Physics 1C: Midterm 1

There are 170 points on the exam, the exam is 10 pages long (including the cover and formula pages), and you have 100 minutes. The exam is closed book and closed notes. The use of any form of electronics is prohibited, except for a basic scientific calculator. To receive full credit, show all your work and reasoning. No credit will be given for answers that simply "appear." If you need extra space, use the backside of the page with a note to help the grader see that the work is continued elsewhere.

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Problem	*Your Score*	*Max Score*
1	30	30
2	29	30
3	20	20
4	29	50
5	7 → 9 EM	40
Total	115 → 117 EM	170

Fundamental Constants

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \quad \& \quad \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$

Kinematics with Constant Acceleration ($s = x$ or y) & Centripetal Acceleration

$$s(t) = s_0 + v_{0s}t + \frac{1}{2}a_s t^2$$

$$v_s(t) = v_{0s} + a_s t$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Electric and Magnetic Force

$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad \text{or} \quad \vec{F}_B = \int_c I d\vec{\ell} \times \vec{B}$$

Here, $d\vec{\ell}$ is an infinitesimal displacement in the direction of the conventional current along the wire defined by curve c.

Magnetic Torque

$$\vec{\tau}_{\text{dip}} = \vec{\mu} \times \vec{B} \quad \text{with} \quad \vec{\mu} = \int \hat{n} A dI$$

$$U_{\text{dip}} = -\vec{\mu} \cdot \vec{B}$$

Gauss's Law

$$\oint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$$

$$\oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

Biot-Savart Law

$$\vec{B} = \frac{\mu_0}{4\pi} I \int_c \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

Here, $\hat{r}$ is a unit vector pointing from $d\vec{\ell}$ to the observation point (with r the distance between these points).

Ampere-Maxwell Law

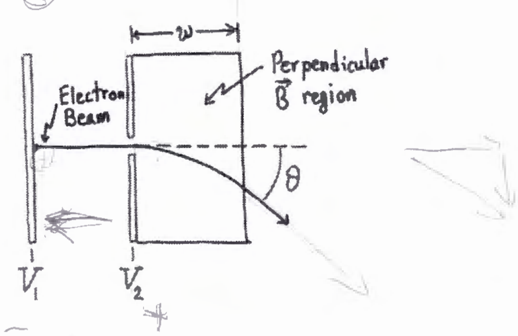
$$\oint_{\partial S} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot d\vec{A} \quad \text{with} \quad I_{\text{encl}} = \int_S \vec{J} \cdot d\vec{A}$$

Faraday's Law and Motional EMF

$$\mathcal{E} = \oint_{\partial S} \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$$

$$\mathcal{E}_{\text{mot}} = \int_c \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}$$

1. In the figure below, an electron beam in a cathode-ray tube is accelerated through a potential difference $\Delta V \equiv V_2 - V_1 > 0$. The beam then passes into a region—of width, w—where there is a perpendicular, constant magnetic field that deflects the electrons. It is found that the electrons are deflected by an angle, θ, upon reaching the edge of the space in which the field is defined. The mass of an electron is m_e and its charge magnitude is e.



(a) Why must we have $V_2 > V_1$? **[3]**

You want to accelerate electrons from V_1 to V_2 passing through the small slit. By making V_2 more positive and V_1 more negative, the $\vec{E}$ will have direction $\leftarrow$, accelerating the e^- from left to right.

3

(b) What must be the direction of the magnetic field in order to cause the electron beam to deflect in the manner shown in the figure above? Briefly explain. **[3]**

According to the right hand ruly the direction of $\vec{B}$ must be into the page ($\otimes$). You can also do $\vec{F}_B = q\vec{v}\times\vec{B}$, but q is negative.

