adjustment**

4.3 (c) **5 / 5**

- ✓ **- 0 pts Correct**
- **- 0 pts** Mistake (will clarify with comments and adjust points below)

4.4 (d) **5 / 5**

- ✓ **- 0 pts Correct**
- **- 0 pts** Mistake (will clarify with comments and adjust points below)

QUESTION 5

Problem 5 10 pts

5.1 (a) **3 / 5**

- **- 0 pts** Correct
- ✓ **- 0 pts Mistake (will clarify with comments and adjust points below)**
- **- 2 Point adjustment**

  -2 did divergence incorrectly

5.2 (b) **3 / 5**

- **- 0 pts** Correct
- ✓ **- 0 pts Mistake (will clarify with comments and adjust points below)**
- **- 2 Point adjustment**

QUESTION 6

Problem 6 10 pts

6.1 (a) 5 / 5

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify with comments and adjust points below)

6.2 (b) 3 / 5

- 0 pts Correct

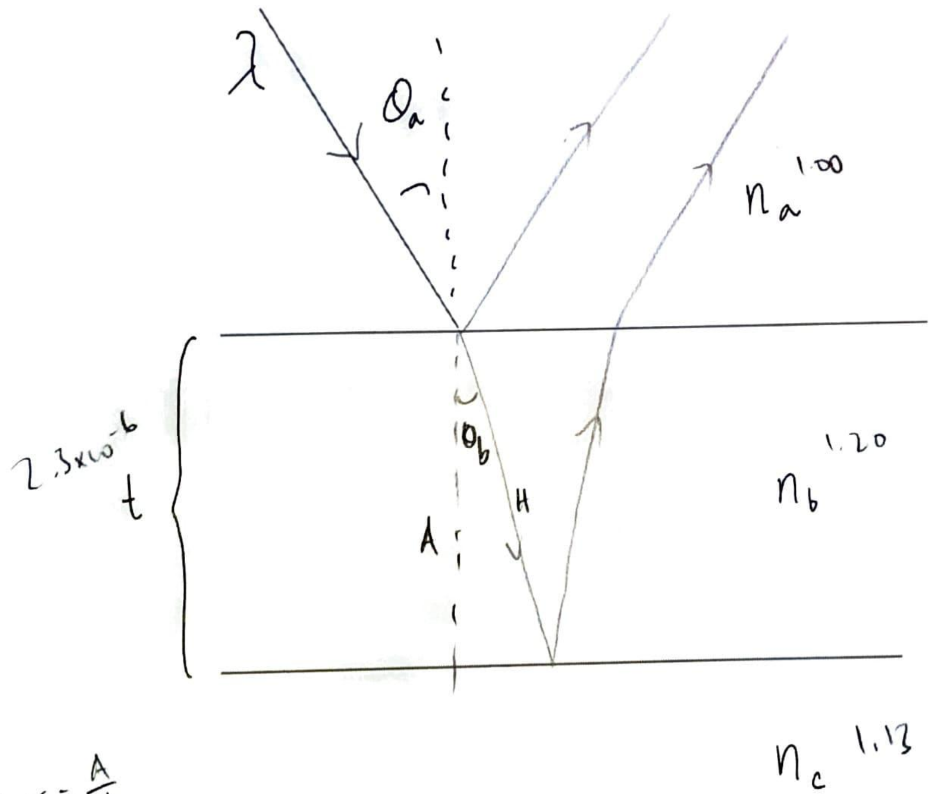
✓ **- 0 pts Mistake (will clarify with comments and adjust points below)**

- 2 Point adjustment

Problem 1

20 points

Consider a thin film of thickness $t = 2.30 \times 10^{-6}$ m and index of refraction $n_b = 1.20$. The film is resting on a material of index of refraction $n_c = 1.13$, and its top face is exposed to air $n_a = 1.00$. What is the shortest wavelength of *visible* light that will interfere destructively when incident on the film at angle $\theta_a = 22.0°$ from the normal? Give your answer in nanometers, to three significant figures. [Note: you may ignore the fact that the wavelength will change upon refraction; this will only very slightly affect the answer.]



