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MIDTERM 1 – PHYSICS 1C – WINTER 2015

Name: Dominic Mortel

Student ID No.: 904 287 174

Signature: DMortel

Specify here if you want your exam returned privately:________

Each question is worth 25 points. The exam is closed book; no notes, calculators or cell phones. If necessary, use the back of the page. If you do use the back of the page, write OVER on the page whose backside you are using. Please make the organization of your answer as clear as possible.

1. Present results for units in terms of kg, m, s and Coulomb, ONLY. (i) What is the Equation for the force produced by a magnetic field on a moving charge? (4 pts.) What are the units of $\vec{B}$ in this expression? (3 pts.) (ii) What is the Equation in a steady state relating the magnetic field to the current density, $\vec{j}$? (3 pts.) What are the units of $\vec{j}$? (3 pts.) What are the units of μ_0, the constant employed in this Equation? (3 pts.) (iii) What is the general Equation relating the curl of the electric field to the magnetic field? (3 pts.) (iv) What is the Equation for the EMF around a loop? (3 pts.) What are the units of the EMF? (3 pts.)

i) $F = Q(\vec{V} \times \vec{B})$ 4

$\frac{kg\,\cancel{m}}{s^{\cancel{2}}} = C\left(\frac{\cancel{m}}{\cancel{s}}\right) B$

$B = \frac{kg}{C \cdot s}$ 3

ii) $\nabla \times \vec{B} = \mu_0 \vec{J}$ 3

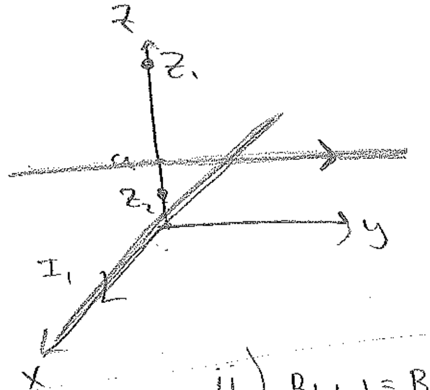
$J = \frac{I}{A} = \frac{C}{s \cdot m^2}$ 3

iii) $\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$ 3

iv) $\mathcal{E} = \frac{-\partial \Phi}{\partial t} = \frac{-\partial}{\partial t} \oint \vec{B} \cdot \partial \vec{A}$ 3

$\frac{\partial}{\partial t}\left(\frac{kg}{C \cdot s} m^2\right) = \frac{kg\, m^2}{C \cdot s^2}$ 3

2. Consider a situation where there are two infinite long wires. Wire #1 lies along the X-axis and current I flows in the $+\hat{x}$ direction. Wire #2 lines parallel to the Y axis and passes through the point (0,0,a); assume that $a > 0$. In this wire, current I flows in the $+\hat{y}$ direction. Consider a point along the Z axis (0,0,z) and take $z > 0$. (i) What is the magnetic field (both direction and magnitude) at (0,0,z) produced by wire #1? (5 pts.) (ii) If $z > a$, what is the magnetic field (both direction and magnitude) at (0,0, z) (5 pts.) (iii) If $0 < z < a$, what is the magnetic field (direction and magnitude) at (0,0,z) produced by wire #2? (5 pts.) (iv) If $z > a$, compute the vector acceleration of a particle located at $(0, 0, z)$, with mass m and charge Q moves with velocity $\vec{v} = V_0\,\hat{z}$ (10 pts.)



