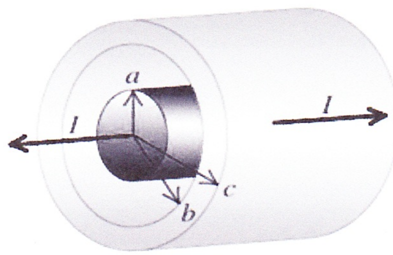


$$0 \le r \le a \qquad I_{en} = \frac{I r^2}{a^2}$$

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{enc} = \mu_0 \frac{I r^2}{a^2}$$

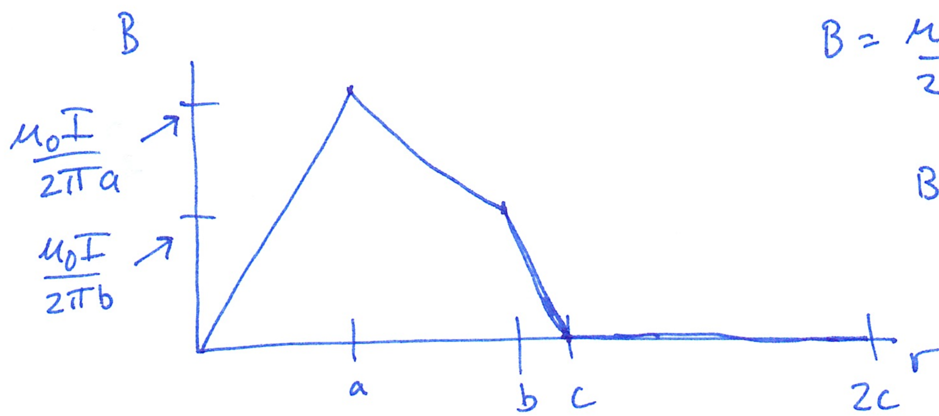
$$B\, 2\pi r = \mu_0 \frac{I r^2}{a^2}$$

$$B = \frac{\mu_0 I r}{2\pi a^2}$$

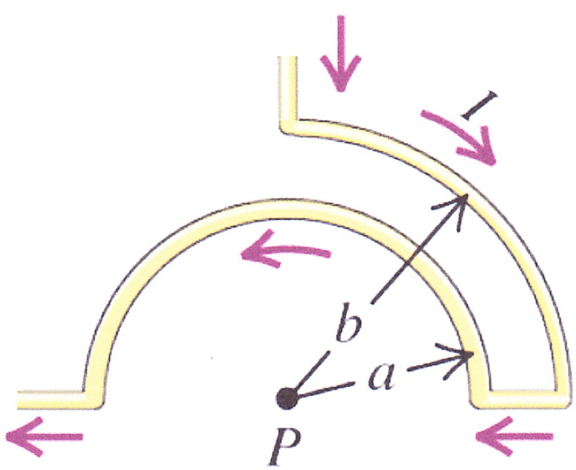
1a. (10 pts) Above is a coax cable carrying a uniform current I in the center conductor and a uniform return current I in the outer conductor. Draw a graph showing the Magnetic Field B(r) from r = 0 to 2c. Evaluate and label the max magnetic field and the field at a, b, and c.

$$B = \frac{\mu_0 I}{2\pi r} \qquad a \le r \le b$$

$$B = 0 \qquad r > c$$



1b. (10 pts) Use the Biot-Savart law to solve for the magnetic field at point P.



$$d\bar{B} = \frac{\mu_0 I}{4\pi} \frac{d\bar{l} \times \hat{r}}{r^2}$$

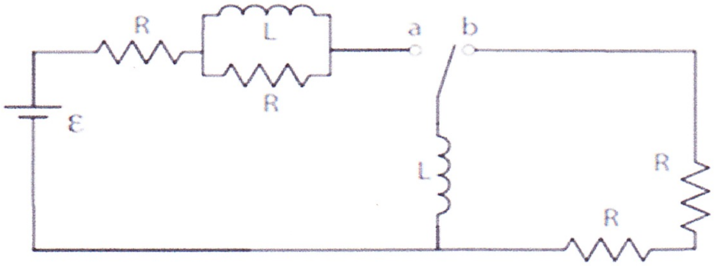
$$B = \frac{\mu_0 I}{4\pi a^2} \int_0^{\pi} a\, d\theta - \frac{\mu_0 I}{4\pi b^2} \int_0^{\pi/2} b\, d\theta$$

$$B = \frac{\mu_0 I}{4\pi} \left[\frac{\pi}{a} - \frac{\pi}{2b} \right]$$

$\bar{B}$ $\odot$ out of Page

$$B = \frac{\mu_0 I}{4} \left[\frac{1}{a} - \frac{1}{2b} \right]$$

Problem 2



Consider the circuit above.