$\cos = \frac{A}{H}$

Find distance travelled in film, d

visible light: 380 - 700 nm

$\theta_b = \arcsin\left(\frac{n_a \sin\theta_a}{n_b}\right)$

$= 18.2°$ (3sf)

$d = \frac{2.3\times10^{-6}}{\cos(18.2)} = 2.42\times10^{-6}$ m

$2t = m\lambda$ ($n_b > n_c$, transmitted wave in n_c faster than in n_b, so no π shift)

$\lambda = \frac{2t}{m}$

$= \frac{2(2.42\times10^{-6})}{m}$ $m = 12$ for $\lambda > 380$

$\lambda = 403$ nm (3sf)

1 Problem 1 15 / 20

- **- 0 pts** Correct
- ✓ **- 0 pts** **Mistake (will clarify with comments and adjust points below)**
- **- 5** **Point adjustment**
  - -5 used 2t when this is not normal incidence

Problem 2

20 points

Consider the thin lens (Lens 1) of focal length $f_1 = -2.00$ cm shown below. An object is placed a distance $d_o = 4.50$ cm to the left of the lens. A second lens (Lens 2, drawn as a single vertical line in the diagram) is then placed $D = 5.00$ cm to the right of Lens 1. What is the focal length Lens 2 must have such that the *final* image of the object seen through the two lenses is inverted and enlarged by a factor of three with respect to the object? Give your answer in centimeters, to three significant figures. Is Lens 2 a converging or diverging lens?

