(First two letters of surname) (Discussion)

LAST Name: Solution

FIRST Name:

University ID:

PHYSICS 1C: Electrodynamics, Optics, and Special Relativity

Fall 2018, Lecture Series 4

Midterm 2—Thursday 15th November

Time allowed: 50 minutes

Answer **all** *questions.*

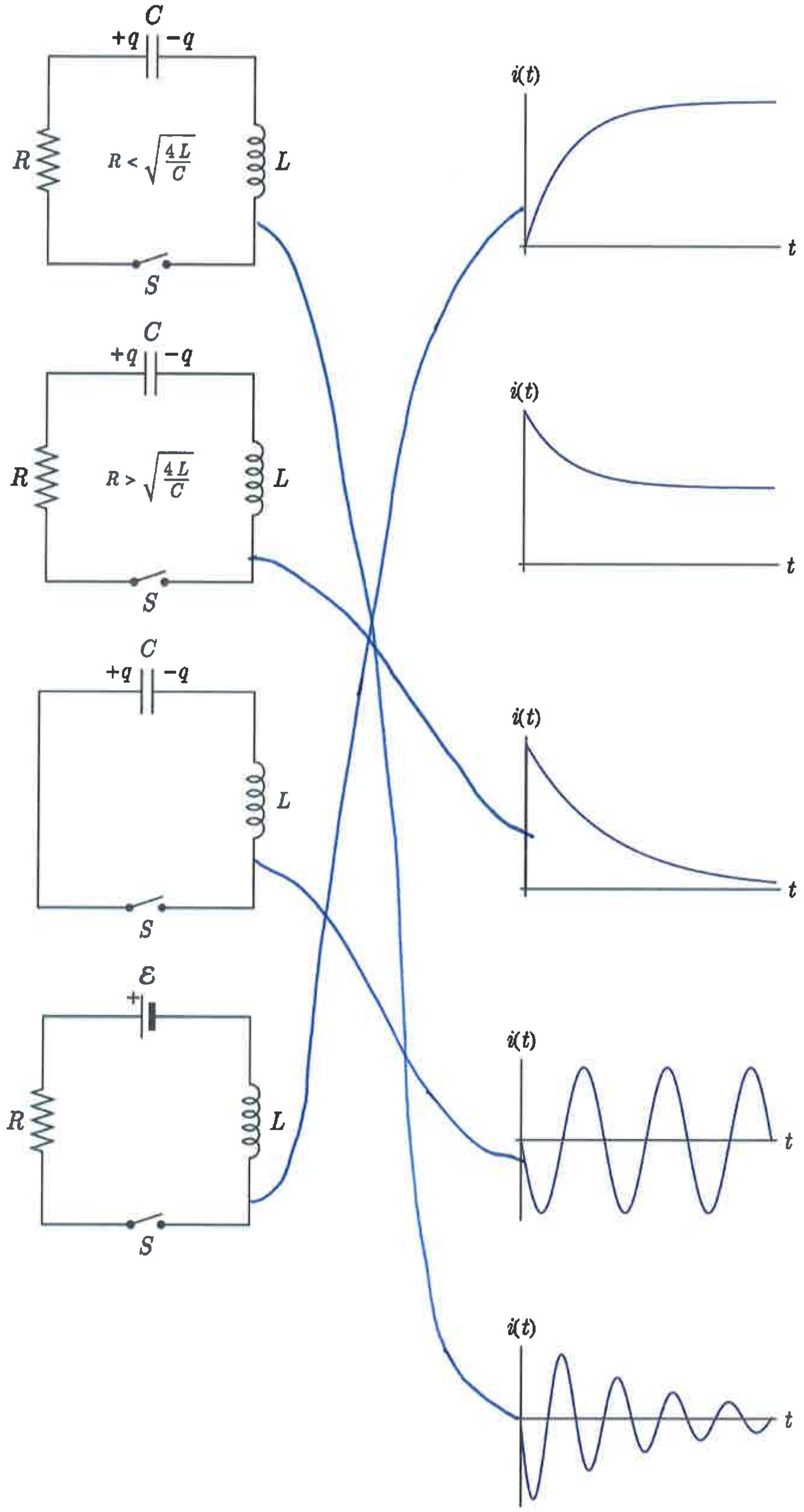
Calculators **are** *permitted in this exam*

The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Question	Points
1	/5
2	/7
3	/23
4	/15

1. The figure below shows four different DC circuits (on the left) and five different plots of current versus time (on the right). Draw a line from each circuit to the plot that shows the corresponding current as a function of time, after the switch at S has been closed. [The capacitor begins fully charged in each case. There will be one plot left over]. [5]



