Physics 1C Final Exam

TOTAL POINTS

97 / 100

QUESTION 1

Problem 1 20 pts

1.1 (a) 7 / 7

✓ **- 0 pts** Correct

- 2 pts missing/incorrect definition of k

- 1 pts define both left&right polarization

- 2 pts incorrect form of electric field equation

- 1 pts error with unit vectors/signs

- 1 pts incorrect amplitude based off initial conditions (should be E0)

1.2 (b) 7 / 7

✓ **- 0 pts** Correct

- 0 pts Correct, based on incorrect (a)

- 2 pts incorrect use of trig identities

- 3 pts no dependence on index of refraction

- 2 pts missing/incorrect conclusion

1.3 (c) 6 / 6

✓ **- 0 pts** Correct

- 0 pts correct, based off incorrect part A/B

- 3 pts incorrect setup

- 1 pts small error

- 2 pts incorrect final answer

QUESTION 2

Problem 2 20 pts

2.1 (a) 7 / 7

✓ **- 0 pts** Correct

- 5 pts Using Galilean results to solve a relativistic problem.

- 2 pts Minor error.

- 5 pts Major error.

- 7 pts No submission.

2.2 (b) 7 / 7

✓ **- 0 pts** Correct

- 5 pts Using Galilean results to solve a relativistic problem.

- 2 pts Minor error.

- 4 pts Did not solve for the velocity.

- 5 pts Major error.

- 7 pts No submission

2.3 (c) 6 / 6

✓ **- 0 pts** Correct

- 3 pts Major error.

QUESTION 3

Problem 3 20 pts

3.1 (a) 10 / 10

✓ **- 0 pts** Correct

- 3 pts Error computing photon energy.

- 5 pts Incorrect reasoning or conclusion.

- 10 pts No submission.

- 3 pts Minor error.

3.2 (b) 10 / 10

✓ **- 0 pts** Correct

- 3 pts Error computing the work function.

- 5 pts Incorrect reasoning or conclusion.

QUESTION 4

4 Problem 4 20 / 20

✓ **- 0 pts** Correct

- 5 pts path difference is integer number of wavelengths

- 5 pts path difference between 1&2 is equal to path difference between 2&3

QUESTION 5

Problem 5 20 pts

5.1 (a) 4 / 7

- **0 pts** Correct
- **5 pts** No loss from inductor and/or capacitor
- **5 pts** This doesn't describe energy loss

✓ **- 3 pts You have only described i2R loss**

5.2 (b) 7 / 7

✓ **- 0 pts Correct**

- **5 pts** Incorrect reasoning

5.3 (c) 6 / 6

✓ **- 0 pts Correct**

- **5 pts** Inductor in dc circuit is just wire. You haven't really explained why the transformer doesn't work.
- **3 pts** Magnetic flux is still produced.

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 1

We have seen in class that the index of refraction of a material can depend on the wavelength of light, a phenomenon known as dispersion. The index of refraction can *also* depend on the polarization of the light. In this problem we consider a material which has different indices of refraction for right- and left-circularly polarized light, $n_{\pm}$, where the plus and minus sign refer to right- and left-circularly polarized light, respectively.

(a): 7 pts

Write down expressions for the electric fields of right- and left-circularly polarized plane waves travelling in the $+\hat{z}$-direction in this material. You may assume that the two waves have the same angular frequency ω, and the same amplitude, and that the electric field for both waves at $t = 0$ and $z = 0$ is equal to $E_0\hat{x}$. [Hint: remember how the index of refraction affects an EM wave. See Lecture 15 for discussion of polarization.]

Given equation as $\vec{E}(z,t)$, when $z=0$ and $t=0$, $\Rightarrow \vec{E}(0,0) = E_0\hat{x}$

Furthermore, given that the two waves have the same ω, to relate the index of refraction, we see that given $k = \frac{2\pi}{\lambda}$ and $\lambda = \frac{\lambda_0}{n}$, $k = \frac{2\pi n}{\lambda_0}$

thus, right circularly polarized light is given by

$$E(z,t) = E_0\left[\cos\left(\frac{2\pi n_+}{\lambda_0}z - \omega t\right)\hat{x} + \sin\left(\frac{2\pi n_+}{\lambda_0}z - \omega t\right)\hat{y}\right]$$

and left circularly polarized light is given by

$$E(z,t) = E_0\left[\cos\left(\frac{2\pi n_-}{\lambda_0}z - \omega t\right)\hat{x} - \sin\left(\frac{2\pi n_-}{\lambda_0} - \omega t\right)\hat{y}\right]$$

Problem 1 continued on next page...

