

(First two letters of surname)(Discussion) LAST Name: Solution

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University ID: _____

PHYSICS 1C: Electrodynamics, Optics, and Special Relativity**Fall 2018, Lecture Series 4****Midterm 1—Thursday 25th October****Time allowed: 50 minutes**

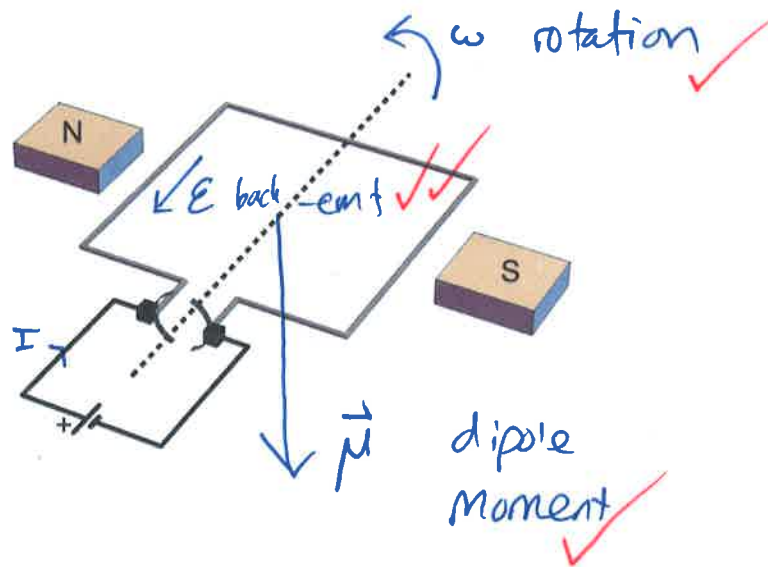
*Answer all questions.**Calculators are permitted in this exam**The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.***Do NOT turn over until told that you may do so.**

Question	Points
1	/4
2	/4
3	/5
4	/14
5	/15
6	/8

1. A simple DC motor consists of a rotating wire loop (rotor), which is connected to a DC voltage supply via a commutator and brush contacts and placed in a static magnetic field, as shown in the figure below. Indicate on the figure:

- The direction of the magnetic dipole moment of the wire loop
- The direction of rotation of the loop about the axis
- The direction of **either** the back-torque or the back-emf experienced by the motor. [It only experiences one of these].

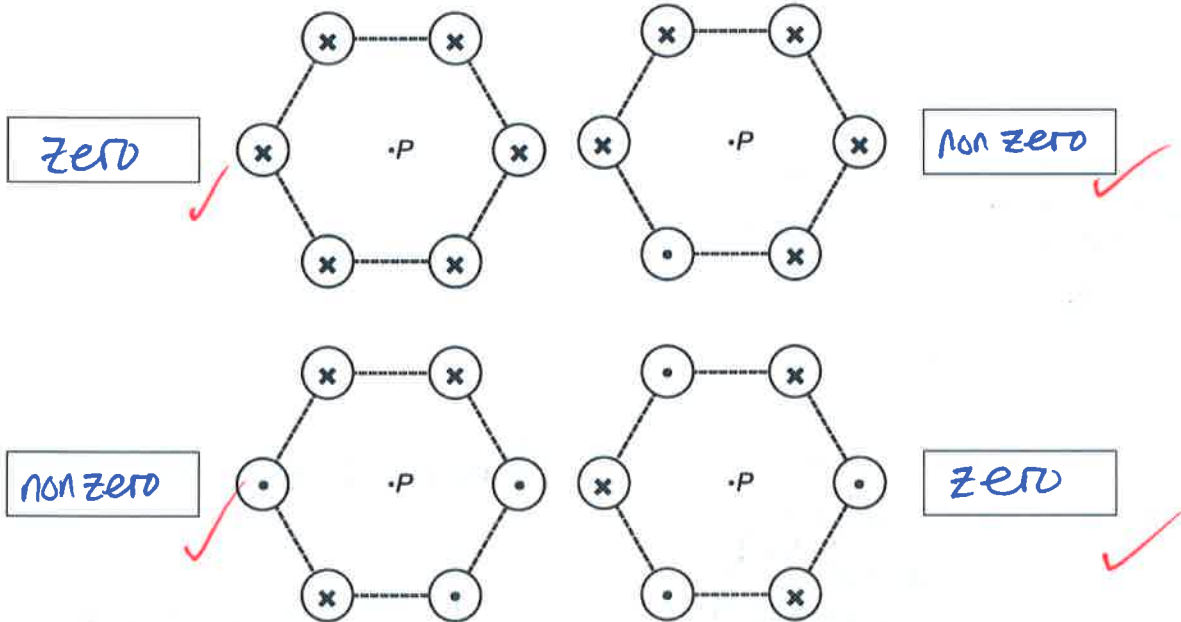
[4]



