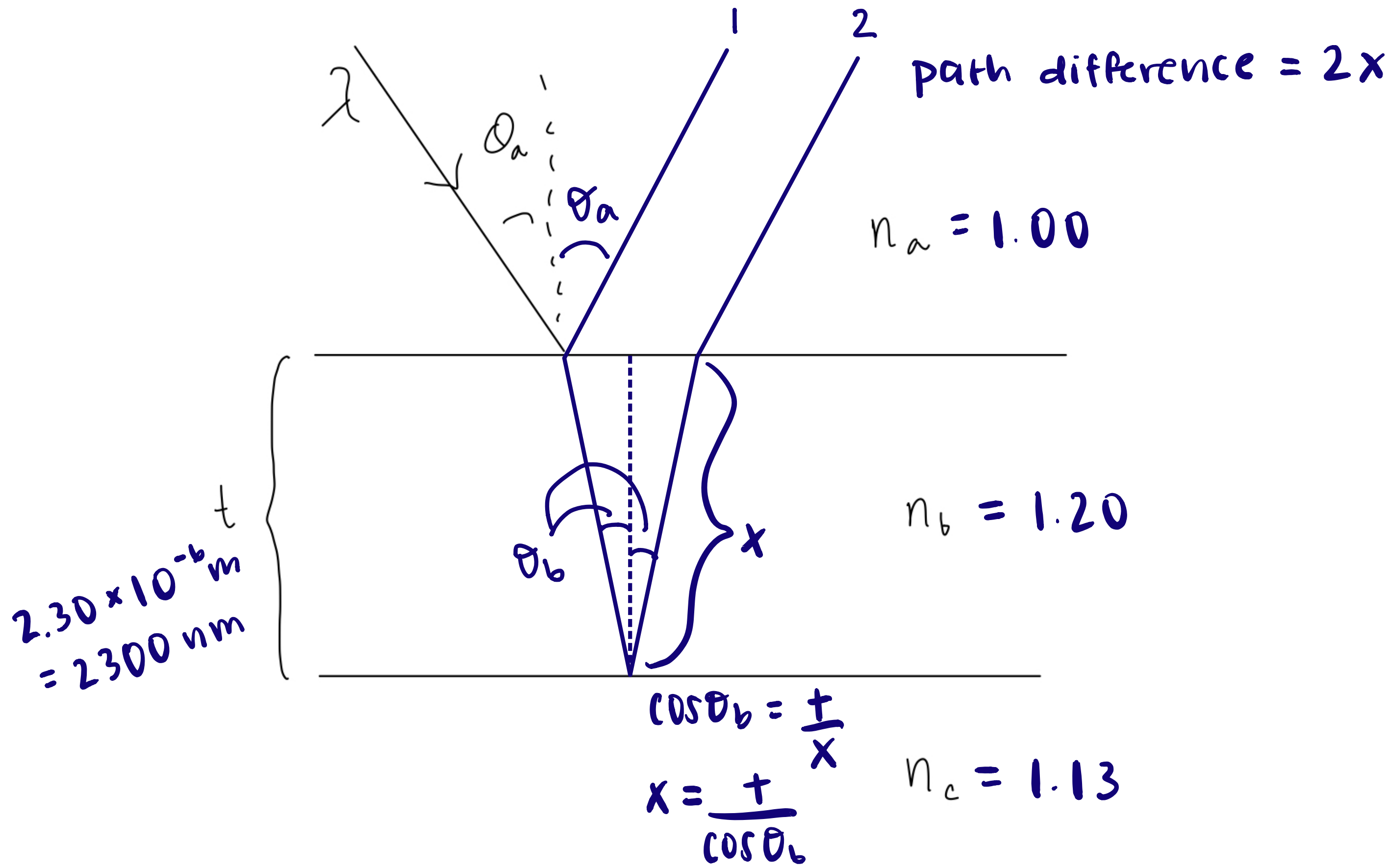
Problem 1

20 points

(equation # in textbook)

Consider a thin film of thickness $t = 2.30 \times 10^{-6}$ m and index of refraction $n_b = 1.20$. The film is resting on a material of index of refraction $n_c = 1.13$, and its top face is exposed to air $n_a = 1.00$. What is the shortest wavelength of *visible* light that will interfere destructively when incident on the film at angle $\theta_a = 22.0°$ from the normal? Give your answer in nanometers, to three significant figures. [Note: you may ignore the fact that the wavelength will change upon refraction; this will only very slightly affect the answer.]