1.1 (a) **7 / 7**

- ✓ **- 0 pts** Correct
- **- 2 pts** missing/incorrect definition of k
- **- 1 pts** define both left&right polarization
- **- 2 pts** incorrect form of electric field equation
- **- 1 pts** error with unit vectors/signs
- **- 1 pts** incorrect amplitude based off initial conditions (should be E0)

You must show your work to receive credit. An answer written down with no work will receive no credit.

(b): 7 pts

Linearly-polarized light can be written as the sum of right- and left-circularly polarized light. Add the right- and left-circularly polarized waves you found in part (a), and check that the resulting wave is linearly polarized. [Hint: how does the electric field behave at a fixed location along the z-axis?] You may find the following trig identities useful:

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) \pm \sin(y) = 2\sin\left(\frac{x \pm y}{2}\right)\cos\left(\frac{x \mp y}{2}\right)$$

$$\vec{E}(z,t) = E_0\left[\cos\left(\frac{2\pi n_+}{\lambda_0}z - \omega t\right)\hat{x} + \cos\left(\frac{2\pi n_-}{\lambda_0}z - \omega t\right)\hat{x} + \sin\left(\frac{2\pi n_+}{\lambda_0}z - \omega t\right)\hat{y} - \sin\left(\frac{2\pi n_-}{\lambda_0}z - \omega t\right)\hat{y}\right]$$

$$= E_0\left[2\cos\left(\frac{\frac{2\pi}{\lambda_0}(n_+ + n_-)z - 2\omega t}{2}\right)\cos\left(\frac{\frac{2\pi}{\lambda_0}(n_+ - n_-)z}{2}\right)\hat{x} + 2\sin\left(\frac{\frac{2\pi}{\lambda_0}(n_+ - n_-)z}{2}\right)\cos\left(\frac{\frac{2\pi}{\lambda_0}(n_+ + n_-)z - 2\omega t}{2}\right)\hat{y}\right]$$

$$\boxed{\vec{E}(z,t) = 2E_0\cos\left(\frac{\pi}{\lambda_0}(n_+ + n_-)z - \omega t\right)\left[\cos\left(\frac{\pi(n_+ - n_-)}{\lambda_0}z\right)\hat{x} + \sin\left(\frac{\pi(n_+ - n_-)}{\lambda_0}z\right)\hat{y}\right]}$$

if we were to fix some value of $z = z_0$ we would find that the electric field does not increase or decrease in the $\hat{x}$ or $\hat{y}$ directions since the inner terms of the expression above underlined in red and green do not increase in time and would be constants if z was fixed, leading to a resulting wave that is linearly polarized. In this case, these inner terms determine the direction in which the linearly polarized wave points

1.2 (b) **7 / 7**

- ✓ **- 0 pts** Correct
- **- 0 pts** Correct, based on incorrect (a)
- **- 2 pts** incorrect use of trig identities
- **- 3 pts** no dependence on index of refraction
- **- 2 pts** missing/incorrect conclusion

You must show your work to receive credit. An answer written down with no work will receive no credit.

(c): 6 pts

The direction of linear polarization rotates as the wave propagates through the material. What is the shortest length of material that could be used to rotate the polarization by 90°?