3

(c) Provide a detailed description of why the magnetic field cannot change the speed of this electron beam. **[6]**

According to the formula, $\vec{F}_B = q\vec{v}\times\vec{B}$, $\vec{F}_B$ will always be perpendicular to both $\vec{v}$ and $\vec{B}$. Thus the acceleration will also be $\perp$ to $\vec{v}$. In this case, you have no tangential acceleration, and the acceleration will only change the direction for $\vec{v}$. Since the magnitude of $\vec{v}$ cannot be change, speed can not be changed by $\vec{B}$.

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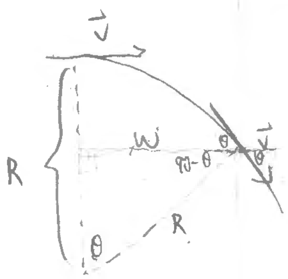
(d) In terms of the given quantities, determine the magnitude of the magnetic field. **[18]**

$$\text{Energy} = qV = e\Delta V = KE = \tfrac{1}{2} m_e v^2$$

$$e\Delta V = \tfrac{1}{2} m_e v^2 \rightarrow \frac{2e\Delta V}{m_e} = v^2 \rightarrow v = \sqrt{\frac{2e\Delta V}{m_e}}$$

$$\vec{F}_B = q\vec{v}\times\vec{B} \rightarrow \|\vec{F}_B\| = \|q\vec{v}\vec{B}\| = eB\sqrt{\frac{2e\Delta V}{m_e}}$$

↳ centripetal force



$$F = \frac{mv^2}{R} \rightarrow FR = mv^2$$

$$W = R\sin\theta \rightarrow R = \frac{W}{\sin\theta}$$

$$eBR\sqrt{\frac{2e\Delta V}{m_e}} = \cancel{m_e}\left(\frac{2e\Delta V}{\cancel{m_e}}\right)$$

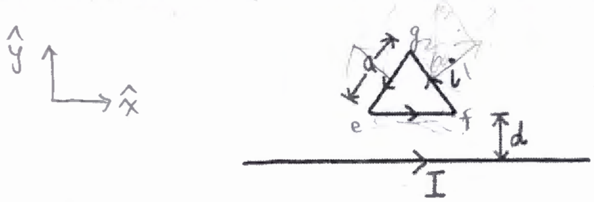
$$B = \frac{2e\Delta V}{eR\sqrt{\frac{2e\Delta V}{m_e}}} = \frac{2\cancel{e}\Delta V}{\cancel{e}\frac{W}{\sin\theta}\sqrt{\frac{2e\Delta V}{m_e}}}$$

$$\boxed{B = \frac{2\Delta V}{\frac{W}{\sin\theta}\sqrt{\frac{2e\Delta V}{m_e}}}}$$

18

29

2. An equilateral, triangular loop, of side-length a, carries a current, i, that is provided by its own power supply (not shown). The loop is placed a distance, d, away from a very long, straight wire that is carrying a current, I, as in the figure below. Determine the force on the loop. **[30]**

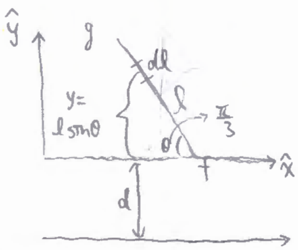


(e, f, and g are vertices of the triangle)

for $\overline{ef}$ segment, two parallel wire, $\vec{B} = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi d}$

$\vec{F_B} = \int I d\vec{l} \times \vec{B} = \boxed{-ia \frac{\mu_0 I}{2\pi d} \hat{y}}$

for $\overline{gf}$ segment, $\vec{B} = \frac{\mu_0 I}{2\pi r}$



$\vec{F_B} = \int_0^a i\, d\vec{l} \times \vec{B} = \int_0^a i\, d\vec{l} \times \frac{\mu_0 I}{2\pi(l\sin\theta + d)}$ $\quad\left(\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right)$

$\|\vec{F_B}\| = \int_0^a \frac{i\mu_0 I}{2\pi(\frac{\sqrt{3}}{2}l + d)} dl$

$= \frac{i\mu_0 I}{2\pi} \int_0^a \frac{1}{\frac{\sqrt{3}}{2}l + d} dl = \frac{i\mu_0 I}{2\pi} \left(\frac{2}{\sqrt{3}} \cdot \ln\left(\frac{\sqrt{3}}{2}l + d\right)\right)\Big|_0^a$

