21S-PHYSICS1C-1 Alternate Final exam

DARREN ZHANG

TOTAL POINTS

60 / 60

QUESTION 1

1 Problem 1 10 / 10

✓ + 2 pts (a) Correct Biot-Savart's law

✓ + 1 pts (a) Correct argument

✓ + 2 pts (b) Taking cross product correctly

✓ + 2 pts (b) Correct integration of B

✓ + 1 pts (b) Correct direction

✓ + 2 pts (c) Correct limits

QUESTION 2

2 Problem 2 10 / 10

✓ + 1 pts (a) Seed value

✓ + 2 pts (b) Correct formula for the magnetic flux

✓ + 1 pts (b) Correct flux at $$t=0$$

✓ + 1 pts (c) Correct time-dependent flux

✓ + 1 pts (c) Correct formula for EMF

✓ + 1 pts (c) Correct result

✓ + 2 pts (d) Correct expression for the maximum voltage

✓ + 1 pts (d) Correct numerical value

QUESTION 3

3 Problem 3 10 / 10

✓ + 1 pts a) correct answer $$\lambda = c/f = 469\textrm{ m}$$

✓ + 1 pts b) energy per second per area = maximum average power over total area

✓ + 1 pts b) area is $$ 2 \pi R^2 $$

✓ + 1 pts b) final result is $$50 \textrm{ kW}/(2\pi\cdot100^2\textrm{ m}^2) = 0.796 \textrm{ W/m}^2$$

✓ + 1 pts c) $$E_0 = \sqrt{2 \mu_0 c I} = \sqrt{2I/\varepsilon_0 c}$$

✓ + 1 pts c) $$E_{\mathrm{rms}} = E_0/\sqrt{2}$$

✓ + 1 pts c) correct answer $$E_{\mathrm{rms}} = 17.3 \textrm{ N/C} $$

✓ + 1 pts d) $$V_{\textrm{rms}} = E_{\textrm{rms}}d$$ where $$d$$ is length of antenna