To rotate the polarization of the linearly polarized wave by 90°

$$\vec{E}(z,t) = 2E_0 \cos\left(\frac{\pi}{\lambda_0}(n_+ + n_-)z - \omega t\right)\left[\cos\left(\frac{\pi(n_+ - n_-)}{\lambda_0}z\right)\hat{x} + \sin\left(\frac{\pi(n_+ - n_-)}{\lambda_0}z\right)\hat{y}\right]$$

→ these values would have to be $\frac{\pi}{2}$

$$\Rightarrow \frac{\pi(n_+ - n_-)}{\lambda_0}z = \frac{\pi}{2}$$

$$\Rightarrow \boxed{z = \left|\frac{\lambda_0}{2(n_+ - n_-)}\right|}$$

↑ shortest length of material

1.3 (c) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 0 pts** correct, based off incorrect part A/B
- **- 3 pts** incorrect setup
- **- 1 pts** small error
- **- 2 pts** incorrect final answer

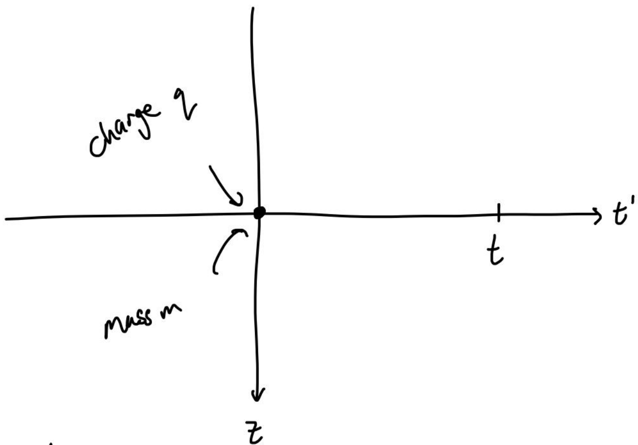
You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 2

You place a particle of charge q and mass m at rest at the origin. At time $t' = 0$, you turn on a uniform electric field $\vec{E} = E\hat{z}$, which you turn off at time $t' = t$. (For this problem, ignore gravity; imagine you are out in space far from any large objects).

(a): 7 pts

What is the momentum of the particle at time t? Take relativistic effects into account, i.e. imagine that the field is on long enough to accelerate the particle to speeds comparable to the speed of light.



Useful Eq

$\vec{F} = q\vec{E}$

$\vec{F} = \frac{dp}{dt}$

Work

$\int_0^p dp' = \int_0^t F\,dt'$

$p = \vec{F}t$

using $\vec{F} = q\vec{E}$

$\Rightarrow p = q\vec{E}t$

$p = qEt\,\hat{z}$

Problem 2 continued on next page...

2.1 (a) **7 / 7**

- ✓ **- 0 pts** Correct
- **- 5 pts** Using Galilean results to solve a relativistic problem.
- **- 2 pts** Minor error.
- **- 5 pts** Major error.
- **- 7 pts** No submission.

You must show your work to receive credit. An answer written down with no work will receive no credit.

(b): 7 pts

What is the velocity of the particle at time t?

Useful eq

$\vec{p} = qEt\hat{z}$

$\vec{p} = mv\gamma$

Work

$$qEt = mv\gamma$$

$$\Rightarrow qEt = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} mv$$

$$\Rightarrow \left(\sqrt{1-\frac{v^2}{c^2}}\right) qEt = mv$$

$$\Rightarrow \left(1-\frac{v^2}{c^2}\right) q^2E^2t^2 = m^2v^2$$

$$\Rightarrow q^2E^2t^2 - \frac{q^2E^2t^2v^2}{c^2} = m^2v^2$$

$$\Rightarrow q^2E^2t^2 = \left(\frac{q^2E^2t^2}{c^2} + m^2\right)v^2$$

$$\Rightarrow v = \sqrt{\frac{(qEt)^2}{\left(\frac{qEt}{c}\right)^2 + m^2}}$$

$$\Rightarrow v = \sqrt{\frac{(qEt)^2}{\frac{(qEt)^2 + (mc)^2}{c^2}}}$$

$$\boxed{\vec{v} = \sqrt{\frac{c^2}{1 + \left(\frac{mc}{qEt}\right)^2}}\,\hat{z}}$$

We can see that if $t \to \infty$, $\vec{v} = c$, and if $t \to \emptyset$, $\vec{v} = \emptyset$, which makes sense

2.2 (b) **7 / 7**

- ✓ **- 0 pts** Correct
- **- 5 pts** Using Galilean results to solve a relativistic problem.
- **- 2 pts** Minor error.
- **- 4 pts** Did not solve for the velocity.
- **- 5 pts** Major error.
- **- 7 pts** No submission

You must show your work to receive credit. An answer written down with no work will receive no credit.