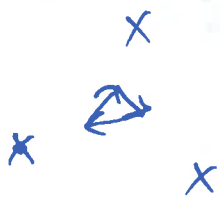
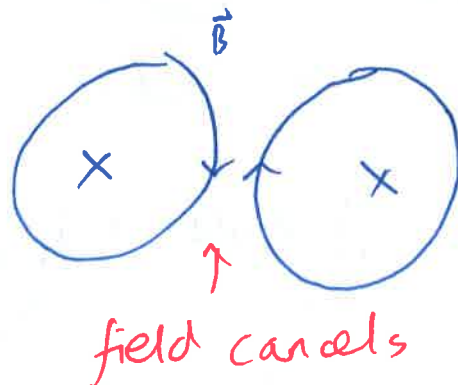
$$\vec{\mu} = I\vec{A}$$



2. Six long, parallel power lines each carry identical current I . A cross-sectional diagram of these lines is a regular hexagon. For each of the four cases below, identify whether the magnetic field at the centre of the hexagon (at point P) is zero or nonzero (write 'zero' or 'nonzero' in the corresponding box). You do not need to calculate any fields.



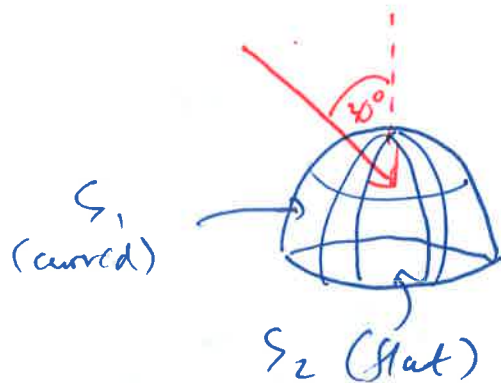
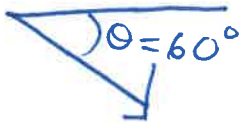
$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$



← field cancels for 3 points arranged in an equilateral triangle

3. The dome of the Griffith Observatory may be modelled as a hemisphere with diameter 24 m. In Los Angeles, the Earth's magnetic field has approximate magnitude $B = 50 \times 10^{-6}$ T and an inclination (angle below the horizontal) of about 60° . Use this information to estimate the total magnetic flux which passes through the curved surface of the dome.

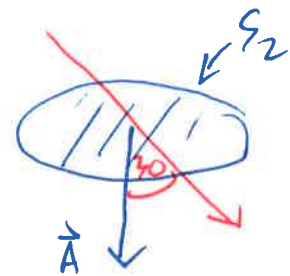
[5]



Gauss' Law for magnetism:

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \checkmark$$

$$\Rightarrow 0 = \iint_{S_1} \vec{B} \cdot d\vec{A} + \iint_{S_2} \vec{B} \cdot d\vec{A}$$



$$\Rightarrow \Phi_1 = - \iint_{S_2} \vec{B} \cdot d\vec{A} \quad \checkmark$$

$$= - \iint B dA \cos \theta$$

$$= - B (\pi R^2) \cos(30) \quad \checkmark$$

$$= - (50 \times 10^{-6}) (\pi) (12^2) \frac{\sqrt{3}}{2}$$

$$= - (3.6 \times 10^{-3}) \times \pi \times \sqrt{3}$$

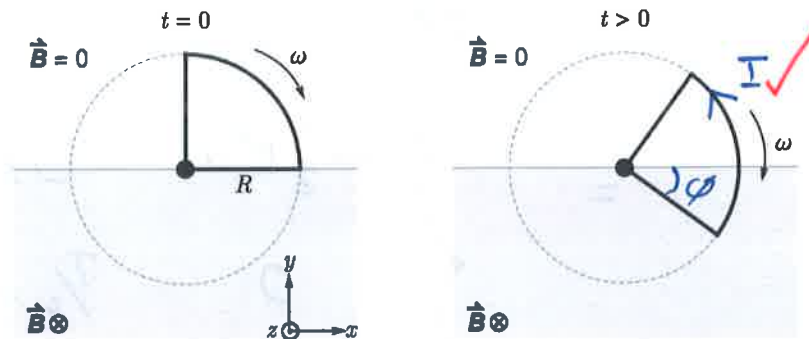
$$= (-) \underline{1.96 \times 10^{-2} \text{ Wb (3 s.f.)}} \quad \checkmark \checkmark$$

[sign not important here]

[2×10^{-2} is fine]

[Wb or Tm^2 fine]

4. A wire loop in the shape of a quadrant of a circle with radius R is rotating with angular velocity ω as shown in the figure below. When the loop is in the lower half of the figure, it experiences a magnetic field $\vec{B} = -B\hat{k}$.



- (a) Find an expression for the magnetic flux through the loop as a function of time $\Phi_B(t)$, for one complete period of revolution (i.e. for $0 \leq t \leq T = 2\pi/\omega$). Hence find an expression for the emf induced in the loop as a function of time $\mathcal{E}(t)$. ~~Indicate the direction of the induced current by adding an arrow to the figure above.~~ *Indicate current direction for figure on right.*
- [Hint: Your answers will be defined piecewise. You may ignore any discontinuities between different pieces when differentiating. You can use the next page for extra space.]

[10]

① Loop = quadrant $\vec{A}_{\text{total}} = -\frac{\pi R^2}{4} \hat{k}$ (choose \parallel to \vec{B})

② Flux. Area of loop in field varies with time between 0 and $\frac{\pi R^2}{4}$.

For angle $\phi = \omega t$, $A = \frac{1}{2} R^2 \phi = \frac{R^2 \omega t}{2}$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}(t) = \begin{cases} \frac{1}{2} R^2 \omega t & 0 \leq t \leq \frac{\pi}{2\omega} = \frac{T}{4} \\ \frac{\pi R^2}{4} & \frac{T}{4} \leq t \leq \frac{T}{2} \\ \frac{\pi R^2}{4} - \frac{1}{2} R^2 \omega (t - \frac{T}{2}) & \frac{T}{2} \leq t \leq \frac{3T}{4} \\ 0 & \frac{3T}{4} \leq t \leq T \end{cases}$$

(3) Faraday: $\mathcal{E} = - \frac{d\Phi_B}{dt}$ ✓

$$= \begin{cases} -\frac{1}{2}R^2\omega & 0 \leq t \leq T/4 \\ 0 & T/4 \leq t \leq T/2 \\ +\frac{1}{2}R^2\omega & T/2 \leq t \leq 3T/4 \\ 0 & 3T/4 \leq t \leq T \end{cases}$$

✓✓✓

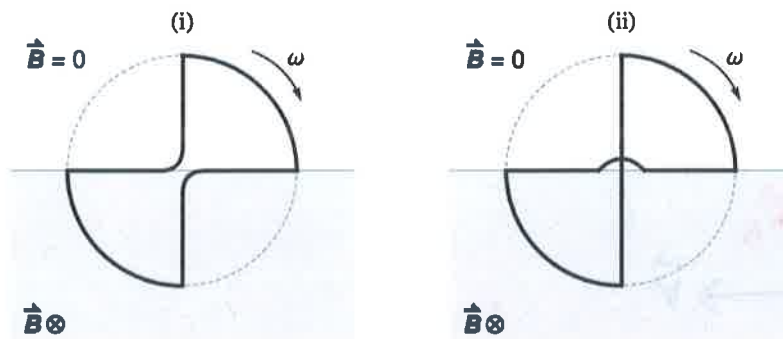
(4) Lenz. Current initially anticlockwise