2a. (5 pts) I throw the switch to position "a" and wait a long time. What is the current flowing from the battery?

$$I = \frac{\varepsilon}{R}$$

L's act like R = 0
Short circuit

2b. (5 pts) I then throw the switch to position "b" and reset time t = 0. Write the differential equation that describes the current I.

$$-L\frac{dI}{dt} - I\,2R = 0$$

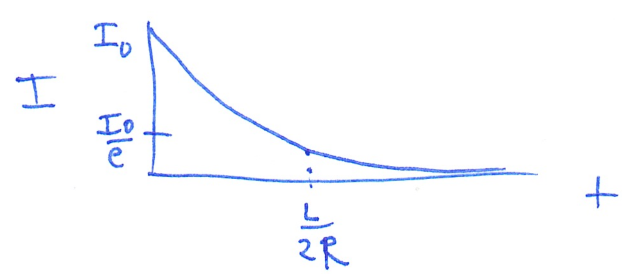
$$\frac{dI}{dt} = -\frac{2R}{L} I$$

2c. (5 pts) Solve the equation to give I(t) =

$$\frac{dI}{I} = -\frac{2R}{L}dt \qquad \ln I = -\frac{2R}{L}t$$

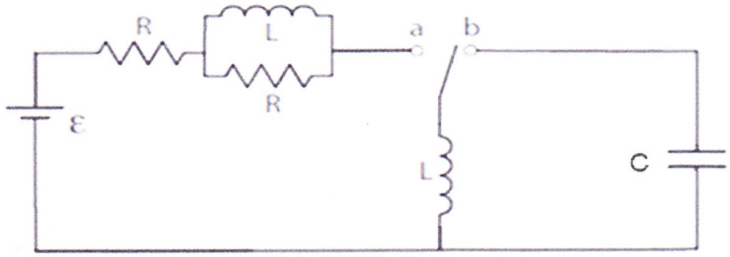
$$I = I_0 e^{-\frac{2R}{L}t}$$

2d. (5 pts) Plot the current I as a function of t, and mark any time constants on the graph.



$$\tau = \frac{L}{2R}$$

Now, before we throw the switch to "b", we replace the resistors R with capacitor C as shown in the diagram below. Again, after a long time with the switch in position "a", we connect to "b" at t=0 so only the inductor and capacitor are in the circuit.



2e. (5 points) Write down the differential equation for the charge Q on the capacitor.

$$-L\frac{dI}{dt} - \frac{q}{C} = 0 \qquad -L\frac{d^2Q}{dt^2} - \frac{Q}{C} = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{Q}{LC}$$

2f. (5 points) Solve for Q(t)

Assume $Q = Q_0 \sin \omega t$

Since at $t=0$ there is max current through L but no Q on C

$$\frac{d^2Q}{dt^2} = -\omega^2 Q_0 \sin \omega t = -\frac{Q \sin \omega t}{LC}$$

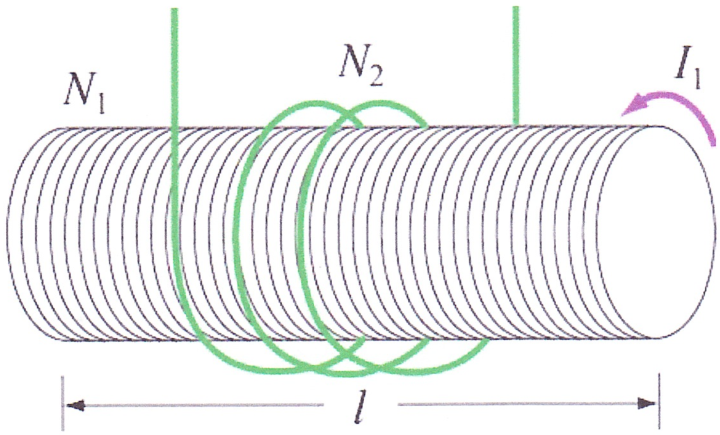
$$-\omega^2 = -\frac{1}{LC}$$

$$\omega^2 = \frac{1}{LC} \qquad Q = Q_0 \sin\left(\frac{1}{\sqrt{LC}}\right)t$$

To find Q_0 $\qquad \frac{1}{2}LI_0^2 = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q_0^2}{C}$

$$LI_0^2 = \frac{Q_0^2}{C} \qquad Q_0 = \sqrt{LC}\, I_0 = \sqrt{LC}\,\frac{\mathcal{E}}{R}$$

3a. (10 pts) Coil 1 is a long straight solenoid with N_1 windings and cross sectional area A.