(c): 6 pts

In the particle's rest frame, it is unstable and has a lifetime of τ. What is the apparent lifetime of the particle from the perspective of an observer who observes the particle moving uniformly with the above velocity?

Useful Eq

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

(Δt: apparent time; Δt_0: rest frame time)

Givens

$$\Delta t_0 = \tau$$

$$\vec{v} = \sqrt{\frac{c^2}{1+\left(\frac{mc}{qEt}\right)^2}}\ \hat{z}$$

Work

$$\Delta t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{\Delta t = \frac{\tau}{\sqrt{1-\frac{1}{1+\left(\frac{mc}{qEt}\right)^2}}}}$$

2.3 (c) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 3 pts** Major error.

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 3

In the photoelectric effect, light striking the surface of a conductor ejects electrons from the conductor. If we shine light on the anode of a capacitor which has some voltage applied across it, the ejected electrons will be accelerated towards the cathode, and we establish a **photocurrent**. Using this principle, you would like to build a laser tripwire.

You put a lightbulb (with resistance 10 Ω), capacitor, and DC voltage source $\mathcal{E} = 9$ V in series, and point a ruby laser of wavelength 694 nm (red light) and intensity $I = 1$ W/m^2 at the anode of the capacitor. The anode has workfunction $\phi = 1.3$ eV. Your idea is that as long as the laser is actively shining on the anode, the light bulb will light; if an object obstructs the laser, the light bulb will turn off, and you will notice the light flicker.

(a): 10 pts

With the given parameters, with the laser shining on the anode, will current flow in your circuit? Justify your answer.

What we want is that the energy of the photons being fired should be greater than the amount of energy needed to eject an electron from the anode. IF Energy of ruby laser $\geq \phi$ work function then only will a photocurrent be established

Useful Eq

$$E = \frac{hc}{\lambda}$$

Givens

$h = 6.62607015 \times 10^{-34}$ J/s

$c = 3.00 \times 10^{8}$ m/s

$\lambda = 6.94 \times 10^{-7}$ m

$$\text{Energy of photon} = \frac{6.62607015 \times 10^{-34} \cdot 3.00 \times 10^{8}}{6.94 \times 10^{-7}}$$

$E = 2.86429545 \times 10^{-19}$ J

↓ converted to eV

$E \approx 1.788$ eV

Thus, since $E > \phi$, $1.788\text{eV} > 1.3\text{eV}$, current will flow in the circuit

3.1 (a) **10 / 10**

✓ **- 0 pts** **Correct**

- 3 pts Error computing photon energy.

- 5 pts Incorrect reasoning or conclusion.

- 10 pts No submission.

- 3 pts Minor error.

You must show your work to receive credit. An answer written down with no work will receive no credit.

(b): 10 pts

One potential problem with your tripwire is that the laser is in the visible spectrum, and a potential burgler might see and avoid it. Will current still flow if you replace the ruby laser with an argon laser, with wavelength 1090 nm (infrared), and double the intensity to 2 W/m? Why or why not?

Given that the big conclusion of the photoelectric effect experiment was that increasing or decreasing intensity doesn't affect the ejection of electrons, we find the following

Useful Eq

$$E = \frac{hc}{\lambda}$$

Givens

$h = 6.62607015 \times 10^{-34} \frac{J}{s}$

$c = 3.00 \times 10^{8} \frac{m}{s}$

$\lambda = 1.09 \times 10^{-6} m$

$$E = \frac{6.62607015 \times 10^{-34} \cdot 3.00 \times 10^{8}}{1.09 \times 10^{-6}}$$

$$E = 1.82368903 \times 10^{-19}$$

↓ converted to eV

$$E \approx 1.138 \text{ eV}$$

Thus, since $E < \phi$, $1.138 \text{ eV} < 1.3 \text{ eV}$, current will NOT flow in the circuit

3.2 (b) **10 / 10**

✓ **- 0 pts** Correct

- 3 pts Error computing the work function.