1 ✓✓
2 ✓
3 ✓
4 ✓

2. If magnetic monopoles were ever discovered experimentally, Maxwell's Equations would need to be modified. Write down the new form that the equations would take in this case, briefly explaining any new terms you add. [You do not need to include the correct coefficients; just the qualitative form of the changes is sufficient]. [7]

ME1: $\oiint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$ ✓ (unchanged)

(may be implied)

ME2: $\oiint \vec{B} \cdot d\vec{A} = (\mu_0) q_m$ ✓✓ (coefficient not important)

ME3: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} + (\mu_0) i_m$ ✓

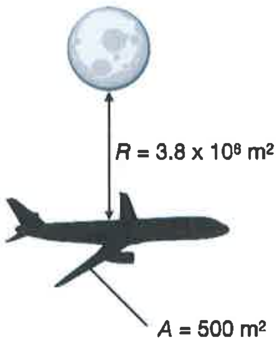
ME4: $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ (unchanged) ✓

New terms: $(\mu_0) q_m$ = magnetic (monopole) charge

$(\mu_0) i_m$ = magnetic (monopole) current

3.

(a) You take a 12:30am flight to Africa, the moonlit wings reflecting the stars. Estimate the radiative force on the wings of the plane due to the light from the moon, assuming the moon is full and is directly overhead, and that the totally reflecting wings have an area of 500 m^2 perpendicular to the rays of light from the moon. The full moon emits radiation with a total power of 1.6×10^{15} W from its illuminated side, and the average distance between the Earth and the Moon is 3.8×10^8 m. [Ignore effects due to the atmosphere]. [7]



Intensity of moon's radiation at Earth's surface : $I = \dfrac{\text{Power}}{2\pi R^2}$ ✓✓ area of hemisphere

Radiation pressure (totally reflecting, average) $p = \dfrac{2I}{c}$ ✓

Radiative force : $F = pA$ ✓

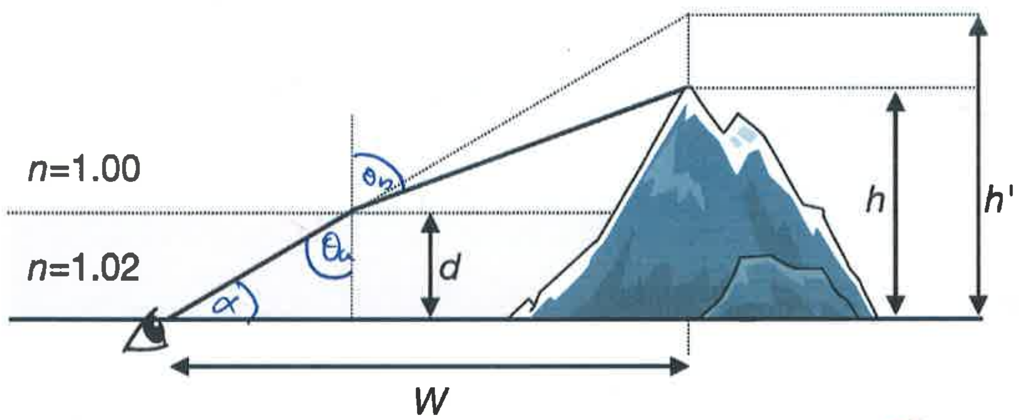
$$\Rightarrow \quad F = \left(\frac{2}{c}\right)\left(\frac{\text{Power}}{2\pi R^2}\right) A$$

$$= \frac{2 \times (1.6 \times 10^{15})(500)}{(3 \times 10^8)(2\pi)(3.8 \times 10^8)^2} = 5.8782\ldots \times 10^{-9}\,\text{N}$$

$$= \underline{5.88 \times 10^{-9}\,\text{N}} \ (3\,\text{s.f.})$$ ✓✓

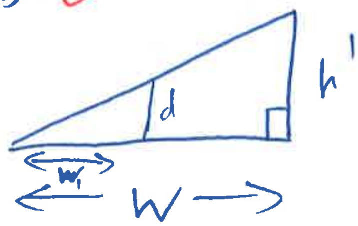
(b) The next day, you observe Mount Kilimanjaro rising above the Serengeti, at a horizontal distance of $W = 10$ km away. However, a dense layer of air with refractive index $n = 1.02$ and depth $d = 3$ km is distorting the view, making the mountain appear taller than it really is (see the path of a ray in the figure below). You observe its height to be $h' = 6130$ m. What is its true height h? [Assume that you are viewing the mountain from ground level. You may continue on the next page.]