i) $\oint B \cdot dl = \mu_0 I$

$B \cdot 2\pi z = \mu_0 I$

$B = \dfrac{-\mu_0 I}{2\pi z}\,\hat{y}$ 5

ii) $B_{total} = B_{wire\,1} + B_{wire\,2}$

$B_{wire\,1} = \dfrac{-\mu_0 I}{2\pi z}\,\hat{y}$

$B_{wire\,2} = \oint B \cdot dl = \mu_0 I$

$B\,2\pi(z-a) = \mu_0 I$

$B = \dfrac{\mu_0 I}{2\pi(z-a)}\,\hat{x}$

5

$B_{total} = \dfrac{\mu_0 I}{2\pi}\left(\dfrac{1}{z-a}\hat{x} - \dfrac{1}{z}\hat{y}\right)$

iii) $\oint B \cdot dl = \mu_0 I$

$B \cdot 2\pi(a-z) = \mu_0 I$ 5

$B = \dfrac{-\mu_0 I}{2\pi(a-z)}\,\hat{x}$

iv) $\vec{F} = m\vec{a} = Q(\vec{v} \times \vec{B})$ 10

$\vec{a} = \dfrac{Q}{m}(\vec{v} \times \vec{B})$

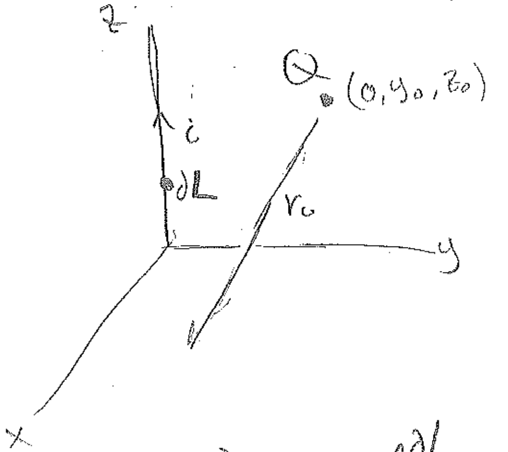
$\vec{v} \times \vec{B} \begin{vmatrix} i & j & k \\ 0 & 0 & V_0 \\ \dfrac{\mu_0 I}{2\pi(z-a)} & -\dfrac{\mu_0 I}{2\pi z} & 0 \end{vmatrix}$

$\vec{v} \times \vec{B} = \dfrac{V_0 \mu_0 I}{2\pi z}\hat{x} + \dfrac{V_0 \mu_0 I}{2\pi(z-a)}\hat{y}$

$\vec{a} = \dfrac{Q}{m}\dfrac{V_0 \mu_0 I}{2\pi}\left(\dfrac{1}{z}\hat{x} + \dfrac{1}{z-a}\hat{y}\right)$

$\oint B \cdot dl = \mu_0 I$

3. A straight section of wire of length dL located at the origin carries a current i in the $+\hat{z}$ direction. Consider a charge, Q, located at $(0, y_0, z_0)$ moving with velocity $\vec{v} = v_0\,\hat{x}$. (i) What is the magnetic field at the charge caused by the wire? (20 pts.) (ii) What is the vector force on the charge? (5 pts.)



$$d\vec{B} = \frac{\mu_0 I}{4\pi}\,\frac{d\vec{l} \times \vec{r}}{r^3} \qquad \vec{r} = \langle 0, y_0, z_0 - dl$$

$$d\vec{l} \times \vec{r} = \begin{vmatrix} i & j & k \\ 0 & 0 & dl \\ 0 & y_0 & z_0 - dl \end{vmatrix} \qquad |\vec{r}| = \sqrt{y_0^2 + (z_0 - dl}$$

$$d\vec{l} \times \vec{r} = -y_0\, dl\, \hat{x}$$

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$$d\vec{B} = \frac{\mu_0 I}{4\pi}\int_0^{dL} \frac{-y_0\, dl}{\left(y_0^2 + (z_0 - dl)^2\right)^{3/2}}\,\hat{x} = B = -\frac{\mu_0 I\, \cancel{y_0}}{4\pi\, y_0^2}\int_0^{dL} \frac{dl}{\left(1 + \left(\frac{z_0 - dl}{y_0}\right)^2\right)^{3/2}}$$

$$= -\frac{\mu_0 I}{4\pi y_0^2}\int_0^{dL} dl\left(1 - \frac{3}{2}\left(\frac{z_0 - dl}{y_0}\right)^2\right)\hat{x} \qquad \frac{1}{(1+f)^{3/2}} \approx 1 - \frac{3}{2} f$$

ii)