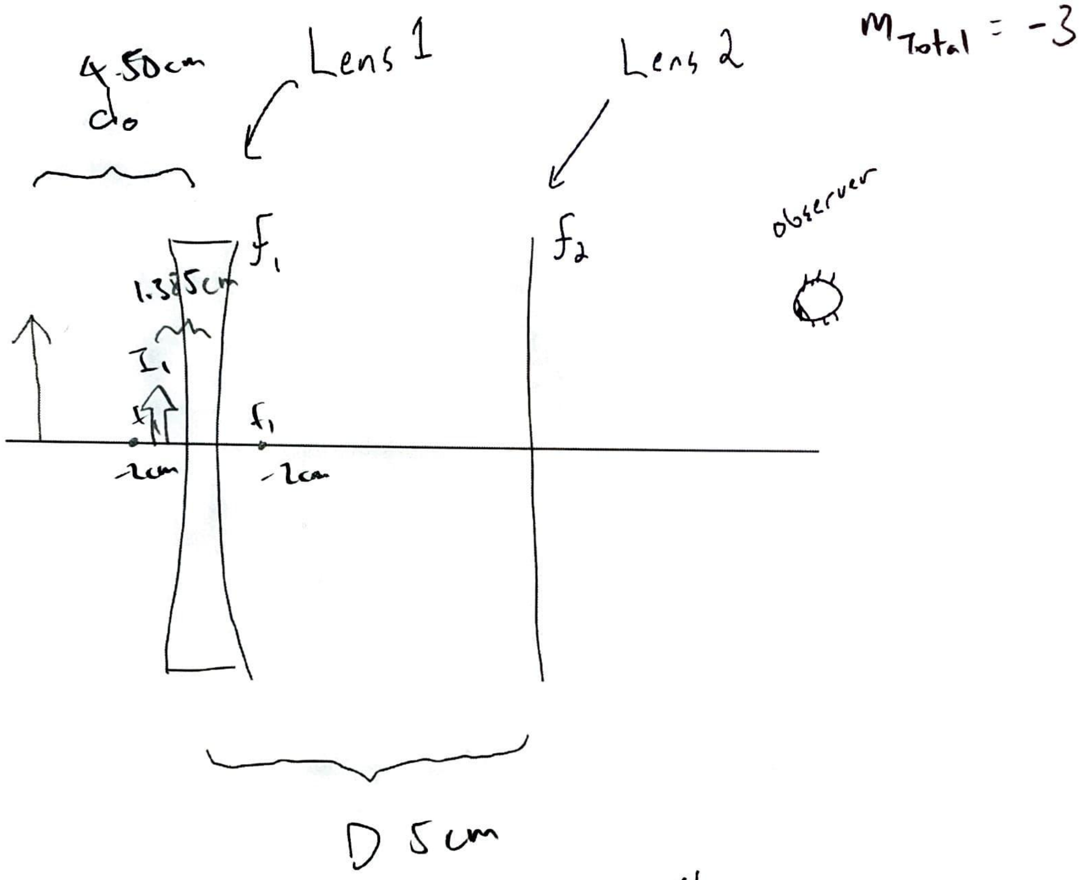


Image 1:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{4.5\text{cm}} + \frac{1}{s'} = \frac{1}{-2\text{cm}}$$

$$\frac{1}{s'} = \frac{1}{-2} - \frac{1}{4.5}$$

$$s' = -1.385\ldots \text{cm}$$

$$M_1 = -\frac{s'}{s}$$

$$= -\frac{-1.385\ldots}{4.5}$$

$$= 0.3077\ldots \left(\frac{4}{13}\right)$$

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$M_{Tot} = M_2 M_1$, $M_2 = \frac{M_T}{M_1} = \frac{-3}{4/13} = -9.75$

Image 2:

$S_2 = 5 + 1.385... = 6.385...$ cm

$M_2 = -\frac{S_2'}{S_2}$; $-9.75 = -\frac{S_2'}{6.385}$; $S_2' = 62.25$ cm (+ve)

$\frac{1}{f_2} = \frac{1}{S} + \frac{1}{S'} = \frac{1}{6.385...} + \frac{1}{62.25} = \frac{43}{249}$

$f_2 = 5.79$ cm (3sf)

converging lens ($f > 0$)

2 Problem 2 **20 / 20**

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify with comments and adjust points below)

Problem 3

20 points

Consider an infinitely long cylindrical wire of radius R centered on the z axis. The current density in the wire varies with the distance ρ from the axis as $\vec{J}(\rho) = J_0\left(1 - \frac{\rho}{R}\right)\hat{z}$, where J_0 is a constant.

(a): 10 points

Find the magnetic field everywhere in space. Recall that current through a surface S is defined as $I = \int_S \vec{J} \cdot d\vec{A}$.

Current density only depends on radial distance, so magnetic field is constant 360° (not radially)

For $r > R$.

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$\vec{B} \int d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{A}$

$B 2\pi r' = \mu_0 \int_S J_0 (1 - \frac{r}{R})\hat{z}\, dA$

$= \mu_0 \int_0^R J_0 (1 - \frac{r}{R}) r\,dr\,d\theta$

$= \mu_0 J_0 \int_0^R r - \frac{r^2}{R}\, dr\,d\theta$

$= \mu_0 J_0 2\pi \left[\frac{r^2}{2} - \frac{r^3}{3R}\right]_0^R$

$= \mu_0 J_0 2\pi \left[\frac{R^2}{2} - \frac{R^3}{3R}\right]$

$= \mu_0 J_0 2\pi \left[\frac{R^2}{6}\right]$

$= \frac{\mu_0 J_0 \pi R^2}{3}$

$B = \frac{\mu_0 J_0 R^2}{6 r'}$

For $r < R$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$B \int dl = \mu_0 \int_S J \cdot dA$

$= \mu_0 \int J_0 (1 - \frac{r}{R})\hat{z}\, dA$

$B 2\pi r = \mu_0 J_0 \int (1 - \frac{r}{R})\, dA$

$= \mu_0 J_0 \int (1 - \frac{r}{R})\, r\,dr\,d\theta$

$= \mu_0 J_0 2\pi \int r(1 - \frac{r}{R})\, dr$

$= \mu_0 J_0 2\pi \left(\int r\,dr - \frac{1}{R}\int r^2\,dr\right)$

$= \mu_0 J_0 2\pi \left(\frac{r^2}{2} - \frac{r^3}{3R}\right)$

$B = \frac{\mu_0 J_0 2\pi r^2 (\frac{1}{2} - \frac{r}{3R})}{2\pi r}$

$= \mu_0 J_0 r \left(\frac{1}{2} - \frac{r}{3R}\right)$

Problem 3 continued on next page...

