

MIDTERM EXAM #1

READ THIS BEFORE YOU BEGIN

- You are allowed to use only yourself and a writing instrument on this exam.
- If you finish more than 5 minutes before the end of the exam period, please raise your hand and a proctor will collect your exam. Otherwise, please stay in your seat until the end of time is called.
- When the exam is finished, please remain in your seat, pass your exam to the aisle, and the proctor(s) will come around and collect your exam. Once your exam is collected, you may leave the room.
- **Show all work.** The purpose of this exam is primarily to test how you think; you will get more partial credit for a logical, well-thought-out response, and *you will get little or no credit for an answer without convincing reasoning.* Points will be given specifically for the quality of your reasoning which includes clarity and conciseness.
- Please **box all of your final answers** to computational problems.
- You may use the back of any exam paper as room for extra work.

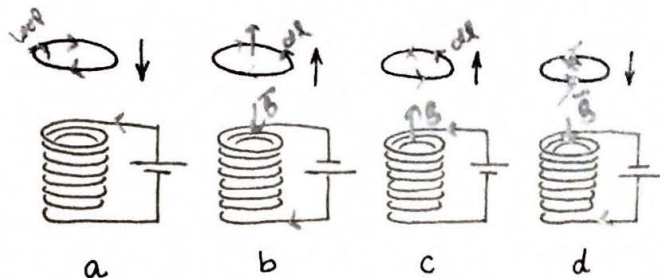
Name _____

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Discussion Section # _____

Problem 1.

Depicted below are four scenarios in which a loop of wire is moving above a coil. Each coil is connected to a battery with polarity indicated. The arrow next to the loop indicates the direction of motion of the loop.



- (12 points) Assuming that in each scenario the loop orientation is defined by a normal vector pointing in the direction of the velocity vector of the loop, determine the sign of the rate of change of the magnetic flux through the loop in each case.
- (12 points) Given that CW and CCW current flow in the loop are defined as follows: determine whether the direction of the induced current in the loop is CW or CCW.

= CW

= CCW

- Since loop moves closer to the coil, B field is increasing. According to Lenz's law, $\mathcal{E} > 0$ *not looking for \mathcal{E} , would give correct answer*
 - loop is moving away, so flux through loop decreases. According to Lenz's law and orientation of loop, $\mathcal{E} < 0$ *+10*
 - loop is moving away, so flux through loop decreases. According to orientation of loop, $\mathcal{E} > 0$
 - Since loop moves closer, downward \vec{B} field increases. According to Lenz's law, $\mathcal{E} < 0$

- CW ✓
 - CW ✓
 - CCW ✓
 - CCW ✓

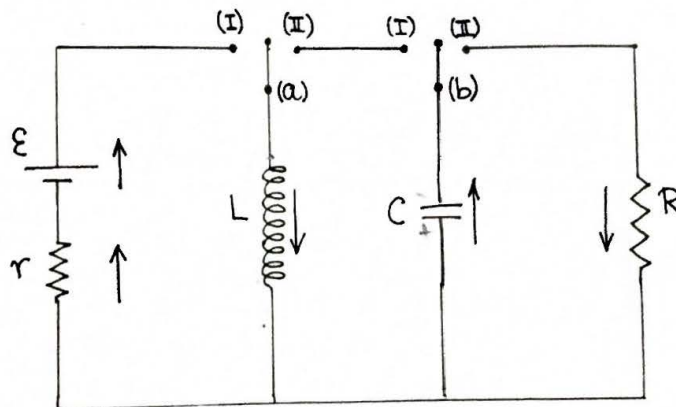
(These answers are based on the above answers and by considering orientation of the loop)

+12

Problem 2.

