9/10

Problem 1 (10 pts): The current in a long solenoid of radius 5 cm and 1200 turns per meter is varied with time at a rate of 4000 A/s. A coil with twelve loops of radius 7 cm and resistance 1.1 Ω surrounds the solenoid. Find the electrical current induced in the loop.

$r_1 = 5\,cm = 0.05\,m$ $\quad A_1 = \pi r_1^2 = \pi \times (0.05\,m)^2 = 7.85 \times 10^{-3}\,m^2$

$n_1 = 1200\,/m$ $\quad \frac{dI_1}{dt} = 4000\,A/s$ $\quad N_2 = 12$

$r_2 = 7\,cm = 0.07\,m$ $\quad A_2 = \pi r_2^2 = \pi \times (0.07\,m)^2 = 1.54 \times 10^{-2}\,m^2$

$\Phi_{21} = \int B \cdot dA_2 = \mu_0 n_1 I_1 A_2$

$M = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2 \mu_0 n_1 A_2 I_1}{I_1} = N_2 \mu_0 n_1 A_2$

$= 12 \times 4\pi \times 10^{-7} \times 1200 \times 1.54 \times 10^{-2}\,m^2$

$= 2.79 \times 10^{-4}$

⊖ sign indicates the direction

$\mathcal{E}_2 = -M \frac{dI_1}{dt} = -2.79 \times 10^{-4} \times 4000 = -1.11\,V$

$I = \frac{\mathcal{E}_2}{R_2} = \frac{1.11\,A}{1.1\,\Omega} = \boxed{1\,A}$

-1

6/10

Problem 2 (10 pts): Electromagnetic rail guns work using Lorentz force to launch high velocity projectiles, by means of a sliding armature that is accelerated along a pair of conductive rails carrying a very large current.

Model such a device by assuming that a metal wire slides without friction on two rails spaced by 0.5 m apart, as in the figure below. The wire carries a projectile, and the combined mass of wire plus projectile is 0.8 kg. Assume there is a constant magnetic field of 0.25 T everywhere between the rails (this is a simplification), and a constant current of 7×10^4 amps flows from the generator G along one rail, across the wire, and back down the other rail.

a) (2 pts) Indicate the direction of force F on the wire on the diagram below.
b) (4 pts) Find the magnitude of the force on the wire.
c) (4 pts) Find the velocity v after 0.20 sec, assuming it to be at rest at t=0.

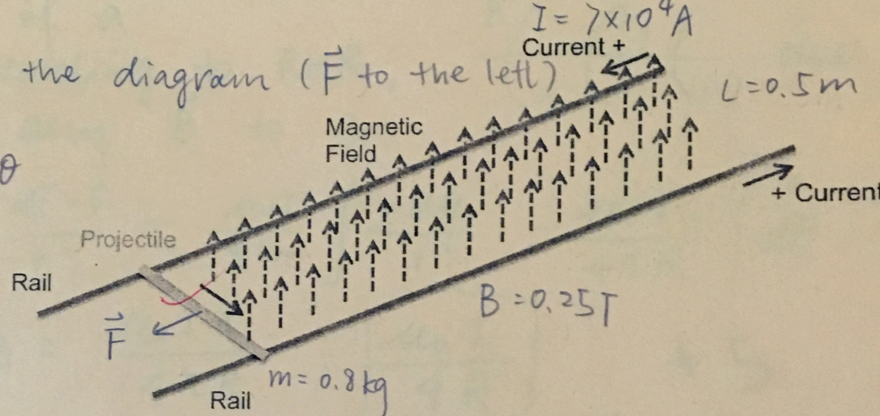
a) as shown in the diagram ($\vec{F}$ to the left)

b) $\vec{F} = I\vec{L} \times \vec{B}$

$F = ILB \sin\theta$

$= 7\times10^4 A \times 0.5 m \times 0.25 T \times 1$

$= \boxed{8750 N}$



c) $F = ma$ $\quad a = \frac{F}{m} = \frac{8750 N}{0.8 kg} = 10937.5 \, m/s^2$

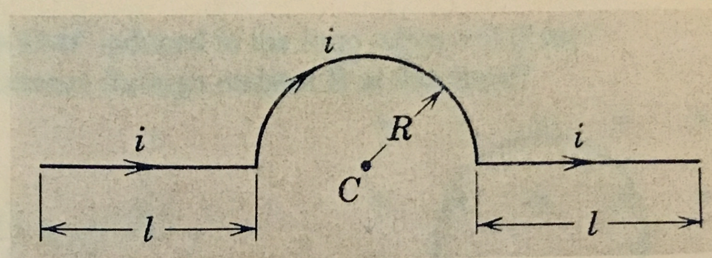
$v = at = 10937.5 \, m/s^2 \times 0.2 s = \boxed{2187.5 \, m/s}$

Problem 3 (10 pts): The wire shown below carries current *I*. What is the magnetic field ***B*** (magnitude and direction) at the center C of the semicircle arising from:

10

a) (3 pts) each straight segment of length *l*
b) (5 pts) the semicircular segment of radius *R*, and
c) (2 pts) the entire wire.