3.1 (a) **10 / 10**

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify with comments and adjust points below)

(b): 10 points

Suppose now that the current density falls off as a function of time:

$$\vec{J}(\rho, t) = J_0 \left(1 - \frac{\rho}{R}\right)\left(1 - \frac{t}{\tau}\right)\hat{z}$$

where τ is a constant and $0 \leq t \leq \tau$. Calculate the electric field everywhere outside the wire, in terms of $\mu_0, \epsilon_0, J_0, R, \tau$, and any spatial coordinates and/or time. [Hint: the electric field will point along the z axis. Use Faraday's law and integrate around a rectangle whose base and top are parallel to the z axis.] [Note: you will only be able to calculate the electric field up to an arbitrary constant; this is ok.]

$$\oint E \cdot dl = -\frac{d\Phi_B}{dt} = -\frac{d\int \vec{B} \cdot d\vec{A}}{dt}$$

$$= -\frac{d}{dt}\int \frac{\mu_0 J_0 R^2}{6r}\left(1 - \frac{t}{\tau}\right) dA$$

looking at flux:

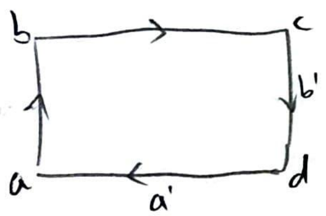
bc and ad = 0.

$$= -\frac{d}{dt}\left(\frac{\mu_0 J_0 R^2}{6}\left(1 - \frac{t}{\tau}\right)\int \frac{1}{r} dA\right)$$

Through rectangle abcd.

$$= -\frac{d}{dt}\left(\frac{\mu_0 J_0 R^2}{6}\left(1 - \frac{t}{\tau}\right)\frac{a'b'}{r}\right)$$

rectangle



$$\oint E \cdot dl = -\frac{\mu_0 J_0 R^2}{6}\frac{a'b'}{r\tau}$$

$$\int_a^b E\,dl + \int_c^d E \cdot dl = -\frac{\mu_0 J_0 R^2}{6}\frac{a'b'}{r\tau}$$

$$E(b-a) + E(d-c) = -\frac{\mu_0 J_0 R^2}{6}\frac{(b-a)(c-b)}{r\tau}$$

$$E = -\frac{\mu_0 J_0 R^2}{6r\tau}\frac{(b-a)(c-b)}{(b+d-a-c)}$$

3.2 (b) **6 / 10**

- 0 pts Correct

✓ **- 0 pts** **Mistake (will clarify with comments and adjust points below)**

- 4 **Point adjustment**

-4 integrated incorrectly

(a): 5 points

Suppose S_1 has been closed for a long time, $(-\infty < t < 0)$, and the emf source drives a steady current. Calculate I_1, I_2, I_3.

Inductor acts like a wire after switch is closed for a long time.

$$I_1 = \frac{V}{R} = \frac{5V}{4.3k\Omega} = 1.16 \times 10^{-3} A \quad (3sf)$$

$$I_2 = 0$$

$$I_3 = 1.16 \times 10^{-3} A \quad (3sf)$$

(b): 5 points

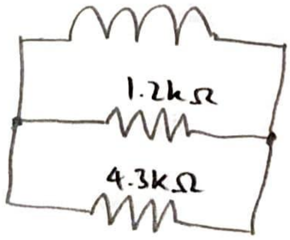
At time $t = 0$, S_1 is opened and S_2 is closed, so that the emf source is no longer connected and current flows through the bottom branch. Calculate I_1, I_2, I_3 just after this happens.