Consider the circuit depicted below. The arrow next to each circuit element denotes the direction in which current is defined to be positive. The labels (a) and (b) indicate two switches, each of which can be turned to one of the positions (I) or (II). The state of the switches for all times is as follows:

- For times $t < 0$, the switches are both open. There is no charge on the capacitor during this period, and there is no current in any circuit element.
- At time $t = 0$, switch (a) is turned to position (I), and switch (b) remains open.
- After a long time $t_1 > 0$, switch (a) is turned to position (II) and switch (b) is simultaneously turned to position (I).
- At time $t_2 > t_1$, after the central LC circuit has undergone two and a quarter cycles of oscillation, switch (b) is turned to position (II).
- At time $t_3 > t_2$, the current in the resistor R is $1/e$ of its magnitude at time t_2



- (8 points)** Determine the time intervals $t_2 - t_1$ and $t_3 - t_2$ in terms of the variables labeled on the diagram.
- (15 points)** At each moment, only one sub-loop of the circuit is "active" in the sense that the sub-loop is connected in a complete circuit. Let $i(t)$ denote the current in the active sub-loop of the circuit at time t . Draw a plot of i versus t for all times $t > 0$, and indicate on your graph the values of the current at the times t_1 , t_2 , and t_3 in terms of the variables labeled on the circuit diagram.
- (5 points)** Plot $q(t)$, the charge on the capacitor, during the interval (t_1, t_2) .

2. a) Angular frequency of oscillation = $\frac{1}{\sqrt{LC}}$ (for LC circuit)

$$\Rightarrow \frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$$

$$T = 2\pi\sqrt{LC} + 1.5$$

$$t_2 - t_1 = 2 \cdot \frac{1}{4} T = \frac{1}{2} T + 1.5$$

$$t_2 - t_1 = \frac{1}{2} \cdot 2\pi\sqrt{LC} = \pi\sqrt{LC}$$

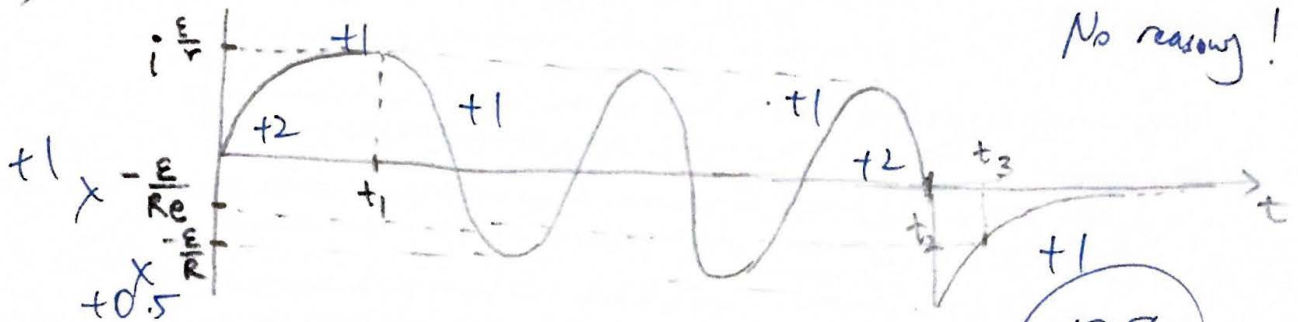
Since the current in the resistor is $\frac{1}{e}$ times, \Rightarrow one time constant has passed.

$$\tau_{RC} = RC + 2$$

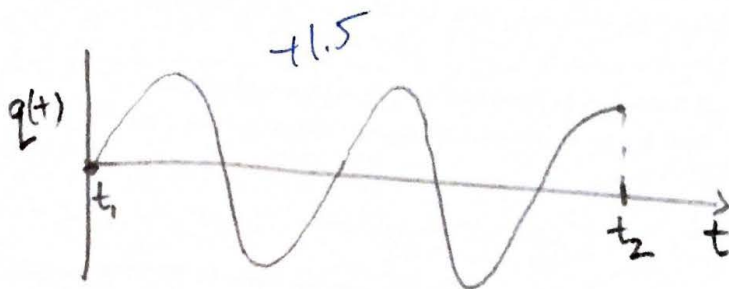
$$\Rightarrow t_3 - t_2 = RC + 2$$

$\frac{8}{8}$

b)



c)

