Problem 1. (28 points)

A converging lens with focal length f is placed a distance f to the left of a diverging lens whose focal length has the same magnitude. An object is place a distance $(3/2)f$ to the left of the converging lens.

a. **(8 points)** If the diverging lens weren't there, where would the image of the object form? Would it be upright or inverted? Would it be real or virtual? What would be its lateral magnification? Draw a ray diagram for this system.

b. **(10 points)** Now consider the full scenario including the diverging lens. Determine the location of the final image for this double lens system, and draw a ray diagram showing what happens to all three principal rays emanating from the tip of the object. Make sure to clearly indicate where the rays or their extensions converge to form the final image. That is the total lateral magnification for this system? Is the final image real or virtual? Is it upright or inverted?

c. **(10 points)** Since we like lenses so much, suppose that we now insert a third converging lens with focal length f at a distance f to the right of the diverging lens. Where will the final image form now? Will it be real or virtual? Will it be upright or inverted? Draw a ray diagram that shows what happens to all three principal rays in this triple lens system.

8/8

(a) we know:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

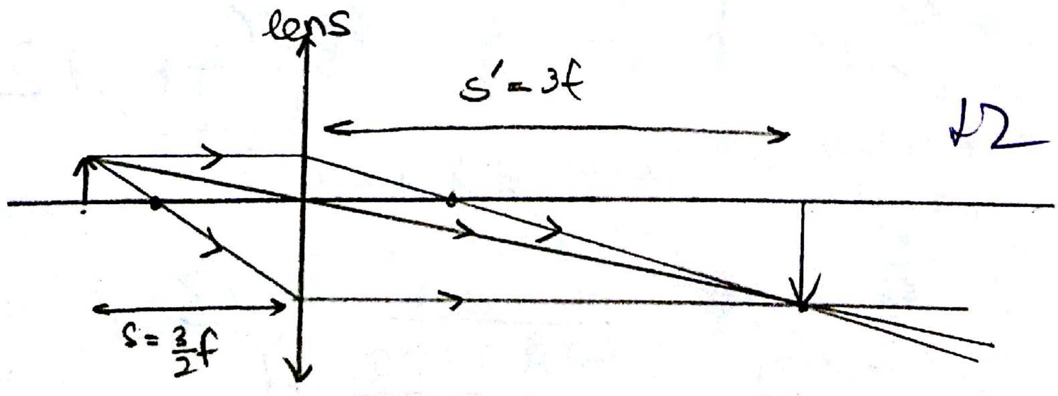
$$\Rightarrow \frac{1}{\frac{3}{2}f} + \frac{1}{s'} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{2}{3f} = \frac{1}{3f} \quad \therefore s' = 3f$$ +2

Therefore image is formed 3f to the right of the lens and will be real. +1

$$m = \frac{-s'}{s} = \frac{-3f}{\frac{3}{2}f} = -3f \times \frac{2}{3f} = -2$$ +2

