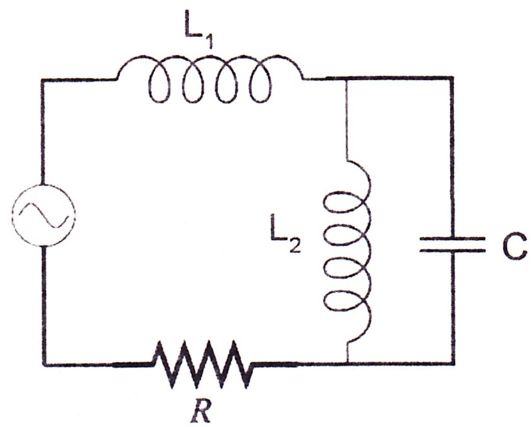
1. Consider the circuit below with an AC voltage source, $V = V_o \cos(\omega t)$



a. (8 pt) What is the total impedance of the circuit, magnitude and phase angle?

$$Z = R + i\omega L_1 + \frac{1}{\frac{1}{i\omega L_2} + i\omega C}$$

$$= R + i\left[\omega L_1 + \frac{1}{\frac{1}{\omega L_2} - \omega C}\right] = R + i\left[\omega L_1 + \frac{\omega L_2}{1-\omega^2 L_2 C}\right]$$

$$|Z| = \left\{R^2 + \left[\omega L_1 + \frac{\omega L_2}{1-\omega^2 L_2 C}\right]^2\right\}^{\frac{1}{2}}$$

$$\phi = \tan^{-1}\left[\frac{\omega L_1 + \frac{\omega L_2}{1-\omega^2 L_2 C}}{R}\right]$$

b. (8 pt) Give an expression for the current I(t) in the circuit.

$$I = \frac{V}{|Z| e^{i\phi}} = \frac{V_0 e^{i(\omega t - \phi)}}{|Z|}$$

Take Real part

$$I = \frac{V_0}{|Z|} \cos(\omega t - \phi)$$

c. (4 pt) What is the average power delivered by the voltage source?

$$P_{AV} = \frac{1}{2}\frac{V_0^2}{|Z|}\cos\phi = \frac{1}{2}\frac{V_0^2 \cos\phi}{\left\{R^2 + \left[\omega L_1 + \frac{\omega L_2}{1-\omega^2 L_2 C}\right]^2\right\}^{\frac{1}{2}}}$$

d. (7 pt) If there is a resonant frequency, what is it?

At resonance, Imaginary part $\Rightarrow 0$

$$\omega L_1 + \frac{\omega L_2}{1-\omega^2 L_2 C} = 0$$

$$\omega L_1 - \omega^3 L_1 L_2 C + \omega L_2 = 0$$

$$L_1 + L_2 = \omega^2 L_1 L_2 C$$

$$\frac{L_1 + L_2}{L_1 L_2 C} = \omega^2$$

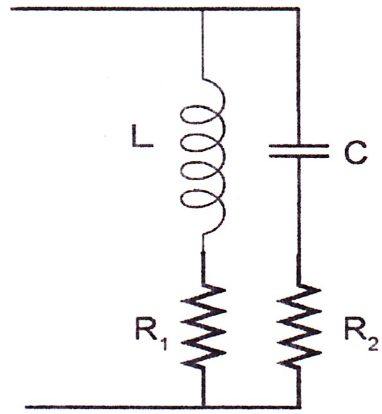
$$\omega = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C}} = \frac{1}{\sqrt{\frac{L_1 L_2 C}{L_1 + L_2}}}$$

Yes, there is a resonant freq

2a. (3 pts) The circuit below is an audio filter. The two wires connect to an audio amplifier. R_1 and R_2 are both 8 Ω speakers. One is a low frequency bass and one a higher frequency tweeter. Which resistor should be the bass speaker?

R_1 should be bass speaker

Inductor blocks High freq.



2b.(5pts) If C = 24 μF and L = 0.6 mH, at what frequency will there be equal current to both speakers?

Set impedances equal

$\omega L = \frac{1}{\omega C}$ $\omega^2 = \frac{1}{LC}$

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(24\times10^{-6})(0.6\times10^{-3})}}$

$2\pi f = \omega = \frac{1}{\sqrt{144\times10^{-10}}} = \frac{1}{12\times10^{-5}}$