✓ + 1 pts d) for 100m, $$V_{\textrm{rms}} = 1.73 \textrm{ V}$$

✓ + 1 pts d) for $$C$$ km, $$V_{\textrm{rms}} = 1.73/(C \cdot 10) \textrm{ V}$$

QUESTION 4

4 Problem 4 10 / 10

✓ + 1 pts a) use thin lens equation

✓ + 1 pts a) use $$f = f_2$$ and $$s' = \infty$$

✓ + 1 pts a) $$s_2 = 10\textrm{ mm}$$

✓ + 1 pts b) use thin lens equation

✓ + 1 pts b) $$f = f_1$$ and $$s' = d - s_2$$

✓ + 1 pts b) correct value

✓ + 1 pts c) reasonable equation

✓ + 1 pts c) correct equation $$M = (25 \textrm{ cm})s_1'/(f_1f_2) $$

✓ + 1 pts c) correct value

✓ + 1 pts d) image is inverted

QUESTION 5

5 Problem 5 10 / 10

✓ + 1 pts a) strong reflection = constructive interference

✓ + 1 pts a) there is a relative phase shift

✓ + 1 pts a) $$2t = (m+1/2) \lambda$$

✓ + 1 pts a) $$\lambda = \lambda_0/n_2$$

✓ + 1 pts a) $$\lambda_0 = 413 \textrm{ nm}$$

✓ + 1 pts b) weak reflection = destructive interference

✓ + 1 pts b) there is a relative phase shift

✓ + 1 pts b) $$2t = m \lambda$$

✓ + 1 pts b) $$\lambda = \lambda_0/n_2$$

✓ + 1 pts b) $$\lambda_0 = 620 \textrm{ nm}$$

QUESTION 6

6 Problem 6 **10 / 10**

- ✓ **+ 1 pts** a) $$\Delta m = 2.80 \cdot 10^{-30} \textrm{ kg}$$
- ✓ **+ 1 pts** b) use energy conservation
- ✓ **+ 1 pts** b) sum of kinetic energies = $$\Delta m \cdot c^2 $$
- ✓ **+ 1 pts** b) numerical value $$2.52 \cdot 10^{-13} \textrm{ J}$$
- ✓ **+ 1 pts** c) $$K = (\gamma-1)mc^2 $$
- ✓ **+ 1 pts** c) solve for $$\gamma = K/mc^2 + 1$$
- ✓ **+ 1 pts** c) correct numerical values $$\gamma_p = 1.00$$, $$\gamma_e = 2.04$$ and $$\gamma_n = \infty$$
- ✓ **+ 1 pts** d) use that $$\gamma = 1/\sqrt{1-v^2/c^2}$$
- ✓ **+ 1 pts** d) solve correctly for $$v = c\sqrt{1-1/\gamma^2}$$
- ✓ **+ 1 pts** d) correct numerical values $$v_p \approx 0$$, $$v_e \approx 2.61 \cdot 10^8 \textrm{ m/s}$$, $$v_n = c$$

Name: Darren Zhang

Student ID #: [redacted]

Signature: Darren Zhang

June 9, 2021

Physics 1C Alternate Final Exam

- You have 3 hours to complete this exam, and it is recommended to finish your work in 2:45 so that you have 15 minutes to upload it to GradeScope. THERE IS A STRICT LATE EXAM POLICY: -10% IMMEDIATELY PLUS -10% EVERY ADDITIONAL 5 MINUTES.

- Numerical values in answers: quote values with 3 significant figures, for example, 1.32 or 9.72.

- Remote exam rules:

    - Keep your cameras on during the exam, including the period in which you submit your answers to GradeScope. You may step away during the first 5 minutes of the exam period if you are printing out the exam, and, if necessary, to scan answers during the last 5 minutes, but send a note to the chat window to explain that is what you are doing.
    - The exam is open notes, open book, you can use any type of calculator, but you must not communicate with classmates or other people during the exam.
    - Please check your chat window occasionally in case there are needed clarifications. You can ask clarifying questions of the professor using the chat window, but be aware that he may be very busy with other students, and you might need to repeat it.

- You MUST sign and date the 2nd page entitled "Academic Integrity – A Bruin's Code of Conduct" in order to receive credit for your work.
- Remember to write down each step of your calculation, and explain your answers fully.

Score :

I.	________/10 points
II.	________/10 points
III.	________/10 points
IV.	________/10 points
V.	________/10 points
VI.	________/10 points
Total	________**/60 points**

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If you engage in these types of unacceptable behaviors in our course, then you will receive a zero as your score for that assignment. If you are caught cheating on an exam, then you will receive a score of zero for the entire exam. These allegations will be referred to the Office of the Dean of Students and can lead to formal disciplinary proceedings. Being found responsible for violations of academic integrity can result in disciplinary actions such as the loss of course credit for an entire term, suspension for several terms, or dismissal from the University. Such negative marks on your academic record may become a major obstacle to admission to graduate, medical, or professional school.

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By submitting my assignments and exams for grading in this course, I acknowledge the above-mentioned terms of the UCLA Student Code of Conduct, declare that my work will be solely my own, and that I will not communicate with anyone other than the instructor and proctors in any way during the exams.

Darren Zhang
Signature

6/10/2021
Date

Darren Zhang
Print Name

[redacted]
UID

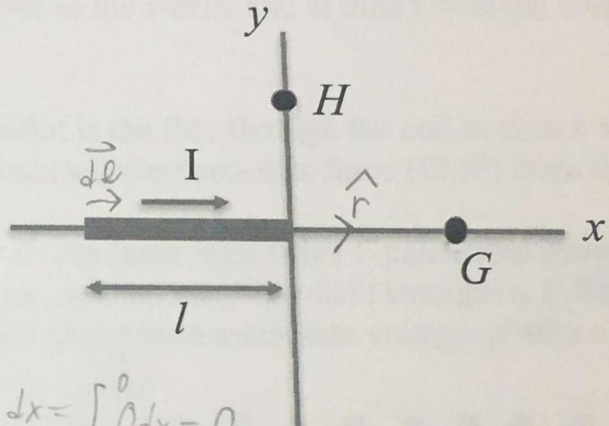
I (10 points) A wire of finite length l carries a current I as shown below.