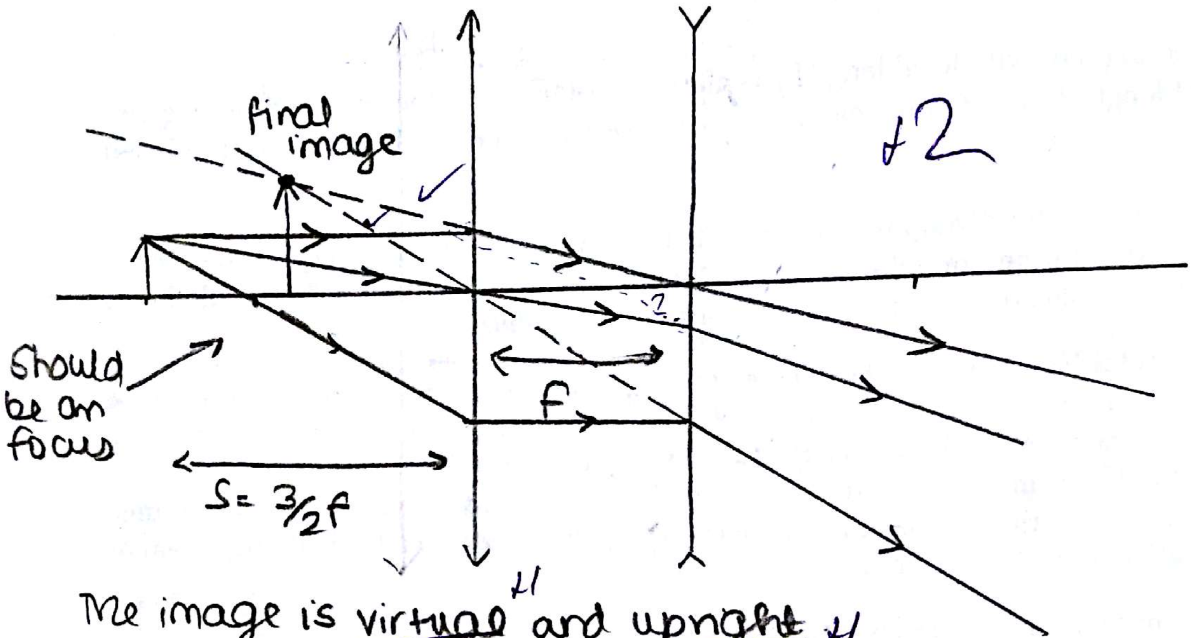
So its magnified by 2 and inverted +1



+2

Scanned by CamScanner

(b)



The image is virtual and upright +1 +1

Since image of first lens is $s' = 3f$.

For the second lens: +1

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{-1}{2f} + \frac{1}{s'} = \frac{-1}{f}$$

$$\Rightarrow \frac{1}{s'} = -\frac{1}{f} + \frac{1}{2f} = \frac{-1}{2f}$$

$$\therefore s' = -2f \quad +2$$

9/10

It is 2f to the left of the second lens.

$m = -\frac{s'}{s} = \frac{+2f}{-2f} = -1 \, m_p$?, $m_p = -2$?