$f = \frac{10^5}{24\pi} = \left(\frac{100}{24\pi}\right)$ KHz

2c. (5 pts) The electric field of an infinite plane wave can be written

$E_y(x,t) = E_0 \cos(kx + \omega t)$

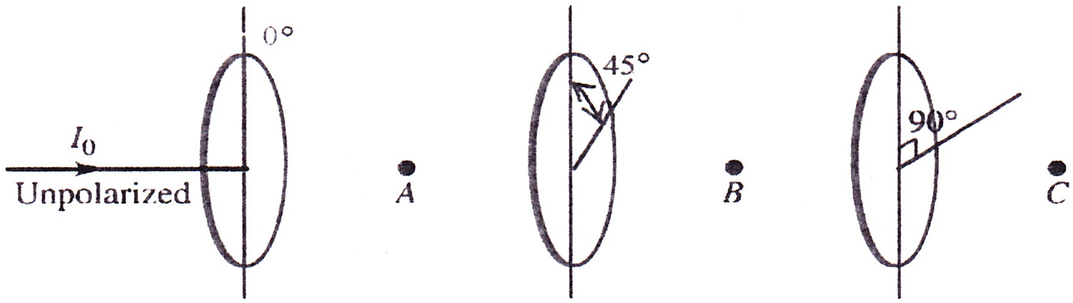
Write an expression for the corresponding magnetic field of the wave.

$B_z = -B_0 \cos(kx+\omega t) = -\frac{E_0}{c}\cos(kx+\omega t)$

use right handed coord system

$\hat{e}_x \times \hat{e}_y = \hat{e}_z$

Then E×B needs to be in −x direction



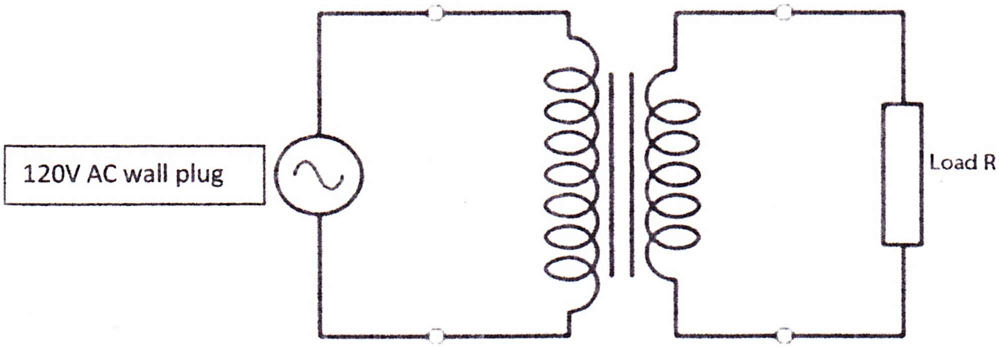
2d. A beam of unpolarized light of intensity I_0 passes through a series of three ideal polarizing filters with their polarizing directions turned to the angles shown above.

What is the light intensity (in terms of I_0) at points A,B.C?

A. (3 pt) $\frac{I_0}{2}$

B. (3 pt) $\frac{I_0}{2} \cos^2 45 = \frac{I_0}{2} \frac{2}{4} = \frac{I_0}{4}$

C. (3 pt) $\frac{I_0}{4} \cos^2 45 = \frac{I_0}{8}$



I hook up a neon sign transformer to a load with resistance R_{load} and plug the primary into a wall outlet. The transformer primary is rated 120 V and the secondary is rated 12,000 V.

3a. (2 pts). What is the ratio of number of windings in the secondary to the number of windings in the primary? $N_s/N_p =$ 100

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{12000}{120} = 100$$

3b. (3 pts) If I_{load} is 50 mA, how much power must the wallplug deliver?

$$P_{sec} = V_S I_S = 12\,kV\ 50\,mA = 600\,W$$

A 160 kW radio AM radio station broadcasts 1500 kHz into a hemisphere above the ground.

3c. (5pt) At 4 km from the station antenna, what is the intensity of the EM wave?

$$S = \frac{160\,kW}{2\pi r^2} = \frac{160\,kW}{2\pi\ 16\times10^6\,m^2} = \frac{1}{2\pi(100)} = \boxed{\frac{1}{200\pi}\ W/m^2}$$

3d. (5 pt) At 4 km from the antenna, what is the maximum electric field in the wave (from 3c.) ?

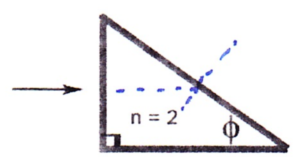
$$S = \frac{E_0^2}{2\mu_0 c} = \frac{1}{200\pi} \ W/m^2$$

$$E_0^2 = \frac{2\mu_0 c}{200\pi} = \frac{2 \cdot 4\pi \times 10^{-7} \cdot 3\times 10^8}{200\pi} = \frac{30 \cdot 4}{100} = \frac{120}{100}$$

$$E_0^2 = 1.2$$

$$\boxed{E_0 = \sqrt{1.2} \ V/m}$$

3e. (5pts) A ray of light in air is incident normally on the face of a prism with index of refraction n = 2.0. What is the largest angle ϕ for which there is total internal reflection in the prism?



angle of incidence $(90 - \phi)$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

set $\theta_2 = 90$

$\sin\theta_2 = 1$

$$2 \sin(90-\phi) = 1$$

$$\sin(90-\phi) = \cos\phi = \frac{1}{2}$$

$$\boxed{\phi = 60^\circ}$$

3f. (5 pts) If instead of air, the prism was surrounded by a fluid with n = 1.4, approximately what would the maximum angle ϕ be for total internal reflection?