+3



a) Since the segments are straight and C is the center of a semicircle, according to RHR, does not attribute any $\vec{B}$ to C.

b) $\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int\frac{r\,d\theta}{r^2} = \frac{\mu_0 I}{4\pi R}\int d\theta = \frac{\mu_0 I\theta}{4\pi r}$

$\theta = \pi$ $\quad B = \frac{\mu_0 I \pi}{4\pi R} = \boxed{\frac{\mu_0 I}{4R}}$ +5

direction: according to RHR, $\vec{B}$ goes into the page

c) $\vec{B}_{total} = B_l + B + B_l = 0 + \frac{\mu_0 I}{4R} + 0$

$= \boxed{\frac{\mu_0 I}{4R}}$

+2

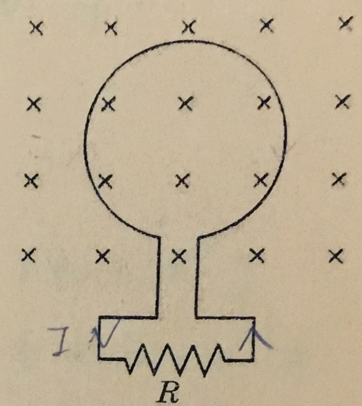
10

Problem 4 (10 pts): In the figure below, the magnetic flux through the loop perpendicular to the plane of the coil and directed into the paper is varying according to the relation

$$\Phi_m = 4t^2 + 7t + 1,$$

where Φ_m is in milli-webers, and t is in seconds.

a) (6 pts) What is the magnitude of the EMF induced in the loop when t=3.0 sec?
b) (4 pts) What is the direction of the current through resistor R at that time?



a) $\mathcal{E} = -\frac{d\Phi_m}{dt} = (-8t - 7) \times 10^{-3}$

$= -(8t+7) \times 10^{-3}$

when $t = 3$

+6 $\mathcal{E} = -(8 \times 3 + 7) \times 10^{-3}$ V

$= |-3.1 \times 10^{-2}$ V$|$

b) B goes into the page

$\Phi_m \uparrow$ as $t \uparrow$, $\frac{d\Phi_m}{dt} \uparrow$ as $t \uparrow$

+4 induced I opposes this change

as shown in the diagram, I

goes from ~~right to left~~

left to right

(10)

Problem 5 (10 pts): At time t=0, the current through a 40.0mH inductor is 30.0 mA and is increasing steadily at the rate of 120 mA/s.

$L = 40\times10^{-3}\,H$ $\quad I_0 = 30\times10^{-3}\,A$ $\quad \frac{dI}{dt} = 120\times10^{-3}\,A/s$

a) (5 pts) What is the energy stored in the inductor at time t=0?

b) (5 pts) How long does it take for the energy to increase by a factor 9 from the initial value?

a) at $t=0$ $\quad I = I_0 = 30\times10^{-3}\,A$

$$U_L = \frac{1}{2} I_0^2 L = \frac{1}{2}\times(30\times10^{-3}\,A)^2 \times 40\times10^{-3}\,H$$

$$= \boxed{1.8\times10^{-5}\,J}$$

+5

b) $U_L' = 9U_L = 9\times1.8\times10^{-5}\,J = 1.62\times10^{-4}\,J$

$$I = \frac{dI}{dt}\times t + I_0 = 120\times10^{-3}\,A/s \cdot t + 30\times10^{-3}\,A$$

$$U_L' = \frac{1}{2} I'^2 L$$

$$I' = \sqrt{\frac{2U_L'}{L}} = \sqrt{\frac{2\times1.62\times10^{-4}\,J}{40\times10^{-3}\,H}} = 0.09\,A$$

$$0.09\,A = 120\times10^{-3}\,A/s\; t + 30\times10^{-3}\,A$$

$$t = \frac{0.09\,A - 30\times10^{-3}\,A}{120\times10^{-3}\,A/s}$$

$$= \boxed{0.5\,s}$$

+5