wave 1:

- shifts by a half cycle because $n_b > n_a$

wave 2:

- does not shift because $n_c < n_b$

so for destructive interference,

$2x = m\lambda$ $(m = 0, 1, 2, \ldots)$ (35.18b)

ignore that λ changes upon refraction

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$$\frac{2t}{\cos\theta_b} = m\lambda$$

$$\lambda = \frac{2t}{m\cos\theta_b}$$

Snell's Law: $n_a \sin\theta_a = n_b \sin\theta_b$ (33.4)

$$\theta_b = \sin^{-1}\left(\frac{n_a \sin\theta_a}{n_b}\right)$$

$$= \sin^{-1}\left(\frac{1.00(\sin 22)}{1.20}\right)$$

$$= 18.19018°$$

$$\lambda = \frac{2(2300)}{m\cos(18.19018)}$$

$$= \frac{4841.975}{m} \quad (m = 1, 2, 3, \ldots)$$

smallest wavelength of visible light is 380 nm

if $m = 13$,

$\lambda = 372$ nm (not visible)

if $m = 12$,

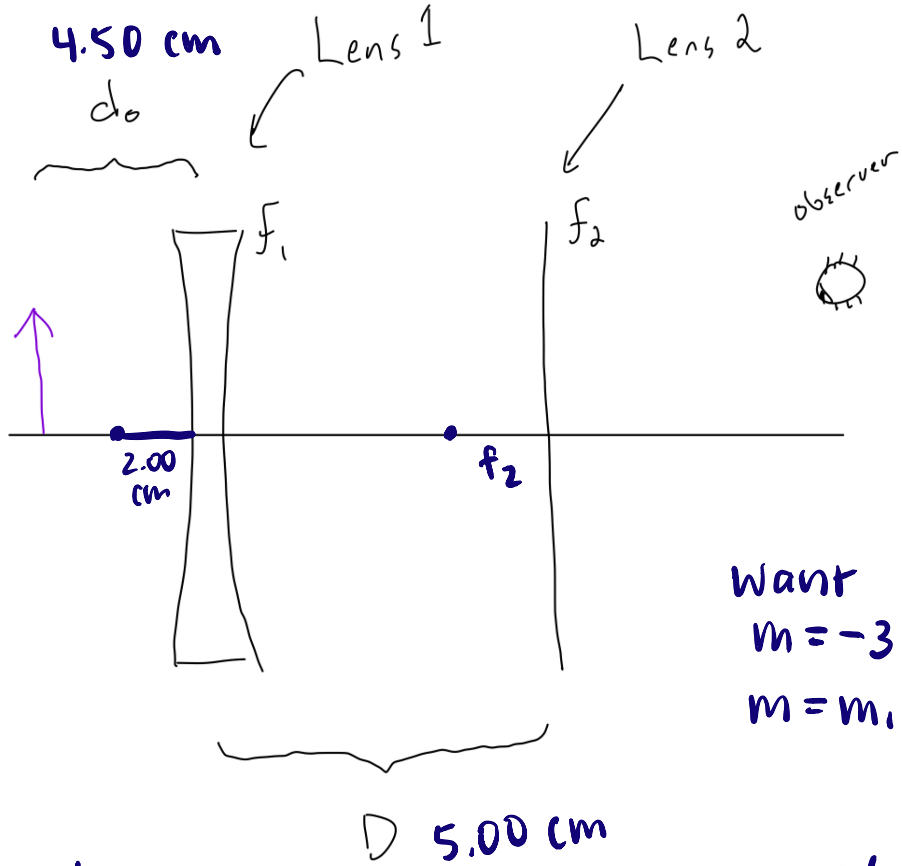
$\lambda = \boxed{403 \text{ nm}}$

Problem 2

20 points

(equation # in textbook)

Consider the thin lens (Lens 1) of focal length $f_1 = -2.00$ cm shown below. An object is placed a distance $d_o = 4.50$ cm to the left of the lens. A second lens (Lens 2, drawn as a single vertical line in the diagram) is then placed $D = 5.00$ cm to the right of Lens 1. What is the focal length Lens 2 must have such that the *final* image of the object seen through the two lenses is inverted and enlarged by a factor of three with respect to the object? Give your answer in centimeters, to three significant figures. Is Lens 2 a converging or diverging lens?