a) (3 pts) Show that for points along the $+x$ axis such as point G below, there is no magnetic field.

b) (5 pts) Find a formula for the magnetic field at point H at $(0, y)$ in the diagram below. A possibly relevant integral is listed below.

$$\int \frac{y}{(x^2+y^2)^{1.5}}\,dx = \frac{x}{y\sqrt{x^2+y^2}} + \text{constant}$$

c) (2 pts) Find the limit of your result from part b) when $l \to \infty$.

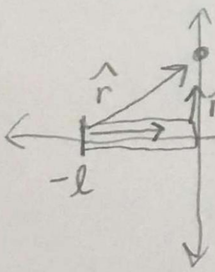


a) $\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,\vec{dl} \times \hat{r}}{r^2}$

$\vec{dl} \times \hat{r} = 0$ b/c $\vec{dl} \parallel \hat{r}$

$\vec{B}_G = \int_{-l}^{0} \frac{\mu_0}{4\pi} \frac{I\,\vec{dl}\times\hat{r}}{r^2} dx = \int_{-l}^{0} \frac{\mu_0}{4\pi} \frac{I\,(0)}{r^2} dx = \int_{-l}^{0} 0\,dx = 0$

b)



$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,\vec{dl}\times\hat{r}}{r^2}$ $\quad \hat{r} = \frac{-x\hat{i} + y\hat{j}}{\sqrt{x^2+y^2}}$ $\quad \vec{dl} = dx(\hat{i})$

$r^2 = x^2 + y^2$

$\frac{\vec{dl}\times\hat{r}}{r^2} = \frac{dx\,\hat{i} \times (-x\hat{i} + y\hat{j})}{(x^2+y^2)^{3/2}} = \frac{y\,dx\,\hat{k}}{(x^2+y^2)^{3/2}}$

$\vec{B}_H = \int_{-l}^{0} \frac{\mu_0 I}{4\pi}\left(\frac{y\,dx\,\hat{k}}{(x^2+y^2)^{3/2}}\right) = \frac{\mu_0 I \hat{k}}{4\pi}\int_{-l}^{0} \frac{y\,dx}{(x^2+y^2)^{3/2}} = \frac{\mu_0 I \hat{k}}{4\pi}\left(\frac{x}{y\sqrt{x^2+y^2}}\right)\Bigg|_{-l}^{0}$

using integral given in problem

$= \frac{\mu_0 I \hat{k}}{4\pi}\left(\frac{0}{y\sqrt{0+y}} - \frac{-l}{y\sqrt{l^2+y^2}}\right) = \frac{\mu_0 I l}{4\pi y \sqrt{l^2+y^2}}\hat{k}$

c) $\lim_{l\to\infty}\left(\frac{\mu_0 I l\,\hat{k}}{4\pi y\sqrt{l^2+y^2}}\right) = \frac{\mu_0 I \hat{k}}{4\pi y} \lim_{l\to\infty} \frac{l}{\sqrt{l^2+y^2}} = \frac{\mu_0 I \hat{k}}{4\pi y}(1) = \frac{\mu_0 I \hat{k}}{4\pi y}$

1 Problem 1 **10 / 10**

- ✓ **+ 2 pts** (a) Correct Biot-Savart's law
- ✓ **+ 1 pts** (a) Correct argument
- ✓ **+ 2 pts** (b) Taking cross product correctly
- ✓ **+ 2 pts** (b) Correct integration of B
- ✓ **+ 1 pts** (b) Correct direction
- ✓ **+ 2 pts** (c) Correct limits

II (10 points):

a) (1 pt) Determine your individual seed value C for subsequent problems in this exam as follows: C is one plus the absolute value of the difference between the last two digits of your student ID number. [For example, if your student ID is 0037623, your seed value will be $C = 1 + abs(2 - 3) = 1 + 1 = 2$.]