→ induces opposite field to
reduce flux through loop

→ induces force to oppose ω

↑ not needed for points

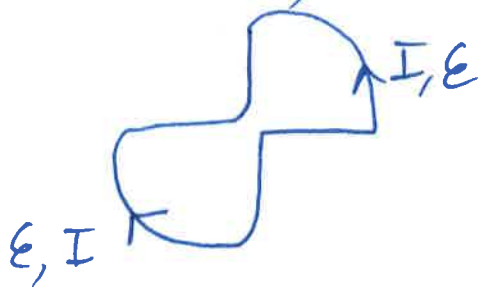
- (c) Two such quadrants are now connected opposite each other using two different connection arrangements, as shown below.



Write down an expression for the (net) induced emf around the complete loop in each case.

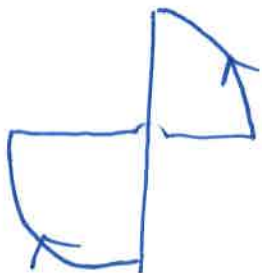
[4]

- (i) Induced emfs are equal and opposite here: flux is increasing in the NE quadrant but decreasing in the SW quadrant:



$$\underline{\underline{\mathcal{E}(t) = 0}} \quad \checkmark\checkmark$$

- (ii) Induced emfs are in the same direction here
 \rightarrow emf is twice the value for a single quadrant



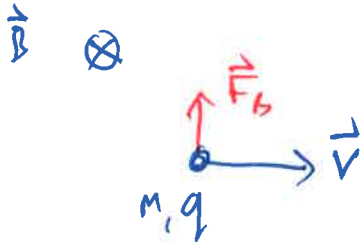
$$\underline{\underline{\mathcal{E}(t) = \begin{cases} -R^2\omega & 0 \leq t \leq T/4 \\ 0 & T/4 \leq t \leq T/2 \\ +R^2\omega & T/2 \leq t \leq 3T/4 \\ 0 & 3T/4 \leq t \leq T \end{cases}}}$$

$\checkmark\checkmark$

5.

- (a) A charged particle with mass m and charge q is moving with speed v in a direction perpendicular to a magnetic field with strength B . By considering the magnetic force acting on the particle, *derive* an expression for the angular cyclotron frequency ω_c experienced by the particle.

[5]



Magnetic force: $\vec{F}_B = q\vec{v} \times \vec{B}$ ✓

This provides centripetal force required for circular motion:

$$F_c = \frac{mv^2}{r} \quad \checkmark \quad (\text{or similar})$$

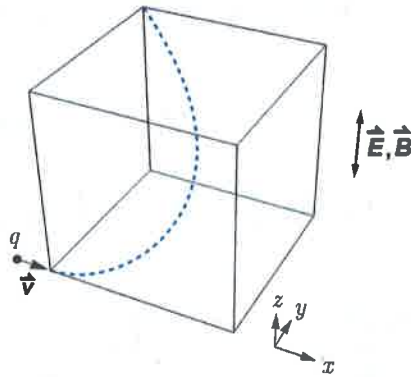
$$\Rightarrow qvB = \frac{mv^2}{r}$$

$$\Rightarrow \frac{v^2}{r} = \frac{qBv}{m}$$

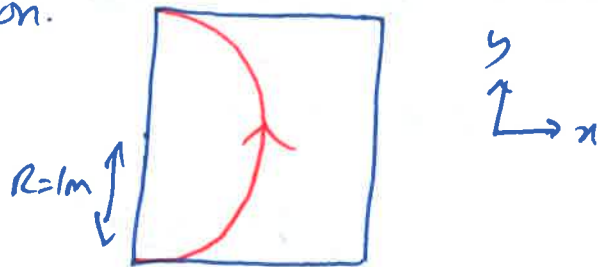
$$v = r\omega \quad \checkmark \quad \Rightarrow \quad \underline{\underline{\omega_c = \frac{qB}{m}}} \quad \checkmark$$