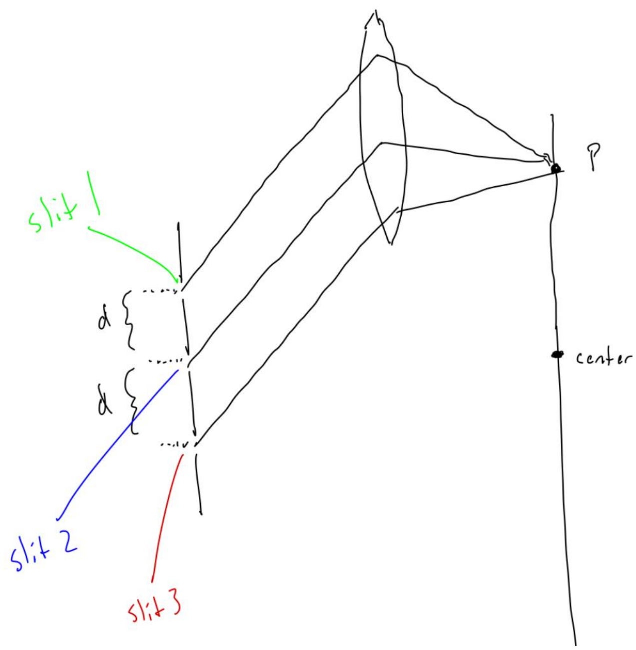
- 5 pts Incorrect reasoning or conclusion.

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 4

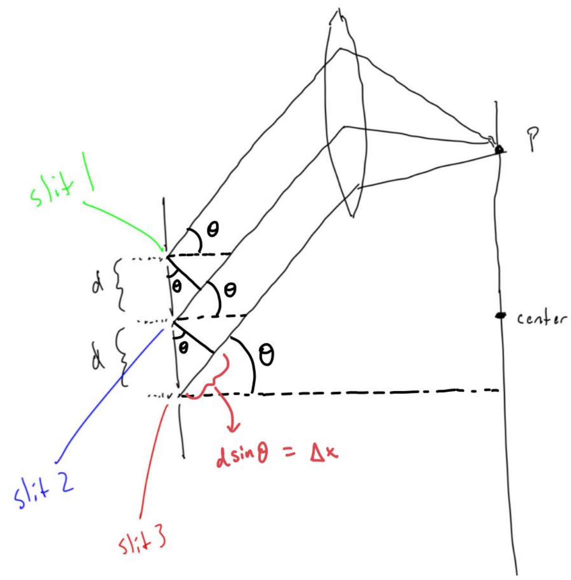
20 points

Consider the following image depicting three thin slits separated by some distance d:



Show that if spot P on the screen is a spot of constructive interference for light of wavelength λ coming through slits 1 and 2, then P is also a spot of constructive interference for light of wavelength λ coming through slits 2 and 3. You may assume that the screen is sufficiently distant that the light rays arriving at P from each slit emerge from the slits parallel to each other. The lens is only there to account for the fact that this page doesn't have infinite horizontal extent.

You must show your work to receive credit. An answer written down with no work will receive no credit.



- Since the screen is a large distance away, the angle θ is ≈ the same for all paths
- thus, the path diff $\Delta x = d\sin\theta$ using trig
- for there to be constructive interference at P Δx must be an integer multiple of the wavelength
  ↳ $\Delta x = m\lambda = d\sin\theta$, $m = 0, 1, 2, 3, \ldots$

→ Since the angle θ is ≈ the same for all paths, $r_2 - r_1 = r_3 - r_2 = d\sin\theta = m\lambda$, which is the condition for constructive interference

→ Given these facts, between slits 1 and 2, there is a path difference of Δx leading to point P for light of wavelength λ and thus since there is a path diff of Δx between slits 2 and 3, then P is also a spot of constructive interference for light with wavelength λ

4 Problem 4 **20 / 20**

- ✓ **- 0 pts** Correct
- **- 5 pts** path difference is integer number of wavelengths
- **- 5 pts** path difference between 1&2 is equal to path difference between 2&3

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 5

Short answer

(a): 7 pts

Both DC and AC circuits dissipate energy through resistors, known as "i^2R loss". AC circuits have an additional mode of energy loss, due to the oscillations of the charge carriers in the circuit. Describe why energy is lost in an AC circuit this way.

