Section A

1. The IKAROS spacecraft has a solar sail with dimensions 14 m×14 m. Assuming the solar sail is totally reflecting, estimate the maximum force it experiences due to radiation pressure from the Sun at the radius of the Earth's orbit.
[The luminosity of the Sun, equal to the total power it emits as radiation, is approximately 3.8×10^{26} W. The radius of the Earth's orbit is approximately 1.5×10^{11} m. The average radiation pressure on a totally reflecting surface is given by $p_{\text{rad}} = 2I/c$, where the symbols have their usual meaning]. [12]

max

$$\text{Pressure} = u = \frac{2S(x,t)}{c} = \frac{2\,E \times B}{c\mu_0} \qquad \frac{2E_0 B_0}{c\mu_0}$$

$$\text{Average intensity} = \frac{E_0 B_0}{2\mu_0} = \frac{3.8 \times 10^{26}}{4\pi (1.5 \times 10^{11})^2}$$

$$\text{max pressure} = \frac{E_0 B_0}{2\mu_0} \times \frac{4}{c} = I \cdot \frac{4}{c}$$

← sorry

$$\approx 1.80 \times 10^{-[illegible]}\ \text{N..}$$

Pressure is right, but you want the force F = PA

+16.

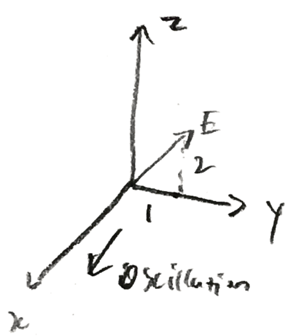
2.

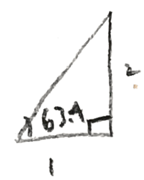
(a) Describe the polarisation state of the plane electromagnetic wave represented by the following electric field equations (where $A > 0$):

$$E_x = 0, \qquad E_y = A\cos(kx - \omega t), \qquad E_z = 2A\cos(kx - \omega t).$$

[Your description should include both the type of polarisation and any relevant details such as direction, etc.] [4]

(b) Determine the direction of oscillation of the corresponding magnetic field. [8]



a). 

$\tan^{-1}(2) \approx 63.4$

4

polarized in $\langle 1,2 \rangle$ direction because $\vec{E}$ is along this vector

63.4° above y-axis in the zy plane.

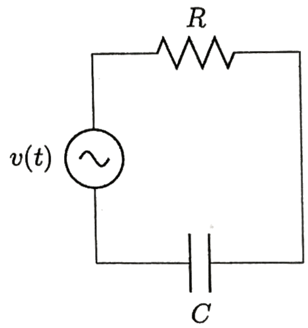
+1

b). Oscillating in $+x$ direction

because $A\cos(kx - \omega t)$ so increasing time t, leads to shifting graph in $+x$ direction.

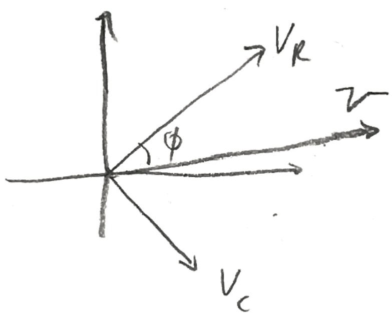
Section B

3. A load consisting of a resistor R in series with a capacitor C is connected to a sinusoidal voltage supply with amplitude V_0 and angular frequency ω.



(a) With the aid of a phasor diagram, or otherwise, obtain the impedance and phase angle of the circuit in terms of ω, R and C. [14]

$L = 0.$



$$\text{Impedance} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\phi = \tan^{-1}\left(\frac{-\frac{1}{\omega C}}{R}\right) = \tan^{-1}\left(-\frac{1}{\omega CR}\right)$$

(b) Calculate the average power dissipated in the resistor in terms of V_0, ω, R and C. What is its value in the limits when i) $\omega CR \gg 1$ and ii) $\omega CR \ll 1$? [11]

$$P_{ave} = \frac{1}{2} V I \cos\phi$$

$$= \frac{1}{2} V_0 \frac{V_0}{Z} \frac{V_R}{V_0}$$

$$= \frac{1}{2} \frac{V_0^2}{Z} \frac{R}{Z} = \frac{1}{2} \frac{V_0^2}{Z^2} R$$

$$P_{ave} = \frac{R V_0^2}{2\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)}$$

i). $P_{ave} = \dfrac{\frac{V_0^2}{R}}{2\left(1+\left(\frac{1}{\omega CR}\right)^2\right)}$ $\quad \omega CR >> 1 \quad \frac{1}{\omega CR} \to 0$