My student ID number SID= [redacted] $1 + |4 - 1| = 1 + 3 = 4$

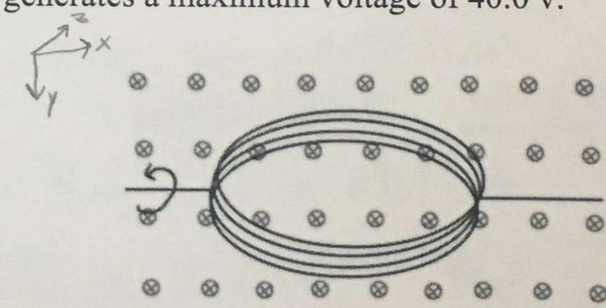
My individual seed value C= 4

A circular coil with diameter d and N loops of wire is the central part of a simple generator. The axis of rotation of the coil is the x-axis, and it rotates at constant frequency f in a constant magnetic field B that is parallel to the z-axis, and at time $t = 0$ the loops of the coil are in the $x - y$ plane.

b) (3 pts) In terms of the variables above, what is the flux through the coil at time $t = 0$?

c) (3 pts) Write down an equation for the induced electromotive force (EMF) from the coil as a function of time, $\varepsilon(t)$.

d) (3 pts) If the diameter of the coil is $d = C$ cm (your seed value C calculated above), the coil consists of $N = 200$ turns of wire, and the magnetic field strength is 0.500 Tesla, find the revolution frequency f that generates a maximum voltage of 40.0 v.



b) $A_{coil} = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

$\Phi_M = BAN = B\frac{\pi d^2}{4}N = \frac{\pi d^2 BN}{4}$

(B is $\parallel$ to area vector at $t=0$)

c) $\varepsilon(t) = \oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_M$

$\Phi_M(t) = NBA\cos\theta = NB\frac{\pi d^2}{4}\cos(\omega t)$ $\omega = 2\pi f$

$\frac{d}{dt}\Phi_m(t) = \frac{NB\pi d^2}{4}\omega(-\sin(\omega t))$

$\varepsilon(t) = -\frac{d}{dt}\Phi_m(t) = \frac{NB\pi d^2\omega}{4}\sin(\omega t) = \frac{NB\pi d^2 2\pi f}{4}\sin(2\pi f t) = \frac{NB\pi^2 d^2 f}{2}\sin(2\pi f t)$

d) $d = 4\text{cm}$ $N = 200$ $B = 0.5\text{T}$

$d = 0.04\text{m}$

$\mathcal{E}_{max} = \frac{NB\pi^2 d^2 f}{2}$

Solve for f.

$f = \frac{2\mathcal{E}_{max}}{NB\pi^2 d^2} = \frac{2(40)}{200(0.5)(\pi^2)(0.04)^2} = 50.66\text{ Hz} \approx 50.7\text{ Hz}$

2 Problem 2 **10 / 10**

- ✓ **+ 1 pts** (a) Seed value
- ✓ **+ 2 pts** (b) Correct formula for the magnetic flux
- ✓ **+ 1 pts** (b) Correct flux at $$t=0$$
- ✓ **+ 1 pts** (c) Correct time-dependent flux
- ✓ **+ 1 pts** (c) Correct formula for EMF
- ✓ **+ 1 pts** (c) Correct result
- ✓ **+ 2 pts** (d) Correct expression for the maximum voltage
- ✓ **+ 1 pts** (d) Correct numerical value

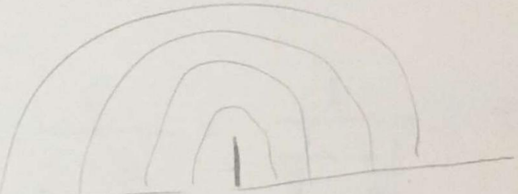
III (10 points): The maximum average power authorized to AM radio stations in the U.S. is 50 kW. Assume that the EM waves are emitted uniformly in the hemisphere above the horizon (nothing downward), and that the frequency *f* of the radio station is 640 kHz. For such a radio station:

a) (1 pt) What is the wavelength of the radio waves?
b) (3 pts) How much energy per second crosses a 1.0 m^2 area above the horizon at a distance 100 m from the broadcasting antenna.
c) (3 pts) What is the RMS magnitude of the **E** field at this point?
d) (3 pts) What are the RMS voltages induced in an antenna above the horizon that is 10.0 cm long at 100 m distance, and at *C* (seed value) km distance? [Assume that the charges and currents induced in the antenna do not significantly alter the EM wave from the AM radio station.]