[10]



Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$

Find θ_a: $\alpha = \tan^{-1}\left(\frac{h'}{W}\right)$



$= \tan^{-1}\left(\frac{6130}{10\,000}\right)$

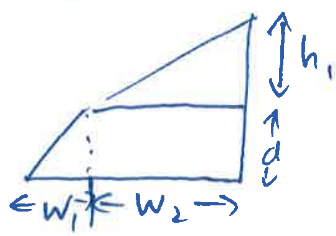
$= 31.5^\circ$

$\theta_a = 90 - \alpha = 58.5^\circ$

Find θ_b: $\theta_b = \sin^{-1}\left(\frac{n_a \sin\theta_a}{n_b}\right)$

$= \sin^{-1}\left(\frac{1.02 \sin(58.5)}{1}\right)$

$= 60.41\ldots^\circ$

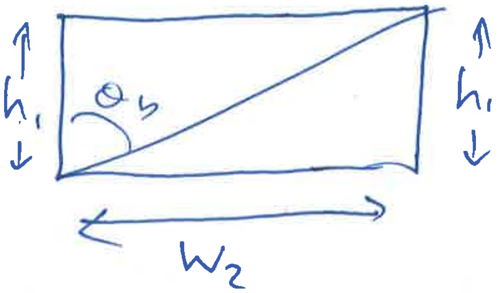


Split h into 2 pieces: $h = d + h_1$

6 Split W into 2 pieces: $W = W_1 + W_2$

$$W_1 = d \tan \theta_a = (3000) \tan (58.5)$$
$$= 4893.96\ldots$$

$$W_2 = W - W_1 = 10000 - 4893.96\ldots$$
$$= 5106.03\ldots$$

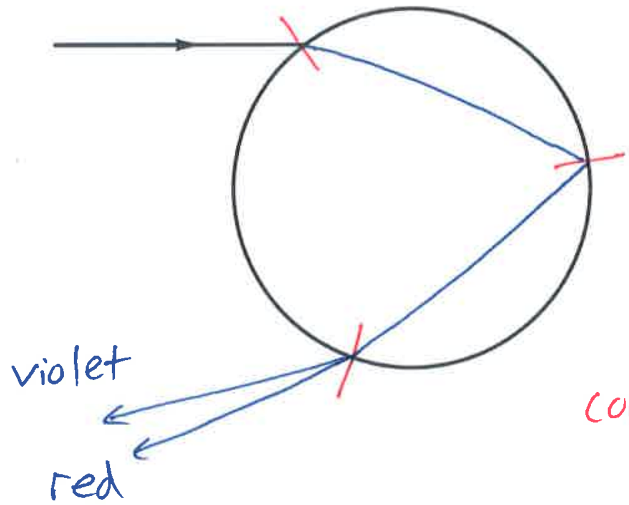


$$h_1 = \frac{W_2}{\tan\theta_b} = \frac{5106.03\ldots}{\tan(60.41\ldots^\circ)}$$

$$= 2898.979\ldots$$

$$h = d + h_1 = 3000 + 2898.97\ldots$$
$$= 5898.97\ldots$$
$$= 5899\text{ m} \quad (4\text{ s.f.})$$
$$= 5900\text{ m} \quad (3\text{ s.f.})$$

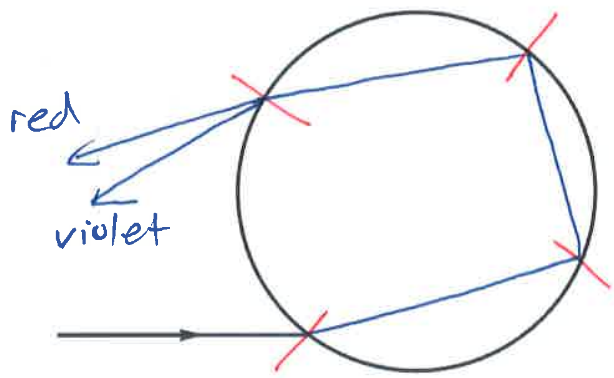
(c) On your final day in Africa, it starts to rain heavily and you observe a double rainbow. The figures below show a ray of white light from the sun striking a spherical raindrop during the formation of a primary rainbow (top) and secondary rainbow (bottom). Draw arrows to show the path of the ray through the droplet in each case. Add rays corresponding to red and violet light leaving the droplet in each figure, indicating their order. [6]