$$\cos\phi = \frac{1.4}{2} = 0.7$$

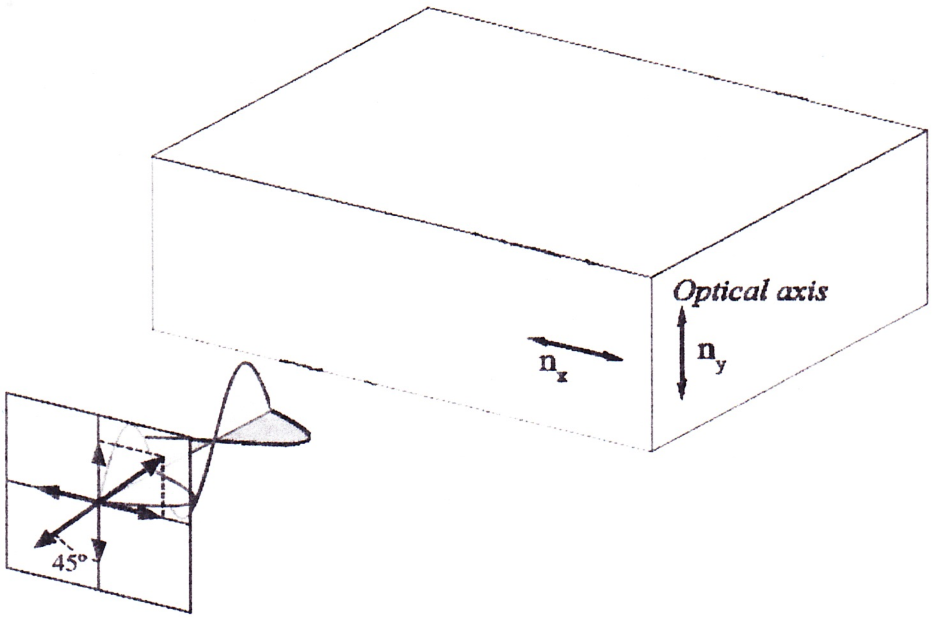
$$\boxed{\phi \approx 45^\circ}$$

4. Quarter wave plate.

An electric field in air E_p , polarized at 45 degrees in the x-y plane and moving in the –z direction can be written as

$$\vec{E}(z,t) = (\hat{x} + \hat{y})E_0 \cos(kz + \omega t)$$

Where $E_0 = E_p \cos 45 = E_p \sin 45$



The linearly polarized light is incident on a birefringent crystal. The birefringent crystal has two different index of refractions, n_x for the electric field aligned with the x-axis and n_y, the index for the electric field along the y-axis.

a. (10 pts) Find the crystal thickness z which will cause a $\pi/2$ (or quarter wave) phase difference for the two polarizations of light with a free space wavelength of λ_0.

$$\phi = kz = \frac{2\pi n}{\lambda_0} z$$

$$\Delta\phi = \frac{2\pi}{\lambda_0} \Delta n z = \frac{2\pi}{\lambda_0}(n_x - n_y) z \quad \text{set equal } \frac{\pi}{2}$$

$$\frac{\pi}{2} = \frac{2\pi}{\lambda_0}(n_x - n_y) z \qquad z = \frac{\pi \lambda_0}{2 \cdot 2\pi (n_x - n_y)} = \boxed{\frac{\lambda_0}{4(n_x - n_y)}}$$

b. (10 pts) (You don't need the correct answer from a. to continue).

Assume the crystal thickness is a ¼ wave plate for a free-space λ_0, shifting the x- and y- components by 90 degrees. Write an expression for the electric field after the light exits the crystal. Assume that $n_x > n_y$ in the crystal.

$n_x > n_y$ means more wavelengths in n_x direction so the x-component advances 90° ahead of y-component

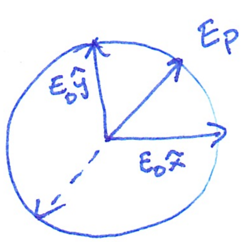
$$E(z,t) = \hat{x} E_o \cos(kz + \omega t) + \hat{y} E_o \sin(kz + \omega t)$$

This is circularly polarized light

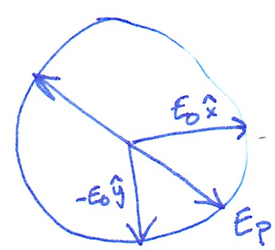
c. (6 pts) A 40 μm layer of cellophane tape acts like a ½ wave plate (180 degree phase difference) for red light passing through it. Show graphically by vector addition that the initial 45 degree linear polarization of the entering wave is rotated by 90 degrees by a half wave plate.

n_y direction delayed by π, turning E_y into $-E_y$

Entering wave



Leaving wave



Polarization rotated 90°