Want
$m = -3$
$m = m_1 m_2$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (34.6)$$

$$m = -\frac{s'}{s} \quad (34.7)$$

lens 1:

$$\frac{1}{4.50} + \frac{1}{s_1'} = \frac{1}{-2.00}$$

$$s_1' = -1.38461 \text{ cm}$$

$$m_1 = \frac{-(-1.38461)}{4.50}$$

$$= 0.30769$$

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lens 2 :

$$\frac{1}{1.38461+5.00}+\frac{1}{S_2'}=\frac{1}{f_2}$$

$$\frac{1}{6.38461}+\frac{1}{S_2'}=\frac{1}{f_2}$$

$$m=m_1m_2$$

$$m_2=\frac{m}{m_1}$$

$$=\frac{-3}{0.30769}$$

$$=-9.75$$

$$m_2=\frac{-S_2'}{S_2}$$

$$S_2'=-m_2S_2$$

$$=-(-9.75)(6.38461)$$

$$=62.25$$

$$f_2=\frac{1}{\frac{1}{6.38461}+\frac{1}{62.25}}$$

$$=\boxed{5.79\text{ cm}}$$

diverging lenses have negative focal lengths, and f_2 is positive, so lens 2 must be a

$\boxed{\text{converging lens}}$

Problem 3

20 points

(equation # in textbook)

Consider an infinitely long cylindrical wire of radius R centered on the z axis. The current density in the wire varies with the distance ρ from the axis as $\vec{J}(\rho) = J_0\left(1 - \frac{\rho}{R}\right)\hat{z}$, where J_0 is a constant.

(a): 10 points

Find the magnetic field everywhere in space. Recall that current through a surface S is defined as $I = \int_S \vec{J} \cdot d\vec{A}$.

because of symmetry, we can use Ampere's Law

$\oint_s \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$ (29.19)

$B \oint_s d\vec{l} = \mu_0 I_{encl}$

$B\, 2\pi\rho = \mu_0 I_{encl}$

$B = \frac{\mu_0 I_{encl}}{2\pi\rho}$

by Right Hand Rule, $\vec{B}$ is in the $+\hat{x}$ direction

if $\rho < R$,

$I = \int_0^\rho J_0\left(1 - \frac{\rho}{R}\right) 2\pi\rho\, d\rho$

$= 2\pi J_0 \int_0^\rho \rho - \frac{\rho^2}{R}\, d\rho$

$= 2\pi J_0\left(\frac{\rho^2}{2} - \frac{\rho^3}{3R}\right)$

$= 2\pi J_0 \rho^2\left(\frac{1}{2} - \frac{\rho}{3R}\right)$

$\vec{B} = \frac{\mu_0\left(\cancel{2\pi} J_0 \rho^{\cancel{2}}\left(\frac{1}{2} - \frac{\rho}{3R}\right)\right)}{\cancel{2\pi\rho}}\hat{x}$

$= \boxed{\mu_0 J_0 \rho\left(\frac{1}{2} - \frac{\rho}{3R}\right)\hat{x}}$

if $\rho > R$,

$I = \int_0^R J_0\left(1 - \frac{\rho}{R}\right) 2\pi\rho\, d\rho$

$= 2\pi J_0 \int_0^R \rho - \frac{\rho^2}{R}\, d\rho$

$= 2\pi J_0\left(\frac{R^2}{2} - \frac{R^2}{3}\right)$

$= \cancel{2}\pi J_0 R^2 \left(\frac{1}{\cancel{2}}\right)_3$

$\vec{B} = \frac{\mu_0 \cancel{\pi} J_0 R^2}{3(2\cancel{\pi}\rho)}\hat{x}$

$= \boxed{\frac{\mu_0 J_0 R^2}{6\rho}\hat{x}}$

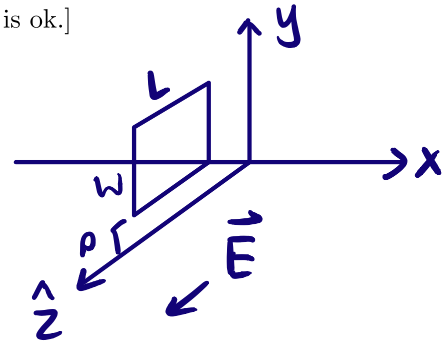
(b): 10 points

Suppose now that the current density falls off as a function of time:

$$\vec{J}(\rho,t) = J_0\left(1-\frac{\rho}{R}\right)\left(1-\frac{t}{\tau}\right)\hat{z}$$