a) $f = 640\text{ kHz} = 640 \times 10^3\text{ Hz}$

$c = f\lambda \qquad \lambda = \frac{c}{f} = \frac{3\times10^8}{640\times10^3} = 468.75\text{ m} \approx 469\text{ m}$

b) Energy per second $= \left\langle \frac{u_{EM}}{Vol} \right\rangle \cdot A \cdot c$



$\left\langle \frac{u_{EM}}{Vol} \right\rangle = \frac{\epsilon_0 E_{max}^2}{2}$

$\langle P \rangle = 50\text{ kW} = 50 \times 10^3\text{ J/s}$

$I = \frac{\text{Power}}{\text{Area}} = \frac{50\times10^3}{4\pi r^2/2} = \frac{50\times10^3}{2\pi(100)^2} = 0.796\text{ W/m}^2 = 0.796\text{ J/s/m}^2$

$I \cdot (1.0\text{ m}^2) = 0.796\text{ J/s}$ — 0.796 J crosses a 1.0 m^2 area every second

~~$I = \frac{E_{max}^2}{2\mu_0 c} \qquad E_{max} = \sqrt{2\mu_0 c I} = \sqrt{2(4\pi\times10^{-7})(3\times10^8)(0.796)} \approx 24.49\text{ V/m}$~~

~~$\left\langle \frac{u_{EM}}{Vol} \right\rangle = \frac{\epsilon_0 E_{max}^2}{2} = \frac{8.854\times10^{-12}(24.49)^2}{2} = 2.656\times10^{-9}\text{ J/m}^3$~~

~~Energy per second $= \left\langle \frac{u_{EM}}{Vol} \right\rangle \cdot A \cdot c = 2.656\times10^{-9}(1.0)(3\times10^8) = 0.797\text{ J/s}$~~

c) $E_{max} = 24.49\text{ V/m}$

$E_{RMS} = \frac{E_{max}}{\sqrt{2}} = \frac{24.49}{\sqrt{2}} = 17.3\text{ V/m}$

d) $V = EL$

at 100 m:

$V_{RMS} = E_{RMS} L = 17.3(0.1\text{ m}) = 1.73\text{ V}$

Additional paper for answer to problem III

d continued)

at 4 km distance

$$I = \frac{\text{power}}{\text{Area}} = \frac{50\times10^3}{2\pi(4000)^2} = 4.97\times10^{-4}\ W/m^2$$

$$I = \frac{E_{max}^2}{2\mu_0 c} \qquad E_{max} = \sqrt{2\mu_0 c I} = \sqrt{2(4\pi\times10^{-7})(3\times10^8)(4.97\times10^{-4})}$$