- (b) A particle with mass $m = 10^{-10}$ kg and charge $q = 2.00 \times 10^{-9}$ C travelling with velocity $\vec{v} = (2 \text{ m s}^{-1}) \hat{i}$ enters a cubic box with dimensions $2\text{m} \times 2\text{m} \times 2\text{m}$, oriented as in the figure below. Inside the box, an electric and magnetic field can each be applied along the positive or negative z -direction. The particle follows the curved path shown by the dashed line in the figure below, entering the box at the corner with coordinates $(0, 0, 0)$ and leaving the box at the corner with coordinates $(0, 2\text{m}, 2\text{m})$. Find the magnitude and direction of the fields applied along the $\pm z$ -direction that lead to this motion.

[Hint: The motion in the xy -plane and the motion in the z -direction can be treated independently. You can use the next page for extra space.]



In xy -plane, particle is acted on by a magnetic force \Rightarrow circular motion.



Force must act to the left initially
 $\Rightarrow \vec{B} \propto -\hat{k}$

Radius of cyclotron orbit (previous page or on Eq. sheet):

$$R = \frac{mv}{qB} = 1\text{m}$$

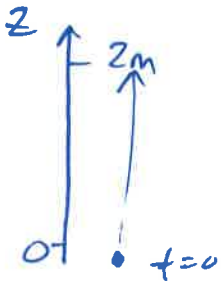
$$\Rightarrow B = \frac{mv}{qR} = \frac{(10^{-10})(2)}{(2 \times 10^{-9})(1)} = 0.1\text{T}$$

$$\Rightarrow \vec{B} = -(0.1\text{T})\hat{k}$$

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In z-direction, particle is accelerated vertically by electric force $\vec{F} = q\vec{E}$.

This leads to acceleration $\vec{a} = \frac{q\vec{E}}{m}$.



The particle must travel $2m$ in the time for half a cyclotron orbit.

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB} \Rightarrow \frac{T}{2} = \frac{\pi m}{qB}$$

Constant acceleration ~~4m~~ $\Delta x = 2m$, $t = \frac{\pi m}{qB}$, $a = \frac{qE}{m}$

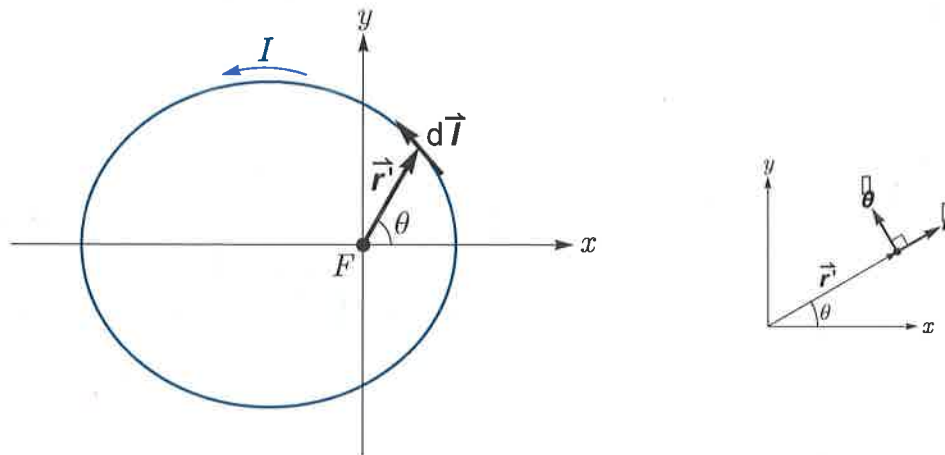
$$\begin{aligned} \Delta x &= \frac{1}{2} a t^2 \\ &= \frac{1}{2} \frac{qE}{m} \frac{\pi^2 m^2}{q^2 B^2} \\ &= \frac{E \pi^2 m}{2 q B^2} \end{aligned}$$