$\therefore$ total $m = (-1)(-2) = \underline{\underline{2}}$ +1 +1

(c) Adding a third lens:- the object is now the final image

10/10

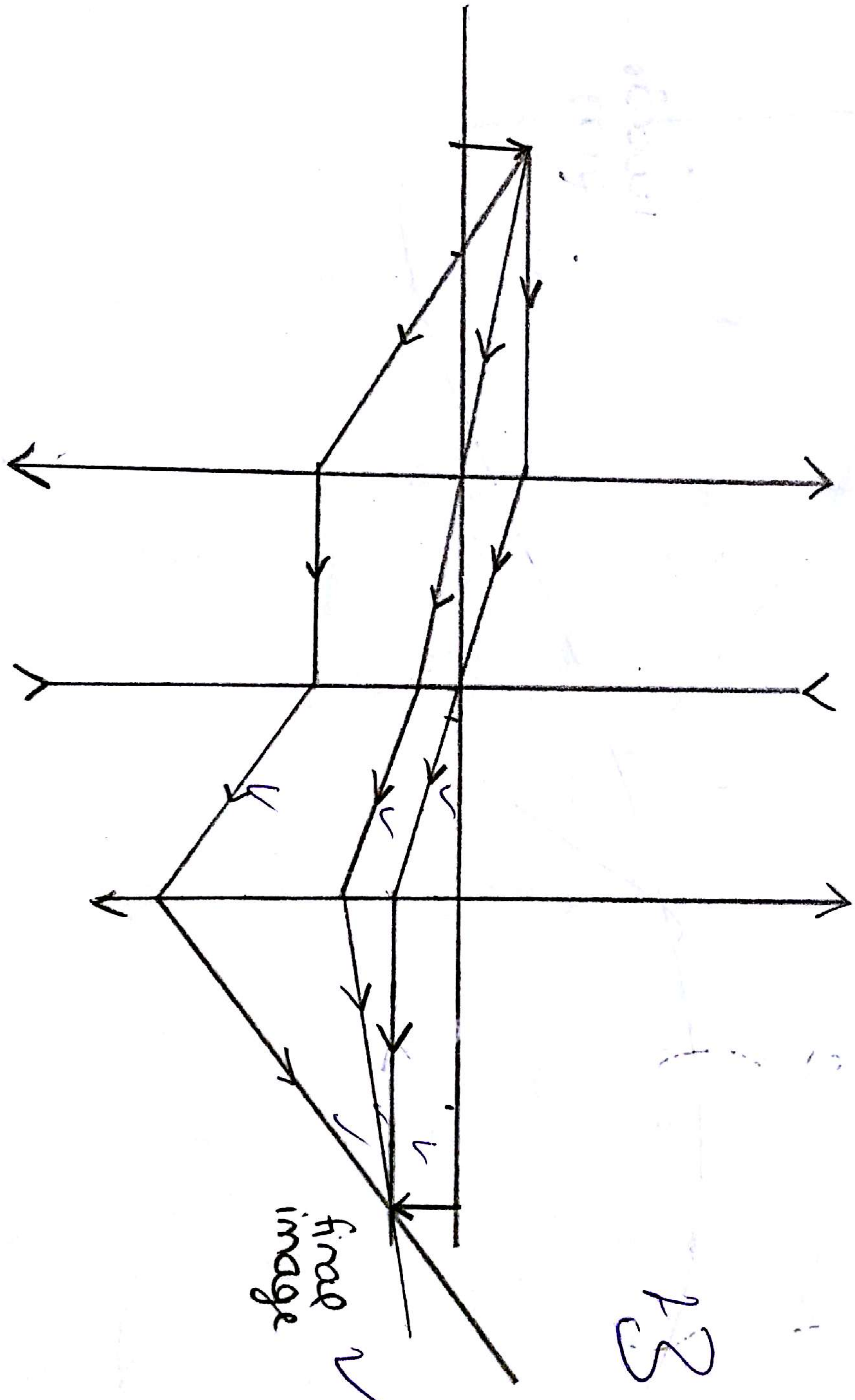
$\therefore s = 3f$ +1

$$\therefore \frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{1}{3f} = \frac{2}{3f} \quad \therefore s' = \underline{\underline{\frac{3}{2}f}} \quad +2$$

Image will be formed $\frac{3}{2}f$ to the right of lens 3.

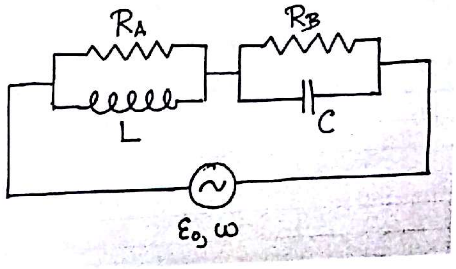
$m = -\frac{s'}{s} = \frac{-3/2f}{3f} = \frac{-1}{2}$ [Therefore it is inverted and real] +3

total $m = -\frac{1}{\cancel{2}} \times \cancel{2} = -1$ +1


final image

Problem 2. (34 points)

Consider the AC circuit depicted below. Make sure all boxed, mathematical answers are written only in terms of the quantities labeled on the diagram.



a. **(6 points)** What is the phase difference between the voltage in the inductor and the voltage in resistor A? What is the phase difference between the current in the inductor and the current in resistor A? Draw a phasor diagram containing the voltage and current phasors for these circuit elements.

b. **(6 points)** What is the phase difference between the voltage in the capacitor and the voltage in resistor B? What is the phase difference between the current in the capacitor and the current in resistor B? Draw a phasor diagram containing the voltage and current phasors for these circuit elements.

c. **(6 points)** Let I_A, I_L, I_B, I_C be amplitudes of the current in resistor A, the inductor, resistor B, and the capacitor respectively. Using physical intuition (no math) about how inductors and capacitors behave at low and high frequencies, what would you expect for the ratios I_L/I_A and I_C/I_B is the low and high frequency limits respectively?

d. **(4 points)** Compute the ratios I_L/I_A and I_C/I_B. Do your mathematical results agree with your physical intuition?

e. **(2 points)** Let i_A, i_L, i_B, i_C denote the time-dependent currents in resistor A, the inductor, resistor B, and the capacitor respectively. What is the relationship between these currents according to the junction rule?

f. **(8 points)** What is the ratio I_A/I_B?

g. **(2 points)** What is the ratio V_A/V_B?