(refr + refl + refr) form: ✓

bending towards/ away from normal: ✓

colour order: ✓

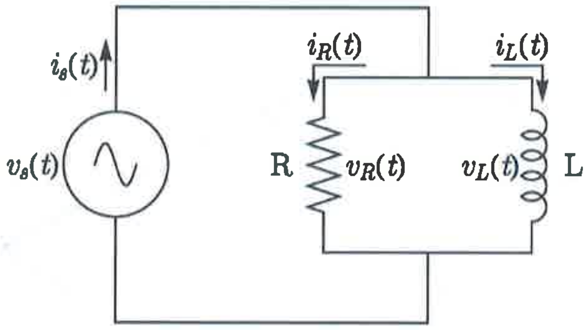


(refr + refl + refl + refr): ✓

bending towards/ away from normal: ✓

colour order ✓

4. The figure below shows an AC circuit with a resistor (with resistance R) and inductor (with inductance L) connected to an AC source *in parallel.* The voltage from the source is $v_s(t)$ and the current from the source is $i_s(t)$. The voltage dropped across the resistor is $v_R(t)$, the voltage dropped across the inductor is $v_L(t)$, while the currents through each component are $i_R(t)$ and $i_L(t)$, as shown in the figure.



(a) Use Kirchhoff's circuit law for currents to obtain an expression relating the current from the source $i_s(t)$ to currents through the resistor and the inductor. Use Kirchhoff's circuit law for voltages to obtain expressions relating the voltage from the source $v_s(t)$ to voltages across the resistor and across the inductor.

[*Hint*: Most of the AC circuits we considered in class were series circuits, whereas this is a parallel circuit. For this reason, the expressions you obtain may look different from what you are used to.] [5]

K1: $\sum_{\text{junction}} i = 0 \Rightarrow i_s(t) = i_R(t) + i_L(t)$

K2: $\sum_{\text{loop}} V = 0 \Rightarrow V_s(t) = V_R(t) = V_L(t)$

loop 1

loop 2

(b) Assume that the voltage from the source takes the sinusoidal form

$$v_s(t) \quad = \quad V_0 \cos(\omega t).$$

Given this input voltage, obtain expressions for the current through the resistor $i_R(t)$ and the current through the inductor $i_L(t)$.

Hence, by drawing a phasor diagram *for currents* and using your answer to part (a) (or otherwise), obtain an expression for ~~$i_s(t)$, the current from the source~~, I_S (amplitude)

What is the impedance of the circuit Z? Does ~~current lead~~ voltage lead or lag? [10]

$$V_R(t) = V_0 \cos(\omega t)$$

$$V_R = i_R R \Rightarrow i_R = \frac{V_R}{R} \Rightarrow \underline{i_R(t) = \frac{V_0}{R}\cos(\omega t)}$$

✓✓

$$V_L(t) = V_0 \cos(\omega t)$$

$$V_L = L\frac{di_L}{dt} \Rightarrow i_L = \frac{1}{L}\int V_L \, dt$$

✓

$$\Rightarrow i_L(t) = \frac{1}{L}\int V_0 \cos(\omega t)\, dt$$

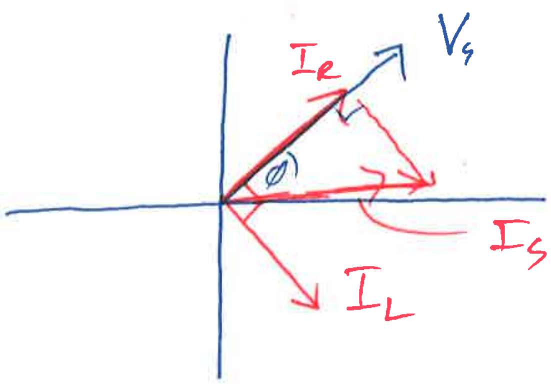
$$= \frac{V_0}{\omega L}\sin(\omega t)$$

either fine

$$= \underline{\frac{V_0}{\omega L}\cos(\omega t - 90^\circ)}$$

✓✓

Current phasors:



✓✓

$$I_s = \sqrt{I_R^2 + I_L^2}$$

$$= \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{\omega L}\right)^2}$$

$$= V_0 \sqrt{\frac{1}{R^2} + \frac{1}{(\omega L)^2}}$$

$$\phi_s = \tan^{-1}\left(\frac{-I_L}{I_R}\right) = \tan^{-1}\left(\frac{-1/\omega L}{1/R}\right)$$

$$= -\tan^{-1}\left(\frac{R}{\omega L}\right)$$

$$i_s(t) = V_0 \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2} \cos(\omega t + \phi_s)$$

not needed for points

$$Z = \frac{V_s}{I_s} = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \frac{1}{(\omega L)^2}}}$$

voltage <u>leads</u> current