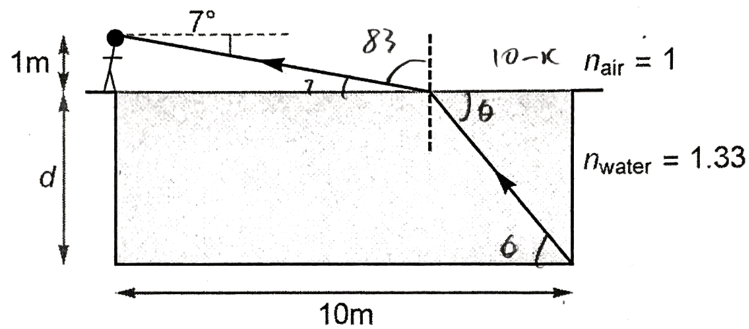
$$P_{ave} \to \boxed{\frac{V_0^2}{2R}}$$

-3

ii) $\omega CR << 1 \quad \frac{1}{\omega CR} \to \infty \quad P_{ave} \to \boxed{0}$

$$\frac{\omega C V_0^2}{2R\omega C\left(1+\left(\frac{1}{\omega CR}\right)^2\right)} \quad \frac{\omega C V_0^2}{2\left(R\omega C + \frac{1}{R\omega C}\right)}$$

4. Ryan is calculating the depth of a rectangular pool full of water. He has measured its length as 10m, and notices that if he looks 7° below the horizontal he can see the bottom corner of the far side of the pool, as shown in the diagram below (ignoring reflected rays).



(a) Assuming Ryan's eyes are 1m above the ground, and that the refractive indices of air and water are $n_{\text{air}} = 1$ and $n_{\text{water}} = 1.33$, calculate the depth d of the pool. [15]

$\sin 83 = 1.33 \sin\theta$

$\theta \approx 48.3$

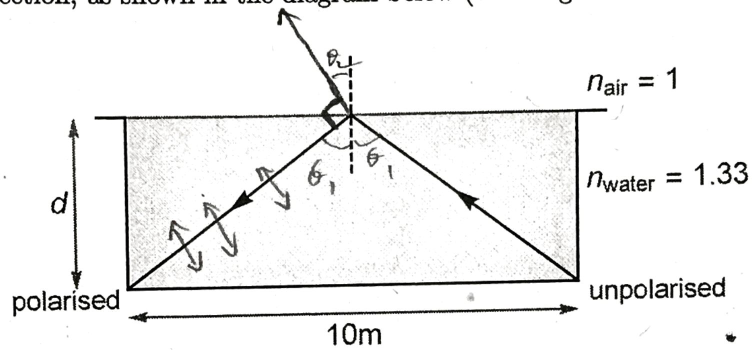
$\tan(7) = \frac{1}{x}$ $\quad x = \frac{1}{\tan(7)} \approx 8.14$ 2

$\tan\theta = \frac{d}{10-x}$ 5

11

$d = (10-x)\tan\theta \approx 2.08$ m

(b) In a different pool, also 10 m in length, Ryan finds that a beam of unpolarised light shone from the bottom corner to the centre of the surface is fully polarised after reflection, as shown in the diagram below (which ignores refracted rays).



In which direction is the reflected light polarised? What is the depth d of this second pool? [10]

$$n_{water} \sin\theta_1 = \sin\theta_2 = \sin(90-\theta_1)$$

$$n_{water} \sin\theta_1 = \cos\theta_1$$

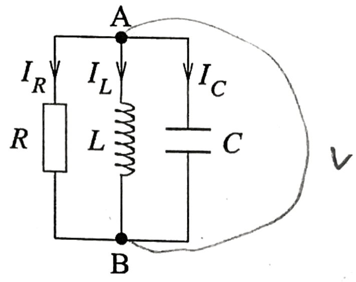
$$\tan\theta_1 = \frac{1}{n_{water}}$$

Polarised perpendicular to the polarised ray in the figure. The $\updownarrow$ shows polarisation direction. X −3