(a) we know: $\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$

we know ⇒ $V_{RA} = V_L = \mathcal{E}_1$ [loop rule]

∴ phase difference in voltage is $\boxed{\text{zero}}$

Since: $V_{RA} = i_R R_A$

$i_R = \frac{V_{RA}}{R_A} = \frac{\mathcal{E}_0}{R_A}\cos(\omega t)$

also:

$$V_L = L\frac{di_L}{dt}$$

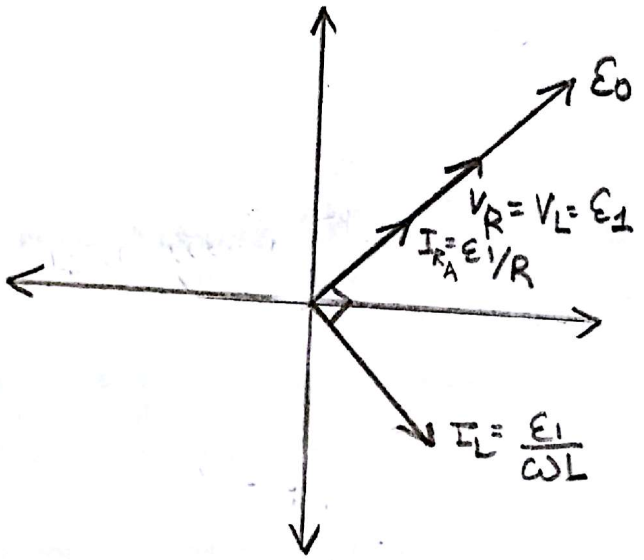
$\cos(90 - \omega t)$

$\cos(\omega t - 90$

$\therefore \frac{di_L}{dt} = \frac{V_L}{L}$ $\therefore i_L = \int \frac{V_L}{L}dt = \frac{\varepsilon_0}{\omega L}\sin(\omega t)$

$\therefore i_L = \frac{\varepsilon_0}{\omega L}\cos(\omega t - \pi/2)$

$\therefore \phi$ diff in currents is $\boxed{-\pi/2}$ where current in inductor lags



ε_1 is some quantity of $\varepsilon_0(t)$

+6

(b) again through loop rule:

$V_{R_B} = V_C = \varepsilon_2$ [zero phase difference in voltages]

however:

$V_{R_B} = i_{R_B}R_B$ $\therefore i_{R_B} = \frac{\varepsilon_0\cos(\omega t)}{R_B}$

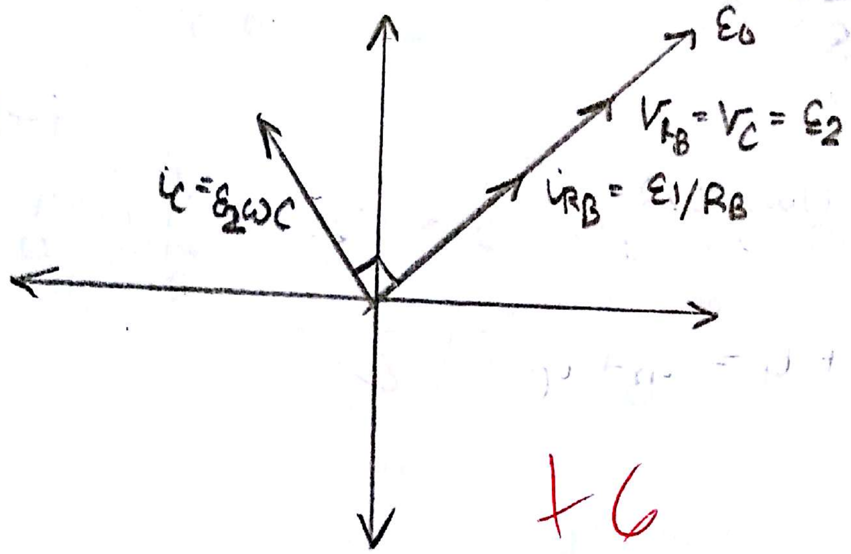
also:

$V_C = \frac{Q}{C}$ $\therefore q = V_C C$

$\therefore \frac{dq}{dt} = i = \frac{d(V_C C)}{dt} = -\varepsilon_0\omega C\sin(\omega t)$