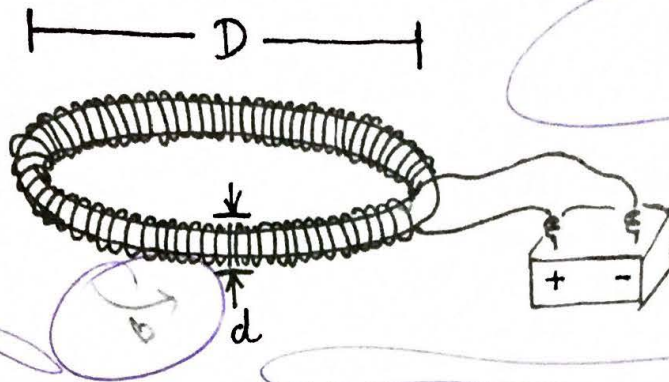


$\frac{1.5}{5}$

No scale on graph!

Problem 3.

A toroidal solenoid of diameter D and of comparatively thin cross-sectional diameter d is connected to a battery with emf \mathcal{E} . There are N turns of wire in solenoid, and the total resistance of the coiled wire is R . Suppose that the solenoid has been hooked up to the battery for a long time so that its current has reached a steady state.



- (3 points) What is the steady state value of the current?
- (2 points) If one is looking down on the solenoid from above, then in which direction is the magnetic field circulating? Clockwise or counterclockwise?
- (8 points) Use Ampere's Law to determine an expression for the magnitude of the magnetic field inside of the solenoid.
- (8 points) In the steady state there is a certain amount of energy U_B stored in the magnetic field of the solenoid. If we were to scale down the whole apparatus to half of its size without changing the emf of the battery, would U_B increase, decrease, or stay the same? If it increases or decreases, then by what factor?

Hint: Find an expression for U_B in terms of the various parameters that characterize this system. How do their values change if we rescale the system size?

a) Steady state value, $I = \frac{\mathcal{E}}{R}$ ^{+1.5} why?

b) Magnetic field is circulating in counter clockwise direction, if viewed from the top ^{+0.5} why?

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c) Using Ampere's law on the loop shown, (circumference of toroid)

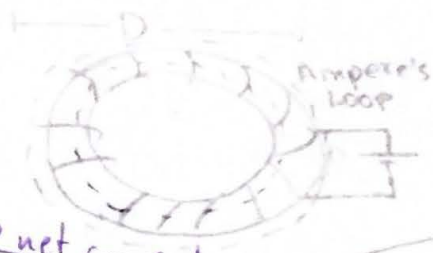
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \quad +2$$

$$B \oint dl = \mu_0 N I H$$

$$B(\pi D) = \mu_0 N I$$

$$\Rightarrow B = \mu_0 \frac{N I}{\pi D} \quad +2$$

Technically your loop encloses no net current



d) Energy Density = $\frac{B^2}{2\mu_0} \quad +0.5$

$$U = \int_{to\ 5} \frac{B^2}{2\mu_0} dV = \frac{B^2}{2\mu_0} \int dV = \frac{B^2}{2\mu_0} \left(\left(\frac{\pi d^2}{4} \right) (\pi D) \right) \quad +1$$

$$U = \frac{\pi^2 d^2 D}{8\mu_0} \left(\frac{\mu_0^2 N^2 I^2}{\pi^2 D^2} \right) = \frac{\mu_0 N^2 I^2}{8} \left(\frac{d^2}{D} \right) \quad +3$$

By decreasing d and D by a factor of 2, H the energy stored will decrease by a factor of 2.

$$\Rightarrow U_{final} = \frac{U_{initial}}{2}$$

Forgot about resistance $R' = \frac{2R}{2}$

Problem	Score
1	22 / 24
2	20 / 28
3	13 / 21
Total	55 / 73

[A large, stylized handwritten signature or mark in blue ink, resembling a large 'Z' or 'E' with a horizontal stroke.]