Turning off an RL circuit,

$$I = I_0 e^{-\frac{R}{L}t},$$

$$t=0 \quad \therefore I = I_0$$

$$I_3 = 1.16 \times 10^{-3} A$$



$$I_2 = \frac{4.3}{1.2+4.3} \times 1.16 \times 10^{-3} A = 9.07 \times 10^{-4} A$$

$$I_1 = \frac{1.2}{4.3+1.2} \times 1.16 \times 10^{-3} A = 2.53 \times 10^{-4} A$$

Problem 4 continued on next page...

4.1 (a) **5 / 5**

✓ **- 0 pts** Correct

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(a): 5 points

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Inductor acts like a wire after switch is closed for a long time.

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$I_2 = 0$

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(b): 5 points

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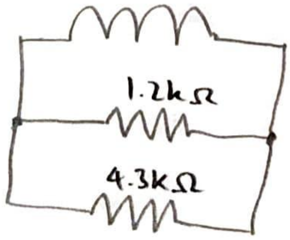


$I_2 = \frac{4.3}{1.2+4.3} \times 1.16 \times 10^{-3} A$

$= 9.07 \times 10^{-4} A$

$I_1 = \frac{1.2}{4.3+1.2} \times 1.16 \times 10^{-3} A$

$= 2.53 \times 10^{-4} A$

Problem 4 continued on next page...

4.2 (b) **3 / 5**

- **- 0 pts** Correct
- ✓ **- 0 pts** Mistake (will clarify with comments and adjust points below)
- **- 2** Point adjustment

(c): 5 points

Calculate I_1, I_2, I_3 in the limit $t \to \infty$.

$I_1 = 0$

$I_2 = 0$ switch S_1 open, S_2 closed

$I_3 = 0$

(d): 5 points

Calculate the total energy dissipated in all resistors during the time period $0 < t < \infty$.

$P = IV = I^2R$

$E = \int P dt$

$= \int I^2 R dt$

$= \int (I_0 e^{-\frac{R}{L}t})^2 R \, dt$

$= R I_0^2 \int_0^\infty (e^{-\frac{R}{L}t})^2 dt$

$= R I_0^2 \left[\frac{-L e^{-\frac{2Rt}{L}}}{2R} \right]_0^\infty$

$= R I_0^2 \cdot \frac{L}{2R}$

$= \frac{1}{2} I_0^2 \cdot L$

$= \frac{1}{2} \times 1.16 \times 10^{-3} A^2 \times 170 \times 10^{-3}$

$= 1.14 \times 10^{-7} J$ (3 sf)

4.3 (c) 5 / 5

✓ - 0 pts Correct

- 0 pts Mistake (will clarify with comments and adjust points below)

(c): 5 points

Calculate I_1, I_2, I_3 in the limit $t \to \infty$.

$I_1 = 0$

$I_2 = 0$ switch S_1 open, S_2 closed

$I_3 = 0$

(d): 5 points

Calculate the total energy dissipated in all resistors during the time period $0 < t < \infty$.

$P = IV = I^2R$

$E = \int P dt$

$= \int I^2 R dt$

$= \int (I_0 e^{-\frac{R}{L}t})^2 R dt$

$= R I_0^2 \int_0^\infty (e^{-\frac{R}{L}t})^2 dt$

$= R I_0^2 \left[\frac{-L e^{-\frac{2Rt}{L}}}{2R} \right]_0^\infty$

$= R I_0^2 \cdot \frac{L}{2R}$

$= \frac{1}{2} I_0^2 \cdot L$

$= \frac{1}{2} \times 1.16\times10^{-3} A^2 \times 170\times10^{-3}$

$= 1.14\times10^{-7} J$ (3 sf)

4.4 (d) **5 / 5**

✓ **- 0 pts** Correct

- 0 pts Mistake (will clarify with comments and adjust points below)

Problem 5

10 points

(a): 5 points

Consider the following vector field:

$$\vec{V}(r,\theta,\phi) = \alpha\frac{1}{r^2}\hat{r},$$

where α is a constant with units T·m^2, r is the distance from the origin, and $\hat{r}$ is a unit vector pointing away from the origin. According to Maxwell's equations, could this represent a magnetic field? Explain.