$$= 0.612\ V/m$$

$$E_{RMS} = \frac{E_{max}}{\sqrt{2}} = \frac{0.612}{\sqrt{2}} = 0.433\ V/m$$

$$V_{RMS} = E_{RMS} L = 0.433(0.1) = 0.0433\ V$$

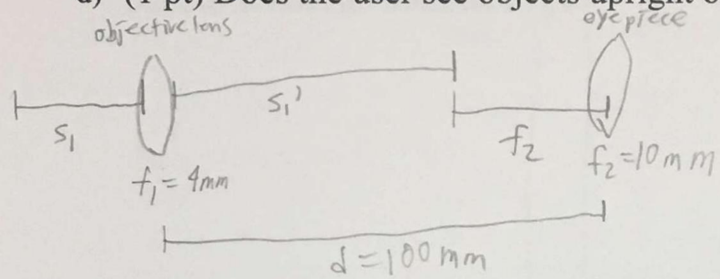
3 Problem 3 **10 / 10**

- ✓ **+ 1 pts** a) correct answer $$\lambda = c/f = 469\textrm{ m}$$
- ✓ **+ 1 pts** b) energy per second per area = maximum average power over total area
- ✓ **+ 1 pts** b) area is $$ 2 \pi R^2 $$
- ✓ **+ 1 pts** b) final result is $$50 \textrm{ kW}/(2\pi\cdot100^2\textrm{ m}^2) = 0.796 \textrm{ W/m}^2$$
- ✓ **+ 1 pts** c) $$E_0 = \sqrt{2 \mu_0 c I} = \sqrt{2I/\varepsilon_0 c}$$
- ✓ **+ 1 pts** c) $$E_{\mathrm{rms}} = E_0/\sqrt{2}$$
- ✓ **+ 1 pts** c) correct answer $$E_{\mathrm{rms}} = 17.3 \textrm{ N/C} $$
- ✓ **+ 1 pts** d) $$V_{\textrm{rms}} = E_{\textrm{rms}}d$$ where $$d$$ is length of antenna
- ✓ **+ 1 pts** d) for 100m, $$V_{\textrm{rms}} = 1.73 \textrm{ V}$$
- ✓ **+ 1 pts** d) for $$C$$ km, $$V_{\textrm{rms}} = 1.73/(C \cdot 10) \textrm{ V}$$

IV (10 points): A microscope has two lenses: an objective lens with focal length $f_1 = C$ mm and an eyepiece with focal length $f_2 = 10.0$ mm. The two lenses are separated by distance $d =$ 100 mm. The user focuses her/his eye at infinity.

a) (3 pts) How far is the eyepiece from the image formed by the objective lens?
b) (3 pts) How far is the object to be viewed from the objective lens?
c) (3 pts) What is the angular magnification of this microscope? Assume the average value for the near point of the human eye is 25 cm as assumed in the textbook.
d) (1 pt) Does the user see objects upright or inverted?

a)



$s_2 = f_2$

so the image formed by the objective lens is 10.0 mm away from the eye piece

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$$

b) $s_1' = d - f_2 = 100\text{mm} - 10\text{mm} = 90\text{mm}$

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1} \qquad \frac{1}{s_1} = \frac{1}{f_1} - \frac{1}{s_1'} \qquad s_1 = \frac{1}{\frac{1}{f_1} - \frac{1}{s_1'}} = \frac{1}{\frac{1}{4} - \frac{1}{90}} = 4.186\text{mm}$$

The object to be viewed is 4.19 mm away from the objective lens

c) $$M = m_1 M_2 = -\frac{s_1'}{s_1}\frac{(250\text{mm})}{f_2} = \frac{-90\text{mm}(250\text{mm})}{4.19\text{mm}(10\text{mm})} = -537.5 \approx -538$$

d) The user sees the objects as inverted.

4 Problem 4 **10 / 10**

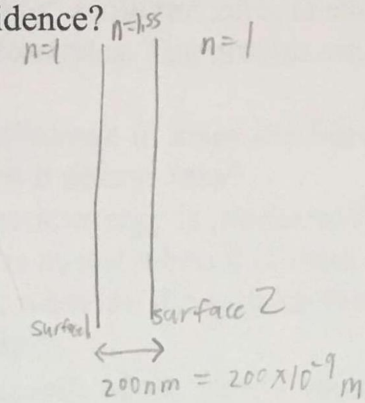
- ✓ **+ 1 pts** a) use thin lens equation
- ✓ **+ 1 pts** a) use $$f = f_2$$ and $$s' = \infty$$
- ✓ **+ 1 pts** a) $$s_2 = 10\textrm{ mm}$$
- ✓ **+ 1 pts** b) use thin lens equation
- ✓ **+ 1 pts** b) $$f = f_1$$ and $$s' = d - s_2$$
- ✓ **+ 1 pts** b) correct value
- ✓ **+ 1 pts** c) reasonable equation
- ✓ **+ 1 pts** c) correct equation $$M = (25 \textrm{ cm})s_1'/(f_1f_2) $$
- ✓ **+ 1 pts** c) correct value
- ✓ **+ 1 pts** d) image is inverted