Energy is lost in an AC circuit this way since the voltage oscillates and thus the current oscillates. This oscillating current changes sinusoidally causing the power to be delivered and extracted. Given that for an arbitrary circuit $v = V\cos(\omega t + \phi)$, power $= iv = I\cos(\omega t) \cdot v\cos(\omega t + \phi)$. The average power is then $P_{av} = \frac{1}{2} IV \cos\phi \Rightarrow I_{rms} V_{rms} \cos\phi$. This phase angle term $\cos\phi$ if maximized, can minimize the dissapative loss i^2R, but if not maximized can lead to additional energy loss because for a given potential diff, a large current is needed to supply a given amount of power. This results in large energy losses. For DC circuits, the power is only determined by $P = IV$, so the energy loss is minimized already. Energy is also lost through the fact that the changing current produces a changing magnetic field, where energy is lost through heat.

5.1 (a) **4 / 7**

- **- 0 pts** Correct
- **- 5 pts** No loss from inductor and/or capacitor
- **- 5 pts** This doesn't describe energy loss
- ✓ **- 3 pts You have only described i2R loss**

You must show your work to receive credit. An answer written down with no work will receive no credit.

(b): 7 pts

A fiber optic cable operates under the principle of total internal reflection. Suppose you've got a fiber optic cable with index of refraction $n_c = 1.2$ used to transmit a beam of light between two elements, that you use in your lab space filled with air (index of refraction $n_a = 1$). If you were to take your device and place it in water (index of refraction $n_w = 1.33$) would the fiber optic cable still work? Why or why not?

Total internal reflection occurs when $n_a > n_b$ where n_a = the index of refraction of the fiber optic cable and n_b = the index of refraction outside the cable.

In both cases, $n_a = n_c$, or the index of refraction of the fiber optic cable. In the first case, $n_b = n_a$ or the index of refraction of the air. Thus, Total Internal Reflection (TIR) occurs since $n_c > n_a$, $(1.2 > 1)$. However, in the second case, $n_b = n_w$ and thus $n_c < n_w$ since $(1.2 < 1.33)$, thus TIR cannot occur. This is further verified by the fact that given the equation for the minimum incident angle θ_{crit} at which TIR occurs:

$$\frac{n_a}{n_b} \sin \theta_{crit} = 1$$

$$\theta_{crit} = \sin^{-1}\left(\frac{n_b}{n_a}\right)$$

Here we see that if $n_b = n_w$ and $n_a = n_c$, we have

$\theta_{crit} = \sin^{-1}\left(\frac{1.33}{1.2}\right)$, and $\sin^{-1}\left(\frac{1.33}{1.2}\right)$ does not exist since the max value $\sin\theta$ can take on is 1, where $\frac{1.33}{1.2} = 1.1$.

Thus, the fiber optic cable would <u>not</u> work since TIR could not occur

5.2 (b) **7 / 7**

✓ **- 0 pts** Correct

- 5 pts Incorrect reasoning

You must show your work to receive credit. An answer written down with no work will receive no credit.

(c): 6 pts

A transformer is a device used to step up or step down the voltage in a circuit, and operates on the principle of mutual induction. We discussed transformers in AC circuits. Would a transformer built in the same way also work in a DC circuit to step up or step down the voltage?

A transformer built with a DC circuit would not work the same way as with an AC circuit. This is because with AC circuits, the transformer takes advantage of the fact that changing voltage produces changing magnetic field and changing magnetic flux. Changing magnetix flux creates an induced current in the second coil. This induced current can be increased or decreased (step up or down) in an AC transformer by increasing or decreasing the amount of turns in the secondary coil. In a DC tranformer, since the voltage is not changing, there is no changing magnetic field / flux and there is no induced current in the secondary coil. Thus, since the DC transformer would be built in the same way as the AC transformer, increasing or decreasing the number of turns in the secondary coil would have no effect since there is no induced current to begin with and thus a transformer built like an AC transformer but in a DC circuit would not work to step up or down the voltage

5.3 (c) **6 / 6**

- ✓ **- 0 pts** Correct
- **- 5 pts** Inductor in dc circuit is just wire. You haven't really explained why the transformer doesn't work.
- **- 3 pts** Magnetic flux is still produced.