$\nabla\cdot B = 0$ for a magnetic field

$$\nabla\cdot V = \left(\frac{\partial}{\partial r}+\frac{\partial}{\partial\theta}+\frac{\partial}{\partial\phi}\right)\cdot\left(\alpha\frac{1}{r^2}\hat{r}\right) = \frac{\partial}{\partial r}\left(\alpha\frac{1}{r^2}\hat{r}\right)$$

$$= \alpha\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) = -2\alpha\frac{1}{r^3} \neq 0$$

∴ According to Maxwell's equations, this cannot be a magnetic field.

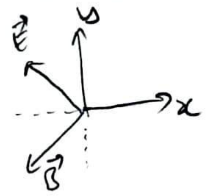
(b): 5 points

Consider the following electric and magnetic fields:

$$\vec{E} = \frac{E_0}{\sqrt{2}}\cos\left(\omega(t-z/c)\right)(\hat{y}-\hat{x})$$

$$\vec{B} = \frac{E_0}{c\sqrt{2}}\cos\left(\omega(t-z/c)\right)(-\hat{y}-\hat{x}).$$

Can these fields constitute a traveling electromagnetic plane wave in vacuum? If yes, prove it. If not, explain why not.



waves are perpendicular.

$$\nabla\times\vec{E} = -\frac{\partial B}{\partial t}$$

$$\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ -\frac{E_0}{\sqrt{2}}\cos(\omega(t-\frac{z}{c})) & \frac{E_0}{\sqrt{2}}\cos(\omega(t-\frac{z}{c})) & 0\end{vmatrix} = \frac{\partial}{\partial z}\left(-\frac{E_0}{\sqrt{2}}\cos\left(\omega(t-\frac{z}{c})\right)\right)\hat{x} + \frac{\partial}{\partial z}\left(-\frac{E_0}{\sqrt{2}}\cos\left(\omega(t-\frac{z}{c})\right)\right)\hat{y}$$

$$= -\frac{E_0}{\sqrt{2}}\left[\frac{\omega\sin\left(\omega(t-\frac{z}{c})\right)}{c}\hat{x} + \frac{\omega\sin\left(\omega(t-\frac{z}{c})\right)}{c}\hat{y}\right]$$

$$= \frac{E_0}{\sqrt{2}c}\omega\sin\left(\omega(t-\tfrac{z}{c})\right)(-\hat{y}-\hat{x})$$

$$\frac{\partial B}{\partial t} = \frac{-E_0}{\sqrt{2}c}\omega\sin\left(\omega(t-\tfrac{z}{c})\right)(-\hat{y}-\hat{x})$$

$$\therefore \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$ ∴ These fields do constitute an em wave in vacuum.

5.1 (a) **3 / 5**

- **- 0 pts** Correct
- ✓ **- 0 pts** **Mistake (will clarify with comments and adjust points below)**

- 2 Point adjustment

-2 did divergence incorrectly

Problem 5

10 points

(a): 5 points

Consider the following vector field:

$$\vec{V}(r,\theta,\phi) = \alpha\frac{1}{r^2}\hat{r},$$

where α is a constant with units T·m^2, r is the distance from the origin, and $\hat{r}$ is a unit vector pointing away from the origin. According to Maxwell's equations, could this represent a magnetic field? Explain.

$\nabla \cdot B = 0$ for a magnetic field

$$\nabla \cdot V = \left(\frac{\partial}{\partial r} + \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \phi}\right)\cdot\left(\alpha\frac{1}{r^2}\hat{r}\right) = \frac{\partial}{\partial r}\left(\alpha\frac{1}{r^2}\hat{r}\right)$$

$$= \alpha\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) = -2\alpha\frac{1}{r^3} \neq 0$$

∴ According to Maxwell's equations, this cannot be a magnetic field.

(b): 5 points

Consider the following electric and magnetic fields:

$$\vec{E} = \frac{E_0}{\sqrt{2}}\cos\left(\omega(t - z/c)\right)(\hat{y} - \hat{x})$$

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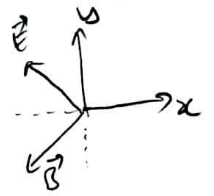

waves are perpendicular.