$, \rho > R$

where τ is a constant and $0 \le t \le \tau$. Calculate the electric field everywhere outside the wire, in terms of $\mu_0, \epsilon_0, J_0, R, \tau$, and any spatial coordinates and/or time. [Hint: the electric field will point along the z axis. Use Faraday's law and integrate around a rectangle whose base and top are parallel to the z axis.] [Note: you will only be able to calculate the electric field up to an arbitrary constant; this is ok.]



$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad (29.10)$$

$\rho > R$, so

$$B = \frac{\mu_0 J_0 R^2}{6\rho}\left(1-\frac{t}{\tau}\right) \quad \text{(from part a)}$$

$$\Phi_B = \int B\cdot dA \quad (27.6), \quad dA = L\,d\rho$$

$$= \frac{\mu_0 J_0 R^2}{6}\left(1-\frac{t}{\tau}\right)\int_{\rho}^{\rho+W}\frac{1}{\rho}\,d\rho$$

$$= \frac{\mu_0 J_0 R^2}{6}\left(1-\frac{t}{\tau}\right)\ln\left(\frac{\rho+W}{\rho}\right)$$

$$-\frac{d\Phi_B}{dt} = \frac{\mu_0 J_0 R^2}{6\tau}\ln\left(\frac{\rho+W}{\rho}\right)$$

only sides ∥ to $\vec{E}$ contribute

$$\oint \vec{E}\cdot d\vec{l} = \frac{\mu_0 J_0 R^2}{6\tau}\ln\left(\frac{\rho+W}{\rho}\right)$$

$\vec{E}$ is constant along the 2 sides

$$\vec{E}\cdot 2L = \frac{\mu_0 J_0 R^2}{6\tau}\ln\left(\frac{\rho+W}{\rho}\right)\hat{z}$$

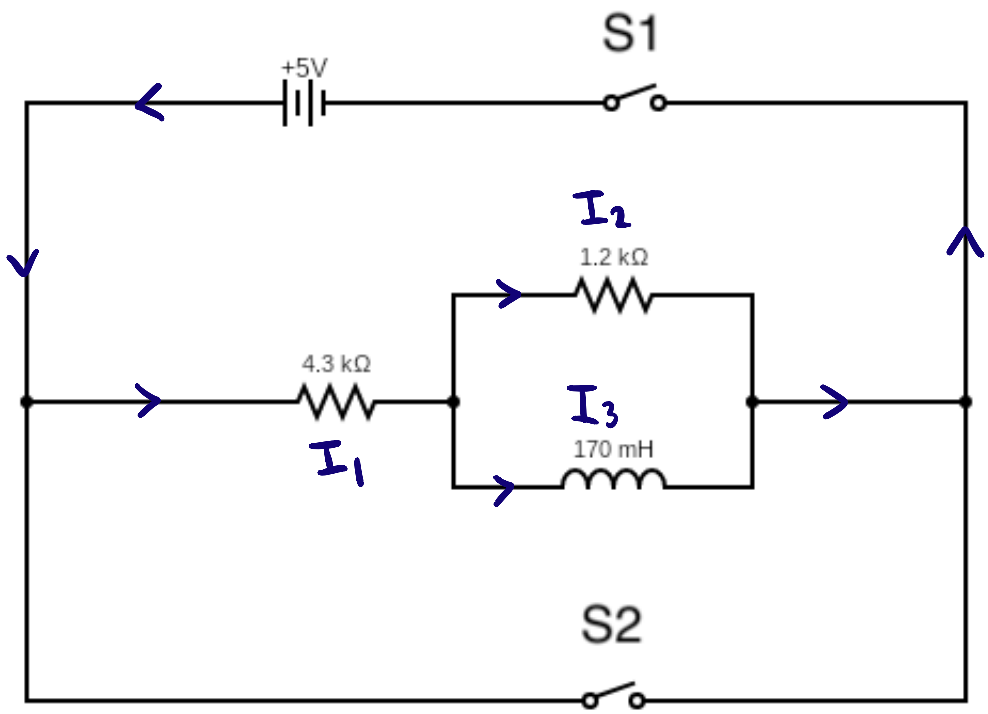
$$= \boxed{\frac{\mu_0 J_0 R^2}{12L\tau}\ln\left(\frac{\rho+W}{\rho}\right)\hat{z}}$$