$= \boxed{\frac{i\mu_0 I}{\pi\sqrt{3}} \left(\ln\left(\frac{\sqrt{3}}{2}a + d\right) - \ln(d)\right)}$ $\quad$ (this has $+\hat{y}$ direction)

for $\overline{ge}$ segment, the result will be the same as $\overline{gf}$ segment

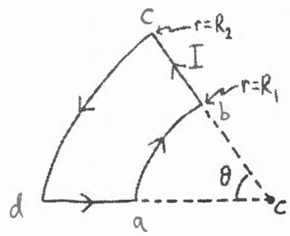
-1

Thus, the net force is $\left(\cancel{2}\frac{i\mu_0 I}{\pi\sqrt{3}} \left(\ln\left(\frac{\sqrt{3}}{2}a + d\right) - \ln(d)\right) - ia\frac{\mu_0 I}{2\pi d}\right)\hat{y}$

direction

20

3. Find the magnetic field (magnitude and direction) at the center, C, of the annular arc shown in the figure below, which is arched by an angle, θ, has inner and outer radii, R_1 and R_2, respectively, and carries a current, I. **[20]**



(a, b, c, d are vertices for explanation purposes)

for $\overline{da}$ and $\overline{bc}$ segments,

since $\vec{B} = \frac{\mu_0 I}{4\pi}\int_c \frac{d\vec{l} \times \hat{r}}{r^2}$ and $d\vec{l}$ in both cases are parallel to $\hat{r}$, $\vec{B}$ are zero

for $\widehat{ab}$ segment

the direction is into the page ($\otimes$) according to right hand rule

$d\vec{l}$ on the arc will equal to $R_1 d\theta$

$r = R_1$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}}{r^2} \text{ (ignore dir. for now)} = \frac{\mu_0 I}{4\pi}\int \frac{R_1 d\theta}{(R_1)^2} = \frac{\mu_0 I}{4\pi R_1}\int_0^\theta d\theta = \frac{\mu_0 I \theta}{4\pi R_1}$$

for $\widehat{cd}$ segment

the direction is out of the page ($\odot$) according to right hand rule

follow similar argument/procedure from $\widehat{ab}$ segment

$$\|\vec{B}\| = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}}{r^2} = \frac{\mu_0 I}{4\pi}\int \frac{R_2 d\theta}{(R_2)^2} = \frac{\mu_0 I}{4\pi R_2}\int_0^\theta d\theta = \frac{\mu_0 I \theta}{4\pi R_2}$$

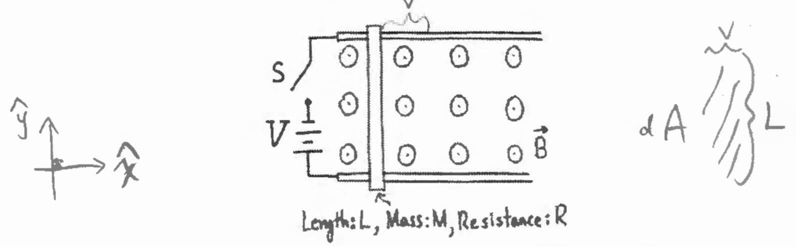
If we let $\hat{z}$ be out of the page ($\odot$)

$$\vec{B} = \left(\frac{\mu_0 I\theta}{4\pi R_2} - \frac{\mu_0 I\theta}{4\pi R_1}\right)\hat{z} = \hat{z}\,\frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$$\boxed{\vec{B} = \hat{z}\,\frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)}$$

✓

4. A bar of length, L, is free to slide without friction on horizontal rails, as shown in the figure below. There is a uniform magnetic field, $\vec{B}$, directed out of the plane of the figure. At one end of the rails there is a battery—with voltage, V—and a switch, S. The bar has mass, M, and resistance, R. Ignore the resistance of the rails and assume the battery is ideal (i.e., has no internal resistance). The switch is closed at time, $t = 0$, with the bar initially at rest.