V (10 points): An exceedingly thin glass bubble is blown; the index of refraction is $n = 1.55$. There is air inside and outside the bubble. The bubble is 200 nm thick. Assume the range of visible light is 380-700 nm.

a) (5 pts) What wavelength(s) of visible light will show the strongest reflection from the near side of the bubble at normal incidence? [Don't worry about the reflection from the far opposite side of the bubble.]

b) (5 pts) What wavelength(s) of visible light will show very little reflection from the bubble at normal incidence?

a)



$m_{surface,1} = \frac{1}{2}$ b/c $n_{air} = 1 < n_{bubble} = 1.55$

$m_{surface,2} = 0 + \frac{2t\,n_2}{n_1 \lambda_1}$

$\Delta m = m_{surface,2} - m_{surface,1} = \frac{2t(1.55)}{\lambda_1} - \frac{1}{2}$

strongest reflection is when there is constructive interference, so when Δm = an Integer

$\Delta m = \frac{3.1t}{\lambda_1} - \frac{1}{2}$ $\quad \Delta m + \frac{1}{2} = \frac{3.1t}{\lambda_1}$ $\quad \lambda_1 = \frac{3.1t}{\Delta m + 1/2} = \frac{3.1(200)}{\Delta m + 1/2} = \frac{620}{\Delta m + 1/2}$

$\lambda_1 = \frac{620}{0 + \frac{1}{2}}, \frac{620}{1 + 1/2}, \frac{620}{2 + 1/2}$

$\lambda_1 = 1240\text{nm}, 413.33\text{nm}, 248\text{nm}$

only 413.33nm is in the visible spectrum

413nm wavelength visible light will be the strongest reflected light

b) very little reflection is when $\Delta m = n + \frac{1}{2}$ where n is an Integer

there is destructive interference, so

$\lambda_1 = \frac{620}{(\frac{1}{2}) + \frac{1}{2}}, \frac{620}{(\frac{3}{2}) + \frac{1}{2}}, \frac{620}{(\frac{5}{2} + \frac{1}{2})}$

$\lambda_1 = 620\text{nm}, 310\text{nm}, 206.67\text{nm}$

Only 620nm is in visible spectrum

The visible light that will show very little reflection is light with wavelength 6.20×10^2 nm

5 Problem 5 10 / 10

- ✓ + 1 pts a) strong reflection = constructive interference
- ✓ + 1 pts a) there is a relative phase shift
- ✓ + 1 pts a) $$2t = (m+1/2) \lambda$$
- ✓ + 1 pts a) $$\lambda = \lambda_0/n_2$$
- ✓ + 1 pts a) $$\lambda_0 = 413 \textrm{ nm}$$
- ✓ + 1 pts b) weak reflection = destructive interference
- ✓ + 1 pts b) there is a relative phase shift
- ✓ + 1 pts b) $$2t = m \lambda$$
- ✓ + 1 pts b) $$\lambda = \lambda_0/n_2$$
- ✓ + 1 pts b) $$\lambda_0 = 620 \textrm{ nm}$$

VI (10 points): A free neutron is an unstable particle: it decays (in about 15 minutes, on average) to a proton, an electron, and a neutrino.

The masses are (values changed from the actual ones for testing purposes):

- Neutron $m_n = 1.6900 \times 10^{-27}$ kg
- Proton $m_p = 1.6863 \times 10^{-27}$ kg
- Electron $m_e = 9 \times 10^{-31}$ kg
- The mass m_o of a neutrino, while not zero, is exceedingly small, more than a million times less that the electron mass. The precise value is actually not yet known.

a) (1 pts) What is the difference in mass Δm between one free neutron and the sum of masses of the particles it decays into?

b) (3 pts) What total kinetic energy is produced by the decay of one neutron?

c) (3 pts) If the neutron is at rest when it decays, and the kinetic energy is shared equally by the three particles, what are the gamma factors of the proton, electron, and neutrino ($\gamma_p, \gamma_e, \gamma_o$, respectively)?