Problem 4

20 points

(equation # in textbook)

Consider the circuit drawn below. Let I_1 be the current flowing through the 4.3 kΩ resistor,, I_2 be the current flowing through the 1.2 kΩ resistor, and I_3 be the current flowing through the 170 mH inductor.



using Kirchhoff's Rules,

1) $I_1 = I_2 + I_3$

2) $\mathcal{E} - I_1 R_1 - I_2 R_2 = 0$

3) $\mathcal{E} - I_1 R_1 - L\frac{dI_3}{dt} = 0$ (30.13)

(a): 5 points

Suppose S_1 has been closed for a long time, $(-\infty < t < 0)$, and the emf source drives a steady current. Calculate I_1, I_2, I_3.

after being closed for a long time, $\frac{dI_3}{dt} = 0$

3) $\mathcal{E} - I_1 R_1 = 0$

$\mathcal{E} = I_1 R_1$

$I_1 = \mathcal{E}/R_1$

$I_1 = \frac{5}{4300} = \boxed{1.16 \times 10^{-3}\ A}$

2) $0 - I_2 R_2 = 0$

$I_2 R_2 = 0$

$I_2 = 0$

$I_2 = \boxed{0}$

$I_3 = \boxed{1.16 \times 10^{-3}\ A}$

1) $I_1 = I_3$

(b): 5 points

At time $t = 0$, S_1 is opened and S_2 is closed, so that the emf source is no longer connected and current flows through the bottom branch. Calculate I_1, I_2, I_3 just after this happens.

now using Kirchhoff's Rules,

3) $-\overbrace{I_1 R_1}^{I} - L\frac{dI_3}{dt} = 0$

$\frac{dI_3}{dt} = \frac{-I_1 R_1}{L}$ $-R_{eq}$

$\frac{dI}{dt} = -\frac{I R_{eq}}{L}$

$\int \frac{dI}{I} = -\int \frac{R_{eq}}{L} dt$

$\ln(I) = -\frac{R_{eq} t}{L}$

$I = I_0 e^{-R_{eq}/L\, t}$ (30.18)

at $t = 0$,

$I = I_0$

I_0 = current reached in part a

$I_1 = \boxed{1.16 \times 10^{-3}\ A}$

$I_2 = \boxed{0}$

$I_3 = \boxed{1.16 \times 10^{-3}\ A}$

(c): 5 points

Calculate I_1, I_2, I_3 in the limit $t \to \infty$.

$$I = I_0 e^{-R_1/L\,t}$$

$$\lim_{t\to\infty} I_0 e^{-R_1/L\,t}$$

$$= I_0 e^{-\infty}$$

$$= I_0(0)$$

$$= 0$$

$$\boxed{I_1 = I_2 = I_3 = 0}$$

(d): 5 points

Calculate the total energy dissipated in all resistors during the time period $0 < t < \infty$.

energy dissipated through both resistors

$$\text{energy dissipated} = I^2 R_{eq} \quad (30.19)$$

$$= R_{eq}\int_0^\infty \left(I_0 e^{-R_{eq}/L\,t}\right)^2 dt$$

$$= R_{eq} I_0^2 \int_0^\infty \left(e^{-R_{eq}/L\,t}\right)^2 dt$$

$$= \cancel{R_{eq}} I_0^2 \left[-\frac{L}{2\cancel{R_{eq}}} e^{-2R_{eq}/L\,t}\right]\Big|_0^\infty$$

$$= -\frac{I_0^2 L}{2}(0-1)$$

$$= \frac{I_0^2 L}{2}$$

$$= \frac{(1.16\times10^{-3})^2(0.17)}{2}$$

$$= \boxed{1.14\times10^{-7}\ \text{J}}$$

Problem 5

10 points

(a): 5 points

(equation # in textbook)

Consider the following vector field:

$$\vec{V}(r,\theta,\phi) = \alpha \frac{1}{r^2}\hat{r},$$

where α is a constant with units T·m^2, r is the distance from the origin, and $\hat{r}$ is a unit vector pointing away from the origin. According to Maxwell's equations, could this represent a magnetic field? Explain.