$F = Q(\vec{v} \times \vec{B})$

$\vec{B}$ = magnetic field from first question

$$\vec{V} \times \vec{B} = \begin{vmatrix} i & j & k \\ V_0 & 0 & 0 \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{v} \times \vec{B} = -V_0 B_z \hat{y} + V_0 B_y \hat{z}$$

$$\vec{F} = Q V_0(-B_z \hat{y} + B_y \hat{z})$$

no $\hat{X}$ component so $F = 0$

5

$$= -\frac{\mu_0 I}{4\pi y_0^2}\int_0^{dL} dl - \frac{3}{2} dl\left(\frac{z_0}{y_0} - \frac{dl}{y_0}\right)^2$$

$$\vec{B} = -\frac{\mu_0 I}{4\pi y_0^2}\int_0^{dL} dl - \frac{3}{2}dl\left(\frac{z_0^2}{y_0^2} - \frac{2 z_0\, dl}{y_0^2} + \frac{dl^2}{y_0^2}\right)\hat{x}$$

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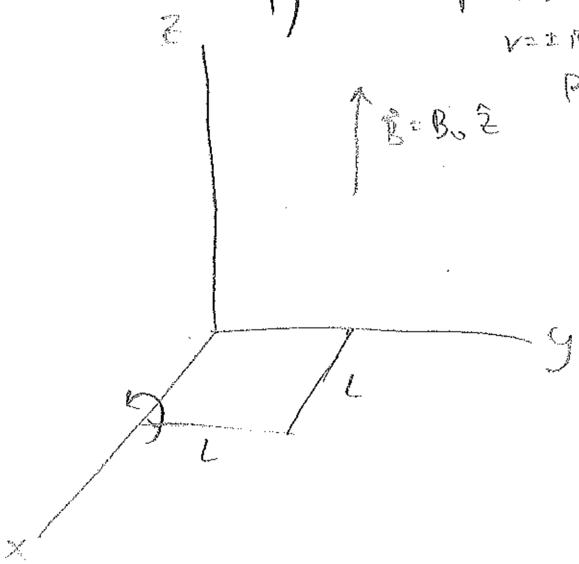
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B\, 2\pi y_0 = \mu_0 I$$

$$B = -\frac{\mu_0 I}{2\pi y_0}\,\hat{X}$$

4. Consider a square loop of side, L, rotating in a uniform magnetic field, $\vec{B} = B_0\,\hat{z}$. Assume that the loop rotates around the X axis with period P, that it initially lies flat in the X, Y plane and rotates upwards above the plane. (i) If the loop is connected to a resistor of value, R, what is the rate at which energy is dissipated in this resistor as a function of time? (20 pts.) (ii) What is the time averaged rate of energy dissipation? (5 pts.)

i) $P = IV$ $\quad V = IR$ $\quad P = \frac{V^2}{R}$ $\qquad \frac{\text{energy}}{\text{time}} = \text{power}$ $\qquad W = \frac{2\pi}{P}$



$$\mathcal{E} = \frac{-\partial \Phi}{\partial t} \qquad \Phi = \oint B \cdot \partial \vec{A}$$

$$\partial A = \partial A \hat{A} \quad \hat{A} = \langle 0, \cos(wt), \sin(wt) \rangle$$

only care about component z.

$$\Phi = \hat{B}_0 L^2 \sin(wt)\,\hat{z}$$

$$\frac{-\partial \Phi}{\partial t} = -wB_0 L^2 \cos(wt)\hat{z} = \mathcal{E}$$

$$P(t) = \frac{B_0^2 w^2 L^4 \cos^2(wt)}{R} \quad \hat{z}$$

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ii) $$\frac{\partial P}{\partial t} = \frac{-2B_0^2 w^3 L^4 \cos(wt)\sin(wt)}{R}$$

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