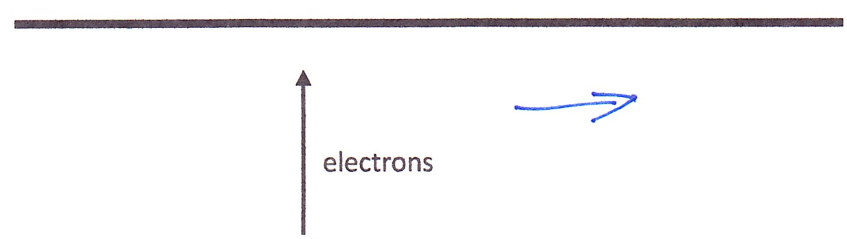


$$\phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

$$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

N_2 windings are wrapped along a short length of the solenoid. What is the mutual inductance M due to the current I_1 in the solenoid.

3b. (5pt) I ⟶



An electron beam approaches a long wire carrying current I to the right as shown. Which way is the electron beam deflected? Draw the direction on the figure.

3c. (5pt) An electron moves with velocity **v** = (4.0 **i** -6.0 **j**) x 10^4 m/s in a magnetic field **B** = (-0.80 **i** + 0.60 **j**) T.

$$F = q \bar{V} \times \bar{B}$$

What is the magnitude and direction of the force?

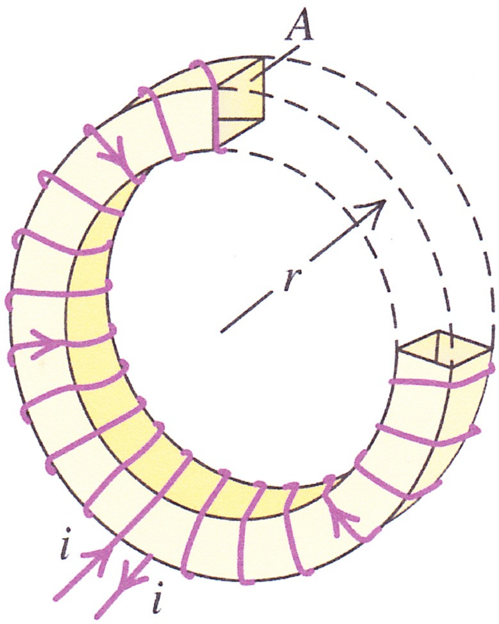
$$v \times B = \begin{vmatrix} i & j & K \\ 4 & -6 & \\ -.8 & .6 & \end{vmatrix} \times 10^4 = (2.4 \times 10^4 - 4.8 \times 10^4)\hat{k}$$

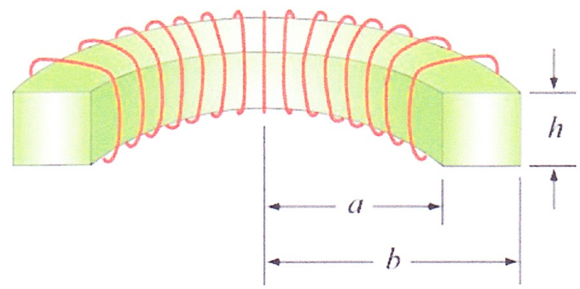
$$= -2.4 \times 10^4 \hat{k}$$

$$F = -e(-2.4 \times 10^4)\hat{k} = 1.6 \times 10^{-19}(2.4 \times 10^4)\hat{k}\ N$$

$$\boxed{F = 3.84 \times 10^{-15}\hat{k}\ N}$$

4. Consider the N toroidal windings shown below.





4a. (6 points) What is the magnetic field B(r) for a ≤ r ≤ b?

$\oint B \cdot dl = \mu_0 I_{en}$

$2\pi r B = \mu_0 I_{en}$

$B = \dfrac{\mu_0 I N}{2\pi r}$

4b. (6 points) What is the magnetic flux through each winding of the toroid?

$\Phi = \int B \cdot dA = \int_a^b \dfrac{\mu_0 N I}{2\pi r} h \, dr = \dfrac{\mu_0 N I h}{2\pi} \ln\left(\dfrac{b}{a}\right)$

4c. (6points) What is the self inductance L of the toroid?

$$L = \frac{N\Phi}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

4d. (6 points) What is the magnetic energy density u_B inside the toroid?

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{\mu_0 N^2 I^2}{8\pi^2 r^2}$$

4e. (6 points) Integrate the magnetic energy density over the volume to get the magnetic energy U_B stored in the toroid.

$$U_B = \int u_B \, dV = \int_a^b \frac{\mu_0 N^2 I^2}{8\pi^2 r^2} 2\pi r h \, dr$$

$$= \mu_0 \frac{N^2 I^2 h}{4\pi} \ln\left(\frac{b}{a}\right)$$

$$= \frac{1}{2} L I^2$$