$$\tan\theta_1 = \frac{5}{d} = \frac{1}{n_{water}}$$

$$d = 5n_{water} \simeq 6.65\text{m}$$ ✓

5. In the parallel resonant circuit below, the voltage across points A and B is V.



The current through the capacitor can be written in terms of V and C using the equation $V = q/C$, so that

$$I_C = \frac{\mathrm{d}q}{\mathrm{d}t} = C\frac{\mathrm{d}V}{\mathrm{d}t}.$$

(a) Obtain similar expressions for the currents I_R and I_L in terms of V, R and L. By using Kirchhoff's circuit law for currents, show that the voltage V satisfies the equation

$$C\frac{\mathrm{d}^2V}{\mathrm{d}t^2} + \alpha\frac{\mathrm{d}V}{\mathrm{d}t} + \beta V = 0, \qquad (\star)$$

where α and β are constants (which you should find). [14]

$I_R = \frac{V}{R}$ $\qquad$ $I_C = C\frac{dV}{dt}$

$V = L\frac{dI_L}{dt}$

$\int V = L\int \frac{dI_L}{dt}\,dt$

$C_1 + \frac{1}{2}V^2 = L I_L$

$I_L = \frac{1}{2L}V^2 + C_1$

where C_1 is some constant.

$I_R + I_L + I_C = 0$ at junction

$\frac{V}{R} + C\frac{dV}{dt} + \frac{1}{2L}V^2 + C_1 = 0$

$\frac{1}{R}\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{1}{L}V = 0$

$\boxed{\alpha = \frac{1}{R} \qquad \beta = \frac{1}{L}}$

(b) Show by substitution, or otherwise, that a solution to this differential equation is given by

$$V(t) = Ae^{-\gamma t}\cos(\omega t),$$

and find expressions for γ and ω in terms of R, L and C.

[You may assume that $4R^2 > L/C$. If you did not complete part (a), you may leave your answer in terms of α and β for full credit. Hint: for the left hand side of Equation ($\star$) to be zero, terms proportional to $\sin(\omega t)$ and terms proportional to $\cos(\omega t)$ should sum to zero independently.]. [12]

$$C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{1}{L}V = 0$$

let $V(t) = Ae^{-\gamma t}\cos(\omega t)$

$$\frac{dV}{dt} = -\omega Ae^{-\gamma t}\sin(\omega t) - A\gamma e^{-\gamma t}\cos(\omega t)$$

$$\frac{d^2V}{dt^2} = -\omega^2 Ae^{-\gamma t}\cos(\omega t) + \omega\gamma Ae^{-\gamma t}\sin(\omega t) + \omega A\gamma e^{-\gamma t}\sin(\omega t)$$
$$+ A\gamma^2 e^{-\gamma t}\cos(\omega t)$$

$$\cos(\omega t)\left(-C\omega^2 Ae^{-\gamma t} + CA\gamma^2 e^{-\gamma t} - \frac{A}{R}\gamma e^{-\gamma t} + \frac{A}{L}e^{-\gamma t}\right)$$

$$+ \sin(\omega t)\left(C\omega\gamma Ae^{-\gamma t} + C\omega A\gamma e^{-\gamma t} - \frac{\omega}{R}Ae^{-\gamma t}\right) = 0$$

$$2C\cancel{\omega}\gamma\cancel{Ae^{-\gamma t}} = \frac{\cancel{\omega}}{R}\cancel{Ae^{-\gamma t}}$$

$$R = \frac{1}{2C\gamma} \qquad \boxed{\gamma = \frac{1}{2CR}} \qquad \boxed{\omega = \sqrt{\frac{1}{CL} - \frac{1}{4C^2R^2}}}$$

$$-C\omega^2 A + CA\gamma^2 - \frac{A}{R}\gamma + \frac{A}{L} = 0 \qquad \omega^2 = \gamma^2 - \frac{\gamma}{CR} + \frac{1}{CL}$$

$$-C\omega^2 + C\gamma^2 - \frac{\gamma}{R} + \frac{1}{L} = 0 \qquad \omega^2 = \frac{1}{4C^2R^2} - \frac{1}{2C^2R^2} + \frac{1}{CL} = \frac{1}{CL} - \frac{1}{4C^2R^2}$$