$$\Rightarrow E = \frac{2 q B^2 \Delta x}{\pi^2 m} = \frac{2 (2 \times 10^{-9}) (0.1)^2 (2)}{\pi^2 (10^{-10})}$$

$$= 0.0810 \dots \text{ V/m}$$

$$= \underline{8.11 \times 10^{-2} \text{ V/m}} \hat{k} \text{ (35 ft.)}$$

6. The figure below shows an elliptical wire carrying an anticlockwise current I .



The point labelled F is known as a focus of the ellipse. The vector \vec{r}' from this focus to a point on the ellipse making an angle θ to the x -axis is given by

$$\vec{r}' = \frac{p}{1 + e \cos(\theta)} \hat{r},$$

where p and e are fixed parameters that describe the ellipse and \hat{r} is a radial unit vector from the focus. The infinitesimal line element of current $d\vec{l}$ at the position \vec{r}' is given by

$$d\vec{l} = \frac{pe \sin \theta d\theta}{(1 + e \cos \theta)^2} \hat{r} + \frac{pd\theta}{1 + e \cos \theta} \hat{\theta},$$

where $\hat{\theta}$ is a unit vector in the direction of increasing θ (and is perpendicular to \hat{r}). The unit vectors \hat{r} and $\hat{\theta}$ are also indicated on the right in the figure above.

Find an expression for the magnitude and direction of the magnetic field produced by the elliptical wire at the point F .

[Hint: The only integration you should need to do is of elementary functions.]

[6]

Note: \vec{r}' in this question is $-\vec{r}'$ from Biot-Savart

$$\text{Biot-Savart: } \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \vec{r}'}{|\vec{r}'|^2} \quad \checkmark$$

$$\hat{r} \text{ and } \hat{\theta} \text{ are perpendicular: } \begin{aligned} \hat{r} \times \hat{r} &= 0 \\ \hat{r} \times \hat{\theta} &= \hat{k} \quad \checkmark \text{ (out of page)} \end{aligned}$$

$$d\vec{l} \times \vec{r}' = \frac{d\vec{l} \times \vec{r}'}{|\vec{r}'|} = \frac{1}{|\vec{r}'|} \left[\frac{pe \sin \theta d\theta}{(1 + e \cos \theta)^2} \hat{r} + \frac{pd\theta}{1 + e \cos \theta} \hat{\theta} \right] \times \left[-\frac{p}{1 + e \cos \theta} \hat{r} \right]$$

$$= \frac{1}{|\vec{r}'|} \left(\frac{\rho d\theta}{1 + e \cos \theta} \hat{\theta} \right) \times \left(- \frac{\rho}{1 + e \cos \theta} \hat{r} \right) \quad \left[\hat{r} \times \hat{r} \text{ term has vanished} \right]$$

$$= \left(\frac{1 + e \cos \theta}{\rho} \right) \left(\frac{-\rho^2 d\theta}{(1 + e \cos \theta)^2} \right) \underbrace{(\hat{\theta} \times \hat{r})}_{-\hat{h}}$$

$$= \frac{\rho d\theta}{1 + e \cos \theta} \hat{h} \quad \checkmark \checkmark$$

BS:

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{\rho d\theta}{(1 + e \cos \theta)} \hat{h} \left(\frac{1 + e \cos \theta}{\rho} \right)^2$$

$$= \frac{\mu_0 I}{4\pi} \int_{\theta=0}^{2\pi} \left(\frac{1 + e \cos \theta}{\rho} \right) d\theta \hat{h}$$

$$= \frac{\mu_0 I}{4\pi} \left[\frac{\theta}{\rho} + \frac{e}{\rho} \sin \theta \right]_{\theta=0}^{\theta=2\pi} \hat{h}$$

$$= \frac{\mu_0 I}{4\pi} \left(\frac{2\pi}{\rho} \right) \hat{h}$$

$$= \frac{\mu_0 I}{2\rho} \hat{h} \quad \checkmark$$