the only Maxwell equation that restricts whether fields can be magnetic fields is

$\nabla \cdot \vec{B} = 0$

$= \frac{1}{r^2}\frac{d}{dr}\left(\cancel{r^2}\alpha \frac{1}{\cancel{r^2}}\right)$ ← divergence in spherical coordinates

$= 0$ ✓

$\boxed{\text{yes, it satisfies the Maxwell equation}}$

(b): 5 points

Consider the following electric and magnetic fields:

$$\vec{E} = \frac{E_0}{\sqrt{2}}\cos\left(\omega(t - z/c)\right)(\hat{y} - \hat{x})$$

$<-1,1,0>$ is in this direction

$$\vec{B} = \frac{E_0}{c\sqrt{2}}\cos\left(\omega(t - z/c)\right)(-\hat{y} - \hat{x}).$$

$<-1,-1,0>$ is in this direction

Can these fields constitute a traveling electromagnetic plane wave in vacuum? If yes, prove it. If not, explain why not.

conditions

1. $\vec{E}$ and $\vec{B}$ are $\perp$ (from $\oint \vec{E} \cdot dA = \oint \vec{B} \cdot dA = 0$)
   $<-1,1,0> \cdot <-1,-1,0> = 1-1 = 0$, so they are $\perp$ ✓
2. $E_0 = cB_0$ (32.4) (from Faraday's Law)
   $\frac{E_0}{\sqrt{2}} = \cancel{c}\frac{E_0}{\cancel{c}\sqrt{2}}$ ✓
3. $B_0 = \epsilon_0 \mu_0 c E_0$ (32.8) (from Ampere's Law)
   this is satisfied in vacuum because $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ ✓

$\boxed{\text{yes, it satisfies Maxwell's equations}}$

Problem 6

10 points

(equation # in textbook)

Consider an LRC series circuit driven with an ac source $v(t) = V_0 \cos(\omega t)$. You may use without proof the impedance and phase of an LRC series circuit.

(a): 5 points

Calculate the total energy dissipated in the resistor over one cycle if the system is driven at its resonant frequency.

at resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (31.32)$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$

$$P = \frac{V^2}{R} \quad (25.18)$$

$$\text{total energy} = \int_0^{2\pi\sqrt{LC}} \frac{\left(V_0 \cos\left(\frac{t}{\sqrt{LC}}\right)\right)^2}{R}\, dt$$

$$= \frac{V_0^2}{R} \int_0^{2\pi\sqrt{LC}} \cos^2\left(\frac{t}{\sqrt{LC}}\right) dt$$

$$= \frac{V_0^2}{R} \left[\frac{t}{2} + \frac{\sqrt{LC}}{4} \sin\left(\frac{2t}{\sqrt{LC}}\right)\right] \Bigg|_0^{2\pi\sqrt{LC}}$$

$$= \frac{V_0^2}{R} \left(\frac{\cancel{2}\pi\sqrt{LC}}{\cancel{2}}\right)$$

$$= \boxed{\frac{V_0^2 \pi \sqrt{LC}}{R}}$$

Problem 6 continued on next page. . .

(b): 5 points

Calculate the total energy dissipated in the resistor over one cycle if the system is driven at *twice* its resonant frequency. Is it greater than or less than the energy found in part (a)?

in this case,

$$\omega = 2\omega_0 = \frac{2}{\sqrt{LC}}$$

$$T = \frac{2\pi}{\omega} = \pi\sqrt{LC}$$

$$\text{total energy} = \int_0^{\pi\sqrt{LC}} \frac{\left(V_0 \cos\left(\frac{2t}{\sqrt{LC}}\right)\right)^2}{R}\, dt$$

$$= \frac{V_0^2}{R}\int_0^{\pi\sqrt{LC}} \cos^2\left(\frac{2t}{\sqrt{LC}}\right) dt$$

$$= \frac{V_0^2}{R}\left[\frac{t}{2} + \frac{\sqrt{LC}}{8}\sin\left(\frac{4t}{\sqrt{LC}}\right)\right]\Bigg|_0^{\pi\sqrt{LC}}$$

$$= \frac{V_0^2}{R}\left(\frac{\pi\sqrt{LC}}{2}\right)$$

$$= \boxed{\frac{V_0^2\pi\sqrt{LC}}{2R}}$$

it is 1/2 of the energy found in part a., so it is

less than the energy found in part a.