(a) In terms of the given quantities, use the magnetic force law, Faraday's Law, and Newton's 2nd Law to find a first-order differential equation for the speed of the bar as a function of the time, t. **[24]**

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The bar will have emf due to the battery

Also emf due to Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{BA}{dt} = -BLv$$

-J voltage ≠ velocity!

Total emf in bar is $\mathcal{E} = V - BLv$; current is $I = \frac{V-BLv}{R}$

$$\vec{F_B} = \int_c I\, d\vec{l} \times \vec{B}$$

$$\|\vec{F_B}\| = ILB = \left(\frac{V-BLv}{R}\right)LB \quad (+\hat{x} \text{ direction})$$

$$\text{acceleration} = \left(\frac{V-BLv}{MR}\right)LB \rightarrow \frac{dv}{dt} = \left(\frac{V-BLv}{MR}\right)LB = v(1-BL)\left(\frac{LB}{MR}\right)$$

$$\rightarrow \frac{dv}{v} = (1-BL)\left(\frac{LB}{MR}\right)dt$$

$$\rightarrow \int_0^v \frac{1}{v}dv = \int_0^t (1-BL)\left(\frac{LB}{MR}\right)dt \rightarrow \ln(v) = (1-BL)\left(\frac{LBt}{MR}\right)$$

$$v = e^{(1-BL)\left(\frac{LBt}{MR}\right)}$$

$$\boxed{v(t) = e^{(1-BL)\left(\frac{LBt}{MR}\right)}}$$

$$\boxed{\frac{dv}{dt} = v(1-BL)\left(\frac{LB}{MR}\right)}$$

for (b)

(b) Explicitly solve this first-order differential equation by separating variables to obtain an expression for the speed as a function of the time, t. Be sure to use the appropriate initial condition, as well as to express the result in terms of the given quantities. **[12]**

$$\frac{dV}{dt} = V(1-BL)\left(\frac{LB}{MR}\right)$$

$$\frac{dV}{V} = (1-BL)\left(\frac{LB}{MR}\right)dt$$

$$\int_0^V \frac{1}{V}\,dV = \int_0^t (1-BL)\left(\frac{LB}{MR}\right)dt$$

$$\ln(V) = (1-BL)\left(\frac{LBt}{MR}\right) + C_1$$

$$V(t) = e^{(1-BL)\left(\frac{LBt}{MR}\right)} + C_2$$

since $V(0)=0$

$$V(0) = e^0 + C_2$$

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$$C_2 = -1$$

$$V(t) = e^{(1-BL)\left(\frac{LBt}{MR}\right)} - 1$$

$$\boxed{V(t) = e^{(1-BL)\left(\frac{LBt}{MR}\right)} - 1}$$

(c) Qualitatively discuss how including each of the following neglected effects would alter your results above. (Only comment on the alteration presented by each, and only each, effect.)

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i. The kinetic-friction force between the bar and the rails. **[4]**

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Kinetic-friction force is constant for same material. This will make the net force equal to $\left(\frac{V-BLV}{MR}\right)LB - F_f$ (in the $+\hat{x}$ direction), $\frac{dV}{dt}$ will be $\left(\frac{V-BLV}{MR}\right)LB - F_f$ and the resulting $V(t)$ will be decrease by a constan C times t.

ii. The internal resistance of the battery. **[4]**

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For internal resistance r, output emf becomes $\mathcal{E} = V - Ir$. Total emf in bar is then $\mathcal{E}_{tot} = V - BLV - Ir$. Solving I gives $\frac{V-BLV}{R} - \frac{Ir}{R} = I \rightarrow \frac{V-BLV}{R} = I + \frac{Ir}{R} = I\left(1+\frac{r}{R}\right)$ $\rightarrow I = \frac{V-BLV}{R(1+\frac{r}{R})}$. This decrease the force by a constant and thus decrease $V(t)$ by a constant times t.

it does not depend on t

iii. The electrical resistance associated with the rails. **[6]**

4/6

(This is nasty...) This will depend on time (t) b/c as the bar moves to the right, there will be more and more rails in the circuit. Say the circuit has resistance per unit length r, then resistance in rails will be $2r\int_0^t V(t)\,dt$. This decrease the current by a function that's depend on t, so the resulting $V(t)$ will be decreased by a function that's depend on t^2.