$= \varepsilon_0\omega C\cos(\omega t + \pi/2)$

$\therefore \phi$ diff of current is $\pi/2$ where the capacitor leads



+6

(c) we know that inductors are low-pass filters and therefore allow a higher current flow for small values of ω

$$\therefore \frac{I_L}{I_A} \rightarrow \infty \text{ as } \omega \rightarrow 0$$

$$\frac{I_L}{I_A} \rightarrow 0 \text{ as } \omega \rightarrow \infty$$

~~+5~~ +6

similarly capacitors are high pass filters:

$$\frac{I_C}{I_B} \rightarrow 0 \text{ as } \omega \rightarrow 0$$

$$\frac{I_C}{I_B} \rightarrow \infty \text{ as } \omega \rightarrow \infty$$

(d) for inductor:

$$I_L = \frac{\varepsilon_1}{\omega L} \qquad I_A = \frac{\varepsilon_1}{R_A}$$

$$\therefore \frac{I_L}{I_A} = \frac{R_A}{\omega L}$$ ✓ this agrees as $\frac{I_L}{I_A} \rightarrow \infty$, for $\omega \rightarrow 0$ and vice versa.

for capacitor:

$$I_C = \varepsilon_2 \omega C \qquad I_B = \frac{\varepsilon_2}{R_B}$$

$\therefore \frac{I_C}{I_B} = R_B \omega C$ [This agrees as the ratio tends to ∞ as $\omega \to \infty$ and zero as $\omega \to 0$]

+1

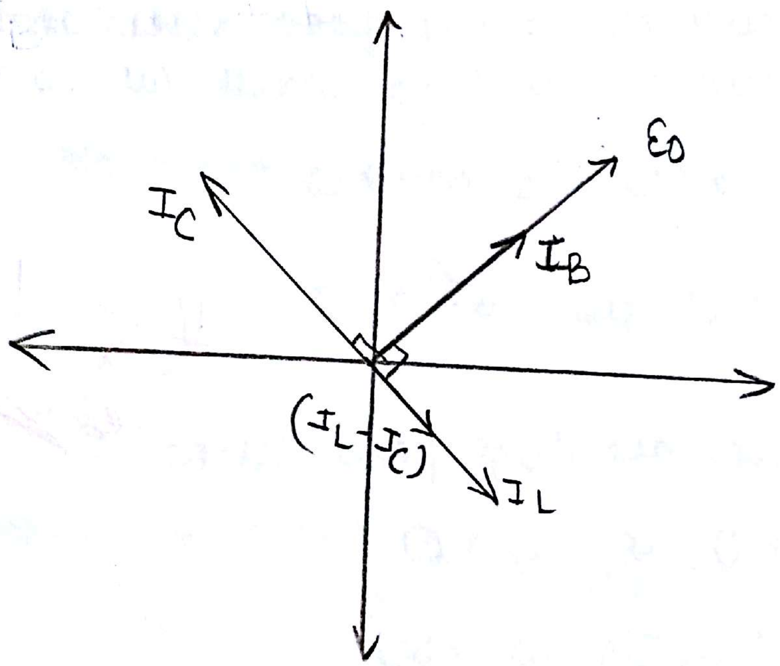
e) According to the junction rule: [between the two elements]

$$i_A + i_L = i_B + i_C$$ +2

f) Since:

$$i_A = i_B + i_C - i_L$$

$\Rightarrow$



I_A is the vector sum

$\therefore I_A^2 = I_B^2 + (I_L - I_C)^2 \quad \therefore I_A = \sqrt{I_B^2 + (I_L - I_C)^2}$

Similarly, we get that:

$$i_B = i_A + i_L - i_C$$

Using phasors:

$I_B^2 = I_A^2 + (I_L - I_C)^2 \quad \therefore I_B = \sqrt{I_A^2 + (I_L - I_C)^2}$

$$\frac{I_A}{I_B} = \frac{\sqrt{I_B^2 + (I_L - I_C)^2}}{\sqrt{I_A^2 + (I_L - I_C)^2}} = \frac{\sqrt{\left(\frac{\mathcal{E}_2}{R_B}\right)^2 + \left(\frac{\mathcal{E}_1}{\omega L} - \mathcal{E}_2 \omega C\right)^2}}{\sqrt{\left(\frac{\mathcal{E}_1}{R_A}\right)^2 + \left(\frac{\mathcal{E}_1}{\omega L} - \mathcal{E}_2 \omega C\right)^2}}$$

Cant do this

where $\mathcal{E}_1, \mathcal{E}_2 = \mathcal{E}_0 \cos(\omega t)$ +2

(d) since $V = IR$

$$\frac{V_A}{V_B} = \frac{I_A R_A}{I_B R_B} = \frac{R_A}{R_B}$$ (answer to part (c))

+2

Problem	Score
1	27 / 28
2	28 / 34
Total	55 / **62**