d) (3 pts) What are the speeds of the proton, electron, and neutrino (v_p, v_e, v_o, respectively)?

a) $\Delta m = m_n - (m_p + m_e + m_o)$

$= 1.69 \times 10^{-27} - (1.6863 \times 10^{-27} + 9 \times 10^{-31} + 0)$

$\Delta m = 2.80 \times 10^{-30}$ kg

b) $KE = \Delta m c^2 = 2.80 \times 10^{-30} \times (3 \times 10^8)^2 = 2.52 \times 10^{-13}$ J

KE = lost rest energy

c) $KE = 2.52 \times 10^{-13}$ J (Each particle has kinetic energy $\frac{KE}{3}$)

$\frac{KE}{3} = (\gamma_p - 1) m_p c^2$

$\gamma_p - 1 = \frac{KE}{3 m_p c^2}$ $\quad \gamma_p = \frac{KE}{3 m_p c^2} + 1 = \frac{2.52 \times 10^{-13}}{3(1.6863 \times 10^{-27})(3 \times 10^8)^2} + 1 = 1.00055$

$\frac{KE}{3} = (\gamma_e - 1) m_e c^2$

$\gamma_e - 1 = \frac{KE}{3 m_e c^2}$ $\quad \gamma_e = \frac{KE}{3 m_e c^2} + 1 = \frac{2.52 \times 10^{-13}}{3(9 \times 10^{-31})(3 \times 10^8)^2} + 1 = 2.04$

$\frac{KE}{3} = (\gamma_o - 1) m_o c^2$

$\gamma_o = \frac{KE}{3 m_o c^2} + 1 = \lim_{m_o \to 0} \frac{2.52 \times 10^{-13}}{3(3 \times 10^8)^2 (m_o)} + 1 = \infty + 1 = \infty$

d) $\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}}$ $\quad \gamma \sqrt{1 - (\frac{v}{c})^2} = 1$ $\quad \gamma^2 (1 - (\frac{v}{c})^2) = 1$ $\quad 1 - (\frac{v}{c})^2 = \frac{1}{\gamma^2}$

$(\frac{v}{c})^2 = 1 - \frac{1}{\gamma^2}$ $\quad \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$ $\quad v = c\sqrt{1 - \frac{1}{\gamma^2}}$

Additional paper for answer to problem VI

d continued) $V = c\sqrt{1 - \frac{1}{\gamma^2}}$

$V_p = (3\times10^8)\sqrt{1 - \frac{1}{(1.00055)^2}} = 9.98\times10^6$ m/s

$V_e = (3\times10^8)\sqrt{1 - \frac{1}{(2.04)^2}} = 2.61\times10^8$ m/s

$V_o = (3\times10^8)\sqrt{1 - \frac{1}{\infty^2}} = 3\times10^8\sqrt{1-0} = 3\times10^8$ m/s

6 Problem 6 **10 / 10**

✓ **+ 1 pts** a) $$\Delta m = 2.80 \cdot 10^{-30} \textrm{ kg}$$

✓ **+ 1 pts** b) use energy conservation

✓ **+ 1 pts** b) sum of kinetic energies = $$\Delta m \cdot c^2 $$

✓ **+ 1 pts** b) numerical value $$2.52 \cdot 10^{-13} \textrm{ J}$$

✓ **+ 1 pts** c) $$K = (\gamma-1)mc^2 $$

✓ **+ 1 pts** c) solve for $$\gamma = K/mc^2 + 1$$

✓ **+ 1 pts** c) correct numerical values $$\gamma_p = 1.00$$, $$\gamma_e = 2.04$$ and $$\gamma_n = \infty$$

✓ **+ 1 pts** d) use that $$\gamma = 1/\sqrt{1-v^2/c^2}$$

✓ **+ 1 pts** d) solve correctly for $$v = c\sqrt{1-1/\gamma^2}$$

✓ **+ 1 pts** d) correct numerical values $$v_p \approx 0$$, $$v_e \approx 2.61 \cdot 10^8 \textrm{ m/s}$$, $$v_n = c$$