5. The figure below shows the electric field inside a cylinder of radius, R (possibly set up by a closely-spaced, parallel-plate-capacitor configuration with circular plates of the same radius). The field strength is changing with time, t, as,

$$\vec{E}(t) = t(\lambda - \mu t)\,\hat{z}, \qquad t\lambda - \mu t^2$$

where μ and λ are constant parameters, with $\lambda > \mu > 0$, and $\hat{z}$ is taken into the page. The electric field outside the cylinder is always zero, and the field inside the cylinder was zero for $t < 0$. Specifically, *the picture shows the electric field profile very shortly after* $t = 0$ (i.e., at $t \gtrsim 0$).

$-\mu t^2 + t\lambda = 0$

$t(-\mu t + \lambda) = 0$

$-\mu t +$

(a) Using dimensional analysis, determine the SI units of the parameters μ and λ. **[3]**

$\vec{E}$ has unit V/m, thus, $(\lambda - \mu t)$ has units V/ms

So, λ has unit $\frac{V}{m \cdot s}$ and μ has unit $\frac{V}{m \cdot s^2}$

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(b) Accurately graph the electric field, E, as a function of time, $t \in (-\infty, \infty)$. **[7]**

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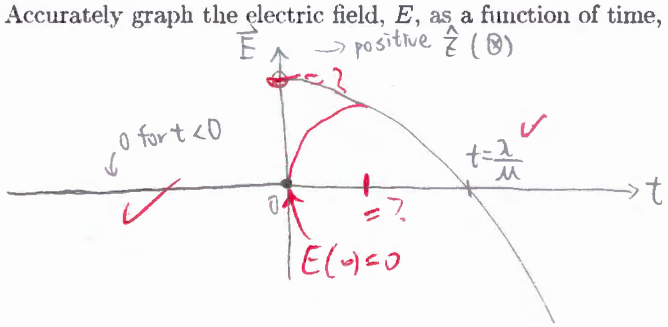

$-\mu t^2 + t\lambda = 0$

$t(-\mu t + \lambda) = 0$

$-\mu t + \lambda = 0$

$\lambda = \mu t$

$t = \frac{\lambda}{\mu}$

(c) Using the outline of the cylinders for each respective part below, draw the magnetic field lines (with appropriate arrowheads to show the field direction) inside and outside the cylinder. *Explain the reasoning behind your drawings in the remaining space below each drawing.* **[10]**

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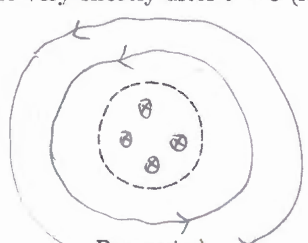
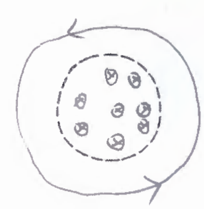
i. At a time very shortly after $t = 0$ (i.e., $t \gtrsim 0$). | ii. At a time much, much later than $t = 0$ (i.e., $t \gg 0$).



Reasoning:

The magnetic field outside is caused by the current attached to the plate. The dir. of I is out of page. The $\vec{B}$ inside is caused by change in $\vec{E}$. To balance the change in $\vec{E}$, $\vec{B}$ must be into the page.



Reasoning: Change in $\vec{E}$ is greater, thus greater $\vec{B}$ inducted. Current is lower, thus lower $\vec{B}$ outside.

(d) In terms of the given quantities, find expressions for the magnetic field strength for $r < R$ and $r \geq R$. **[20]**