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{E_0}{\sqrt{2}}\cos(\omega(t-\frac{z}{c})) & \frac{E_0}{\sqrt{2}}\cos(\omega(t-\frac{z}{c})) & 0 \end{vmatrix} = \frac{\partial}{\partial z}\left(\frac{E_0}{\sqrt{2}}\cos\left(\omega(t-\frac{z}{c})\right)\right)\hat{x} + \frac{\partial}{\partial z}\left(-\frac{E_0}{\sqrt{2}}\cos(\omega(t-\frac{z}{c}))\right)\hat{y}$$

$$= -\frac{E_0}{\sqrt{2}}\left[\frac{\omega\sin(\omega(t-\frac{z}{c}))}{c}\hat{x} + \frac{\omega\sin(\omega(t-\frac{z}{c}))}{c}\hat{y}\right]$$

$$= \frac{E_0}{\sqrt{2}c}\omega\sin\left(\omega(t-\tfrac{z}{c})\right)(-\hat{y}-\hat{x})$$

$$\frac{\partial B}{\partial t} = \frac{-E_0}{\sqrt{2}c}\omega\sin\left(\omega(t-\tfrac{z}{c})\right)(-\hat{y}-\hat{x})$$

$$\therefore \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

∴ These fields do constitute an em wave in vacuum.

5.2 (b) 3 / 5

- **- 0 pts** Correct
- ✓ **- 0 pts** Mistake (will clarify with comments and adjust points below)

- 2 Point adjustment

Problem 6

10 points

Consider an LRC series circuit driven with an ac source $v(t) = V_0 \cos(\omega t)$. You may use without proof the impedance and phase of an LRC series circuit.

(a): 5 points

Calculate the total energy dissipated in the resistor over one cycle if the system is driven at its resonant frequency.

At resonant frequency:

$Z = R$

$P = \frac{V^2}{R} = \frac{V_0^2}{R}\cos^2(\omega t)$

$\omega = 2\pi f = \frac{2\pi}{T}$

One cycle:

let one cycle = T

$E = \frac{V_0^2}{R}\int_0^T \cos^2(\omega t)\, dt$

$= \frac{V_0^2}{R}\int_0^T \cos^2\left(\frac{2\pi}{T}t\right) dt$

$= \frac{V_0^2}{R} \times \frac{T}{2}$

$\omega = \frac{1}{\sqrt{LC}} = \frac{2\pi}{T}$

$T = 2\pi\sqrt{LC}$

$\therefore E_{Tot}$ over 1 cycle

$= \frac{V_0^2}{R} \times \frac{T}{2}$

$= \frac{V_0^2}{R} \times \pi\sqrt{LC}$

Problem 6 continued on next page...

6.1 (a) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 0 pts** Mistake (will clarify with comments and adjust points below)

(b): 5 points

Calculate the total energy dissipated in the resistor over one cycle if the system is driven at *twice* its resonant frequency. Is it greater than or less than the energy found in part (a)?

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad \omega = \frac{2}{\sqrt{LC}}$$

In one cycle, power is delivered to and extracted from both L and C, so $P_{\text{dissipated}} = 0$.

$$P = \frac{V^2}{R}$$

$$= \frac{(V_0 \cos(\omega t))^2}{R}$$

$$= \frac{V_0^2 \cos^2(\omega t)}{R}$$

$$E = \int_0^T P\,dt$$

$$= \frac{V_0^2}{R} \times \frac{T}{2} \quad ; \qquad T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2}{\sqrt{LC}}} = \pi\sqrt{LC}$$

$$= \frac{V_0^2}{R} \times \frac{\pi\sqrt{LC}}{2}$$

∴ less energy dissipated per cycle.

6.2 (b) **3 / 5**

- **- 0 pts** Correct
- ✓ **- 0 pts** Mistake (will clarify with comments and adjust points below)

- 2 Point adjustment